8.2 Margin of Error, Sample Size (old 8.3)

GOALS:
1. Understand the Margin of Error is a measure of sampling error, and is directly proportional to the standard error.
2. Understand how the Margin of Error relates to the confidence interval for a population mean.
3. Recognize that the Margin of Error is decreased when sample sizes are increased.
4. Find sample sizes needed for a desired Margin of Error

Study 8.2, # 44(51), 49(53), 51(55), 53-56(57a-d), 61(59), 85(61)
( old 8.3 problems)

8.3 Margin of Error, Sample Size

What if the interval that you have found is not small enough? What can you do?

eg: medicine - too large a dose dangerous
airplane parts - require precise fit, etc.

Area = 0.9544

What can you do to narrow the interval?

\[ \bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \]
8.3 Margin of Error, Sample Size

What can you do to narrow the interval?

\[ \bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \]

Focus in the \[ + \]

Can you change \( \sigma \)?

NO.  \( \sigma \) is a population parameter.
It is fixed for the population.

CAN:
1. decrease \( z \) by reducing CL - not usually desirable, eg: 99% conf to 90% confident
2. increase \( n \) by increasing the sample size

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8.3 Margin of Error, Sample Size

**Margin of Error**

\[ E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \]

Relationship betw. \( E \) and the standard error?

\[ E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \]

**What is the standard error?**
8.3 Margin of Error, Sample Size

Margin of Error

\[ E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \]

Relationship betw. \( E \) and the \textit{standard error}?

\[ \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} \]

\textit{std. error of the mean}

What is the standard error?

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...from Ch. 7...

7.2 Mean and Standard Dev of Sample Mean

Mean of the Sample Mean

\[ \mu_{\bar{x}} = \mu \]

Standard Deviation of the Sample Mean

\[ \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} \]

Standard Error (of the Mean)
8.3 Margin of Error, Sample Size

\[ \bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \]

G: E = 0.047

a) F: length of CI  
   b) If \( \bar{x} = 0.205 \), F: CI
   c) sketch (number line)

\[ 2(0.047) = 0.094 \]

length of interval

\[ 0.205 \pm 0.047 \]

\[ (0.158, 0.252) \]

\[ 0.158 \leq \mu \leq 0.252 \]

c) sketch
What about the sample size? How large should \( n \) be to get a desired Margin of Error

\[
E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}
\]

Solve the above equation for \( n \):

\[
n = \left( \frac{z_{\alpha/2} \cdot \sigma}{E} \right)^2
\]

Use \( z_{\alpha/2} \) score for desired confidence with \( E \) for desired precision. Round up to next integer.

NOTE: This requires finding the value of \( z_{\alpha/2} \)

Find \( z_{\alpha/2} \)
Use the calculator:
\[
z_{\alpha/2} = \text{invNorm}(\alpha/2,0,1)
\]
8.3 Margin of Error, Sample Size

G: body fat; n=27, $\bar{x} = 22.46\%$, n.d., $\sigma = 4.10\%$

a) F: 95% CI
b) F: E

c) F: explain E in terms of accuracy of estimate

d) F: n $\geq$ E = 1.55% with 99% Confidence

Solve by finding $z_{\alpha/2}$

95% CI: $CL=0.95$, $\alpha=1-0.95=0.05$; $\alpha/2=0.025$; $z_{\alpha/2}=\text{invNorm}(0.025,0,1)=-1.96$

$22.46 \pm 1.96(4.10)/\sqrt{27}$

$22.46 \pm 1.55$ E = 1.55% body fat

$20.91 \leq \mu \leq 24.01$

Solve by using zinterval

STAT/TESTS/ZINTERVAL,

$\sigma=4.10/\bar{x}=22.46/n=27/C-level:.95$

$(20.913,24.007)$ $20.91 \leq \mu \leq 24.01$
For a sample size of $n=27$, we have 95% confidence that the population mean lies within the Margin of Error, 1.55%, of the sample mean of 22.46% body fat.

95% CI: $CL=0.95$, $\alpha=1-0.95=0.05$; $\frac{\alpha}{2}=0.025$; $z_{\frac{\alpha}{2}}=invNorm(0.025,0,1)=-1.96$

$$22.46 \pm 1.96 \left( \frac{4.10}{\sqrt{27}} \right)$$

$$22.46 \pm 1.55$$

$$(20.91, 24.01)$$

$$20.91 \leq \mu \leq 24.01$$

**E = 1.55% body fat**

c) F: explain E in terms of accuracy of estimate

For a sample size of $n=27$, we have 95% confidence that the population mean lies within the Margin of Error, 1.55%, of the sample mean of 22.46% body fat.

d) F: $n \ni E=1.55\%$ with 99% Confidence

Need $z_{\frac{\alpha}{2}}$ for 99% CI: $CL=0.99$, $\alpha=1-0.99=0.01$; $\frac{\alpha}{2}=0.005$; $z_{\frac{\alpha}{2}}=invNorm(0.005,0,1)=-2.58$

$$n = \left( \frac{z_{\frac{\alpha}{2}} \sigma}{E} \right)^2 = \left( \frac{2.58 \times 4.10}{1.55} \right)^2 = 46.57 \therefore n = 47$$

**need a sample of 47 to ensure a margin of error of 1.55% with 99% confidence**