9.4 Hypothesis Tests: one $\mu$, $\sigma$ Known

GOALS:
1. Understand the differences between the critical value and p-value approaches to hypothesis testing.
2. Understand what the p-value is and how to find it.
3. Understand the assumptions of a z-test (same as z-interval).
4. Perform a z-test using either the critical value or the p-value approach.

Study Ch. 9.4, # 73, 77-85
(65, 67-75)

Energy Use

G: BTUs consumed/household/year in US:
$\mu = 92.2$ mill BTU, n.d., $\sigma = 15$ mill BTU
n = 20 household in West US (mill BTUs)

<table>
<thead>
<tr>
<th></th>
<th>104</th>
<th>84</th>
<th>72</th>
<th>95</th>
<th>69</th>
</tr>
</thead>
<tbody>
<tr>
<td>West</td>
<td>80</td>
<td>78</td>
<td>74</td>
<td>76</td>
<td>81</td>
</tr>
<tr>
<td></td>
<td>82</td>
<td>61</td>
<td>94</td>
<td>65</td>
<td>100</td>
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<td></td>
<td>70</td>
<td>65</td>
<td>83</td>
<td>76</td>
<td>84</td>
</tr>
</tbody>
</table>

F: Do households in the West US use a different amount of energy?

Investigate: Compare all US to West on calculator.
1. all US: $y_i = \text{normalpdf}(X,92.2,15)$
2. window: $x_{\min}=0, x_{\max}=130, y_{\min}=0, y_{\max}=0.07$
3. West: STAT/EDIT L1 enter data above
4. STAT PLOT (2nd y=) 1 ON
Type: row 2, col 1 Box Plot
Xlist: L1
Hypothesis Tests: known standard deviation

Energy Use

G: BTUs consumed/household/year in US:
\( \mu = 92.2 \) mill BTU, n.d., \( \sigma = 15 \) mill BTU

F: Do households in the West US use a different amount of energy at 5% significance level?

Using visual display is helpful. The sample certainly looks different. But, a determination that the West US is different is subjective. Need more objective means to draw a conclusion. Use sample data.

Would we conclude differently if the z score from the data is: \( z = -2.5 \) or \( z = -2.0 \) or \( z = -1.25 \)

Need criteria and a consistent approach to arrive at unbiased conclusion.

9.4 Hypothesis Tests: one \( \mu, \sigma \) Known

<table>
<thead>
<tr>
<th>Approach</th>
<th>Critical Value</th>
<th>P-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(table based, can use calculator)</td>
<td>(calculator based, can use table)</td>
<td></td>
</tr>
<tr>
<td>1. State the Null and Alternative Hypotheses: ( H_0, H_a )</td>
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<tr>
<td>2. Decide the significance level, ( \alpha ), and sketch</td>
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<tr>
<td>3. Compute the test statistic: ( z, t, ) etc.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. Find the critical values ( z_{\alpha/2}, z_{\alpha}, t_{\alpha/2}, t_{\alpha} )</td>
<td>Find the P-value</td>
<td></td>
</tr>
<tr>
<td>5. Decision: Rej. ( H_0 ) if test statistic lies beyond critical value in rejection region</td>
<td>Decision: Rej. ( H_0 ) if ( P \leq \alpha )</td>
<td></td>
</tr>
<tr>
<td>6. Interpret results</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

SRS, n.d. or large sample, \( \sigma \) known

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Class Notes Homework
9.4 Hypothesis Tests: one $\mu$, $\sigma$ Known

G: BTUs consumed/household/year in US:
\[ \mu = 92.2 \text{ mill BTU}, \quad \text{n.d., } \sigma = 15 \text{ mill BTU} \]

F: Do households in the West US use a different amount of energy at 5% significance level?

\begin{align*}
\text{G: } \text{BTUs consumed/household/year in US:} & \quad \mu = 92.2 \text{ mill BTU}, \quad \text{n.d., } \sigma = 15 \text{ mill BTU} \\
\text{F: Do households in the West US use a different amount of energy at } & \quad 5\% \text{ significance level?} \\
\end{align*}

\begin{itemize}
  \item 1. State the Null and Alternative Hypotheses: $H_0$, $H_a$
  \item 2. Decide the significance level, $\alpha$, and sketch
  \item 3. Compute the test statistic: $z$, $t$, etc.
  \item 4. Find the critical values
  \item 5. Decision: Reject $H_0$ if test statistic lies beyond critical value in rejection region
  \item 6. Interpret results
\end{itemize}

\textbf{Example:}

1. $H_0$: $\mu = 92.2$  \\
2. $\alpha = 0.05$
3. $z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{79.65 - 92.2}{15/\sqrt{20}} = -3.74$
4. $P = 1.828 \times 10^{-4}$
5. $P = 1.828 \times 10^{-4} < 0.05$
6. Conclusion: Yes, Western households use a different amount of energy.
p = 0.00018 same as 2 times result from normalcdf(-9, -3.74, 0, 1)

