9.4 Hypothesis Tests: one μ, σ Known

GOALS:
1. Understand the differences between the critical value and p-value approaches to hypothesis testing.
2. Understand what the p-value is and how to find it.
3. Understand the assumptions of a z-test (same as z-interval).
4. Perform a z-test using either the critical value or the p-value approach.

Study Ch. 9.4, # 73, 77-85
(65, 67-75)

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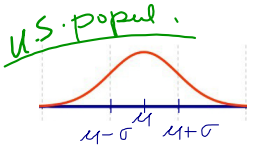
9.4 Hypothesis Tests: one μ, σ Known

Energy Use

G: BTUs consumed/household/year in US:
μ = 92.2 mill BTU, n.d., σ = 15 mill BTU
n = 20 household in West US (mill BTUs)

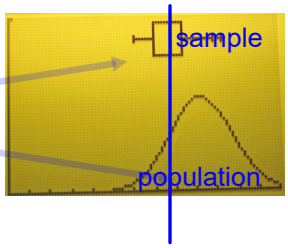
104	84	72	95	69
80	78	74	76	81
82	61	94	65	100
70	65	83	76	84



μ = 92.2 mil BTUs
σ = 15 mil BTUs

F: Do households in the West US use a different amount of energy?

Investigate: Compare all US to West on calculator.
1. **all US:** y_1 = normalpdf(X,92.2,15)
2. window: x_{min}=0, x_{max}=130, y_{min}=0, y_{max} =0.07
3. **West:** STAT/EDIT. L1 enter data above
4. STAT PLOT (2nd y=) 1 ON
   Type: row 2, col 1 Box Plot
   Xlist: L1



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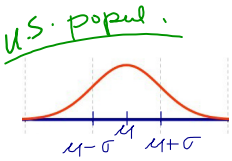
9.4 Hypothesis Tests: one μ, σ Known

Energy Use

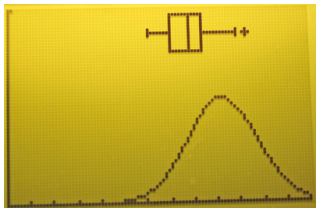
G: BTUs consumed/household/year in US:
μ = 92.2 mill BTU, n.d., σ = 15 mill BTU



n = 20 household in West US (mill BTUs)

104	84	72	95	69
80	78	74	76	81
82	61	94	65	100
70	65	83	76	84

F: Do households in the West US use a different amount of energy at 5% significance level?



Using visual display is helpful.
The sample certainly looks different.
But, a determination that the West US is different is subjective.

Need more objective means to draw a conclusion.

Use sample data.

Would we conclude differently if the z score from the data is: z = -2.5 or z = -2.0 or z = -1.25

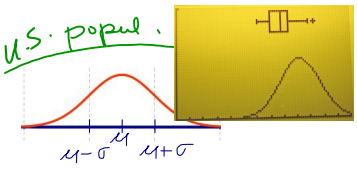
Need criteria and a consistent approach to arrive at unbiased conclusion.

9.4 Hypothesis Tests: one μ, σ Known

Approach	Critical Value (table based, can use calculator)	P-Value (calculator based, can use table)
	SRS, n.d. or large sample, σ known	
1.	State the Null and Alternative Hypotheses: H_0, H_a	
2.	Decide the significance level, α, and sketch	
	$H_a: \mu < \mu_0$ (α) ; $H_a: \mu \neq \mu_0$ ($\alpha/2$, $\alpha/2$) ; $H_a: \mu > \mu_0$ (α)	
3.	Compute the test statistic: z, t, etc.	$z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}}$
4.	Find the critical values $z_\alpha, z_{\alpha/2}, t_\alpha, t_{\alpha/2}$	Find the P-value
5.	Decision: Rej. H_0 if test statistic lies beyond critical value in rejection region	Decision: Rej. H_0 if $P \leq \alpha$
6.	Interpret results	

9.4 Hypothesis Tests: one μ, σ Known

G: BTUs consumed/household/year in US:
μ = 92.2 mill BTU, n.d., σ = 15 mill BTU



n = 20 household in West US (mill BTUs)

104	84	72	95	69
80	78	74	76	81
82	61	94	65	100
70	65	83	76	84

F: Do households in the West US use a different amount of energy at 5% significance level?

Approach	Critical Value (table based, can use calculator)	P-Value (calculator based, can use tab
1.	State the Null and Alternative Hypotheses: H_0,H	
2.	Decide the significance level, α, and sketch	
3.	Compute the test statistic: z, t, etc.	
4.	Find the critical values $z_\alpha, z_{\alpha/2}, t_\alpha, t_{\alpha/2}$	Find the P-valu
5.	Decision: Rej. H_0 if test statistic lies beyond critical value in rejection region	Decision: Rej. H_0 $P \le \alpha$
6.	Interpret results	

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9.4 Hypothesis Tests: one μ, σ Known

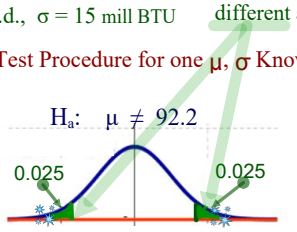
G: BTUs consumed/household/year in US:
μ = 92.2 mill BTU, n.d., σ = 15 mill BTU

F: Do households in the West US use a different amount of energy, at 5% s.l.?

z-Test: Hypothesis Test Procedure for one μ, σ Known

SRS, n.d. or large sample, σ known

1. H_0: $\mu = 92.2$ H_a: $\mu \neq 92.2$



2. $\alpha = 0.05$

3. $z = \dfrac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \dfrac{79.65 - 92.2}{15/\sqrt{20}} = -3.74$

n = 20 household in West US (mill BTUs)

104	84	72	95	69
80	78	74	76	81
82	61	94	65	100
70	65	83	76	84

Calculator instructions

4. $P = 1.828\ (10^{-4}) = 0.00018$

p=.00018 same as 2 times result from normalcdf(-9,-3.74,0,1)

5. $P = 1.828\ (10^{-4}) = 0.00018 < 0.05$
REJECT H_0

6. Conclusion: Yes, Western households use a different amount of energy

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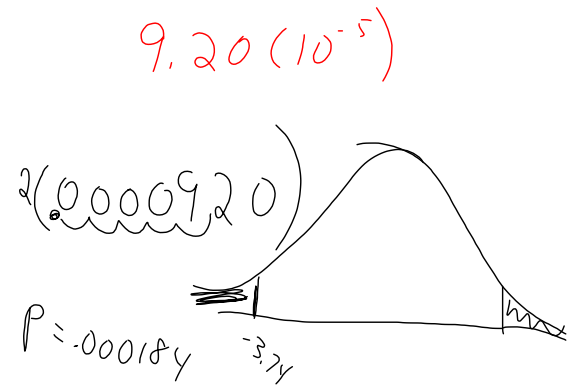
p=.00018 same as 2 times result
from normalcdf(-9,-3.74,0,1)



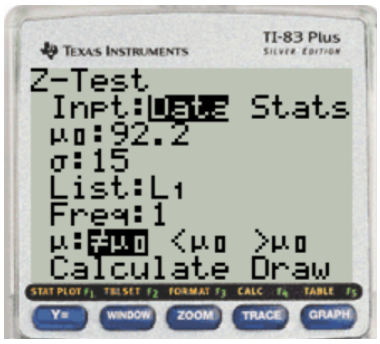
9.4 Hypothesis Tests: one μ, σ Known

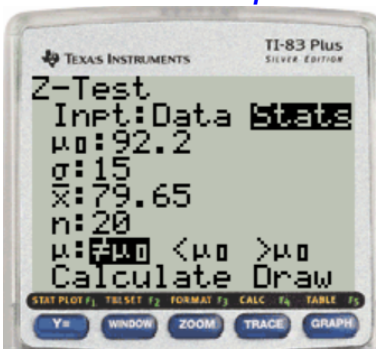
Using the Calculator Functions to Perform z-Test

z-Test for one Mean, σ known (assumption: SRS, normal distribution)

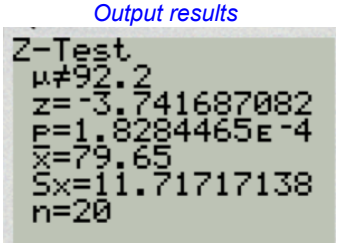
STAT/TESTS/**Z-Test**

Enter data into L1 and use Data as Input OR ***Obtain sample mean and use Stats as Input***