9.4 Hypothesis Tests: one \( \mu, \sigma \) Known

**Using the Calculator Functions to Perform z-Test**

z-Test for one Mean, \( \sigma \) known (assumption: SRS, normal distribution)

STAT/TESTS/Z-Test

Enter data into L1 and use Data as Input

Obtain sample mean and use Stats as Input

Output results
Hypothesis Tests: known standard deviation: Classnotes

9.4 Hypothesis Tests: one μ, σ Known

To Use the Critical Value Approach

p. 421: Hypothesis Test Procedure for one μ, σ Known

For the Energy Problem:

1. \( H_0: \mu = 92.2 \)
2. \( H_1: \mu \neq 92.2 \)
3. \( \alpha = 0.05 \)
4. \( z_\alpha = 1.96 \)
5. \( z_t = -3.73 < z_{0.025} = 1.96 \)
6. Conclude that energy use in the West is different from energy use in general US

Solving Word Problems

1. Read the problem. Try to identify the general type of problem. eg: CI, Hyp Test, specific value, etc.
2. Read the problem again, identifying what is given and what you need to find.
3. Use the Procedure Index to select a procedure.
4. Before beginning a procedure, determine if all assumptions are met.
5. If assumptions are not met, look for a different procedure. (Exception: if srs not met, then write "Assuming srs...")
6. If assumptions are met, follow the procedure including:
   - Draw a sketch to show \( \alpha \) as left-tailed, 2-tailed, or right-tailed.
   - For Hypothesis Tests, include null and alternative hypotheses.
   - Include all equations, substitutions, and answers for the equations. (Indicate calculator or tables.)
   - Decide to reject null hypothesis or not. Explain why.
   - Write a verbal interpretation of your decision.
7. Check: Have you satisfied the to find above?
9.4 Hypothesis Tests: one $\mu$, $\sigma$ Known

Solving Word Problems:
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   - Write a verbal interpretation of your decision.
7. Check: Have you satisfied the conditions to find $\alpha$ above?

Prob. 73

Prob. 105

Calculator instructions

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9.4 Hypothesis Tests: one $\mu$, $\sigma$ Known

G: $\overline{x} = 21$, $n=32$, $\sigma=4$; $H_0$: $\mu=22$, $H_a$: $\mu<22$,

F: Does the sample have a mean less than the population mean?
9.4 Hypothesis Tests: one \( \mu, \sigma \) Known

G: \( \bar{x} = 21, n=32, \sigma=4; H_0: \mu=22, H_a: \mu<22, \alpha=0.05 \)

F: Does the sample have a mean less than the population mean?

\[
\bar{X} - \mu = \frac{21 - 22}{4/\sqrt{32}} = -1.414
\]

\( P = 0.0786 \)

Decision: Do not reject \( H_0 \).

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<td>4. Find the ( P )-value</td>
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<td>5. Decision: ( \text{Rej } H_0 ) if ( P \leq \alpha )</td>
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9.4 Hypothesis Tests: one $\mu, \sigma$ Known

1. $H_0: \mu = 22$, $H_a: \mu \neq 22, \alpha = 0.05$

2. $z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{23 - 22}{2/\sqrt{24}} = 1.2247$

3. $z = 1.2247 < 1.96 = z_{c}$

4. $P = 0.2207 > 0.05 = \alpha$

5. $z_{c}$ is not in critical region:

6. At the 95% CL, there is insufficient evidence to conclude that $\mu \neq 22$
9.4 Hypothesis Tests: one $\mu$, $\sigma$ Known

**G:** $\bar{X} = 52.5$, $n=21$, $\sigma=6.8$; n.d., $\alpha=0.01$

**F:** Is the mean age less than 55 years?

$H_0: \mu____$, $H_a: \mu____$

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$\alpha/2$ $\alpha/2$

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$\sigma=6.8, \bar{x}=52.5$

1. $H_0: \mu = 55$, $H_a: \mu < 55$

2. $z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} = \frac{52.5 - 55}{6.8/\sqrt{21}} = -1.685$

3. $z = -1.685$ is not in critical region

4. $\text{invNorm}(0.01,0,1) = -2.33$

5. $p = \text{normcdf}(-9,-1.685,0,1) = 0.0460$

6. $p = 0.0460 > 0.01 = \alpha$

Data insufficient to rej. $H_0$.  

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9.4 Hypothesis Tests: one $\mu$, $\sigma$ Known

Example from calculator instructions:

**$z$-Test for One Mean, $\sigma$ Known**

(assumptions: SRS, normal distribution)

STAT / TESTS
Z-Test
Inpt: STAT
$\mu$: 75
$\sigma$: 10
$\bar{x}$: 82
n: 23

$H_0$: $\mu = \mu_0$  $H_1$: $\mu < \mu_0$  $H_1$: $\mu > \mu_0$

Calculate  Draw

Result: $z = 3.3571$, $P = 3.9390 \times 10^{-4}$

The P-Value = 3.9390 (10^{-4}) = 0.0003939

The values and alternative hypothesis entered above are an example. Use the values appropriate for your problem.

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$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$