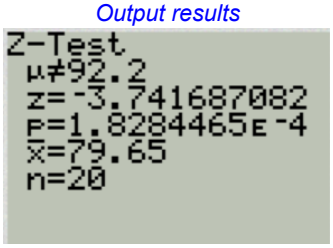




Output results



Output results



9.4 Hypothesis Tests: one μ, σ Known

To Use the **Critical Value Approach**

p. 421: Hypothesis Test Procedure for one μ, σ Known

For the Energy Problem:

$\alpha = 1 - CL$

$\alpha = 1 - 0.95$

1. H_0: $\mu = 92.2$ H_a: $\mu \neq 92.2$
2. $\alpha = 0.05$ CL 95%
3. $z_t = -3.73$
4. $z_{\alpha/2} = ? = z_{0.025} = \pm 1.96$ criteria
5. $z_t = -3.73 < z_{0.025} = -1.96$

invNorm(0.025,0,1= -1.9599 = -1.96

z_t is in critical region

REJECT H_0.

6. Conclude that energy use in the West is different from energy use in general US

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9.4 Hypothesis Tests: one μ, σ Known

Solving Word Problems

1. Read the problem. Try to identify the general type of problem. eg: CI, Hyp Test, specific value, etc.
2. Read the problem again, identifying what is given and what you need to find.
3. Use the Procedure Index to select a procedure.
4. Before beginning a procedure, determine if all assumptions are met.
5. If assumptions are not met, look for a different procedure. (Exception: if srs not met, then write "Assuming srs... ")
6. If assumptions are met, follow the procedure including:
   - Draw a sketch to show α as left-tailed, 2-tailed, or right-tailed.
   - For Hypothesis Tests, include null and alternative hypotheses.
   - Include all equations, substitutions, and answers for the equations. (Indicate calculator or tables.)
   - Decide to reject null hypothesis or not. Explain why.
   - Write a verbal interpretation of your decision.
7. Check: Have you satisfied the to find above?

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9.4 Hypothesis Tests: one μ, σ Known

Prob. 73 Prob. 105

Solving Word Problems

1. Read the problem. Try to identify the general type of problem. eg: CI, Hyp Test, specific value, etc.
2. Read the problem again, identifying what is given and what you need to find.
3. Use the Procedure Index to select a procedure.
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   - Include all equations, substitutions, and answers for the equations. (Indicate calculator or tables.)
   - Decide to reject null hypothesis or not. Explain why.
   - Write a verbal interpretation of your decision.
7. Check: Have you satisfied the to find above?

Calculator instructions

n

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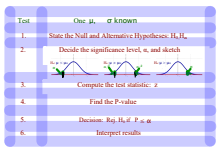
9.4 Hypothesis Tests: one μ, σ Known

G: $\bar{x} = 21$, $n=32$, $\sigma=4$; H_o: $\mu=22$, H_a: $\mu<22$,

F: Does the sample have a mean less than the population mean?



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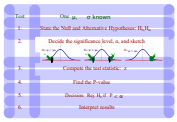
9.4 Hypothesis Tests: one μ, σ Known

①

G: $\bar{x} = 21$, $n=32$, $\sigma=4$; $H_o: \mu=22$, $H_a: \mu<22$,

F: Does the sample have a mean less than the population mean?

② $\alpha = 0.05$



③ $z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} = \frac{21-22}{4/\sqrt{32}} = -1.414$

Z-Test
Inpt:Data Stats
μ0:22
σ:4
x̄:21
n:32
μ:≠μ0 <μ0 >μ0
Calculate Draw

④ $p = 0.0786$

Z-Test
μ<22
z=-1.414213562
p=.0786496525
x̄=21
n=32

⑤ rej? $p = 0.0786 > .05$

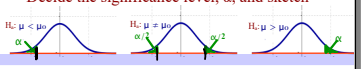
DO NOT rej.

6 Interpret Data is insufficient to rej. H_0. accept that $\mu = 22$.

9.4 Hypothesis Tests: one μ, σ Known

G: $\bar{x} = 23$, $n=24$, $\sigma=4$; n.d.
$H_o: \mu=22$, $H_a: \mu\neq22$, $\alpha=0.05$

F: Is the sample mean different from the pop. mean?

Test	One μ, σ known
1.	State the Null and Alternative Hypotheses: H_0,H_a
2.	Decide the significance level, α, and sketch 
3.	Compute the test statistic: z
4.	Find the P-value
5.	Decision: Rej. H_0 if $P \le \alpha$
6.	Interpret results

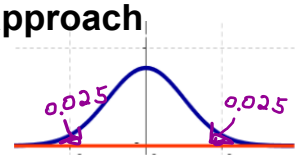
9.4 Hypothesis Tests: one μ, σ Known

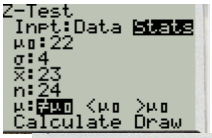
classical approach

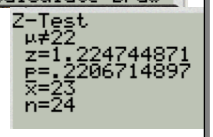
G: $\bar{x}=23, n=24, \sigma=4$

① $H_0: \mu=22, H_a: \mu \neq 22$, ② $\alpha=0.05$

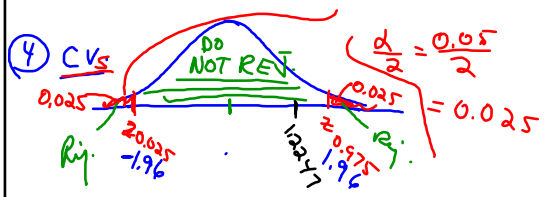
③ $z_t = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} = \frac{23-22}{4/\sqrt{24}} = 1.2247$







④ CVs



⑤ **z_t is not in critical region:**
$z_t = 1.225 < 1.96 = z_c$ Do NOT reject H_O

⑥ At the 95% CL, there is insufficient evidence to conclude that μ ≠ 22

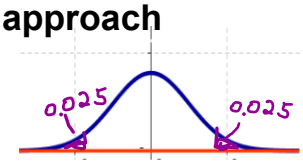
9.4 Hypothesis Tests: one μ, σ Known

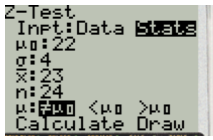
p-value approach

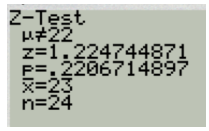
G: $\bar{x}=23, n=24, \sigma=4$

① $H_0: \mu=22, H_a: \mu \neq 22$, ② $\alpha=0.05$

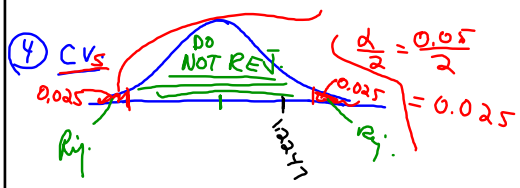
③ $z_t = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} = \frac{23-22}{4/\sqrt{24}} = 1.2247$







④ CVs



⑤ **$p = 0.2207 > 0.05 = \alpha$**
Do NOT reject H_O

⑥ At the 95% CL, there is insufficient evidence to conclude that μ ≠ 22

9.4 Hypothesis Tests: one μ, σ Known

G: $\bar{x}$ = 52.5, n=21, σ=6.8; n.d., α=0.01

F: Is the mean age less than 55 years?

H_o: μ_____, H_a: μ_____

Test	One μ, σ known
1.	State the Null and Alternative Hypotheses: H
2.	Decide the significance level, α, and sket
3.	Compute the test statistic: z
4.	Find the P-value
5.	Decision: Rej. H_0 if $P \le \alpha$
6.	Interpret results

G: n=21, α=0.01, σ=6.8, $\bar{x}$=52.5

?, mean age less than 55 yr.

① H_0: $\mu = 55$ H_a: $\mu < 55$ yr.

$z_{.01} = -1.685$

-2.33

③ $z_t = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} = \frac{52.5-55}{6.8/\sqrt{21}} = -1.685$

④ CVs invNorm(.01,0,1) = -2.326 = -2.33

p= normcdf(-9,-1.685,0,1) =0.0460

p= 0.0460 > 0.01 = α

⑤ z_t is not in critical region Do NOT rej.

⑥ Data insufficient to rej. H_0.

9.4 Hypothesis Tests: one μ, σ Known

Example from calculator instructions:

z-Test for One Mean, σ known

(assumptions: SRS, normal distribution)

STAT / TESTS

Z-Test

Inpt: STAT

μ: 75

σ: 10

$\bar{x}$: 82

n: 23

μ: ≠μ0 <μ0 >μ0

Calculate Draw

Result: z=3.3571, P=3.9390E-4

The P-Value = 3.9390 (10^{-4}) = 0.0003939

The values and alternative hypothesis entered above are an example. Use the values appropriate for your problem.

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$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$