8.2 Margin of Error, Sample Size (old 8.3)

GOALS:

1. Understand the Margin of Error is a measure of sampling error, and is directly proportional to the standard error.
2. Understand how the Margin of Error relates to the confidence interval for a population mean.
3. Recognize that the Margin of Error is decreased when sample sizes are increased.
4. Find sample sizes needed for a desired Margin of Error

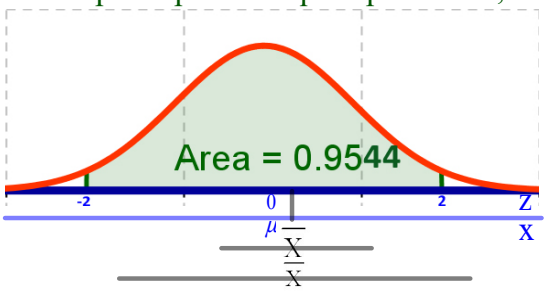
Study 8.2, # 44(51),49(53),51(55),
53-56(57a-d), 61(59), 85(61)
(old 8.3 problems)

Class Notes: Prof. G. Battaly, Westchester Community College, NY

8.3 Margin of Error, Sample Size

What if the interval that you have found is not small enough? What can you do?

eg: medicine - too large a dose dangerous
airplane parts - require precise fit, etc.



What can you do to narrow the interval?

$$\bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

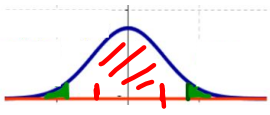
Class Notes: Prof. G. Battaly, Westchester Community College, NY

8.3 Margin of Error, Sample Size

What can you do to narrow the interval?

$$\bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

Focus in the $\pm \; z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$



Can you change σ ?

NO. σ is a population parameter.
It is fixed for the population.

CAN:

1. decrease *z* by reducing CL- not usually desirable, eg: 99% conf to 90% confident
2. increase *n* by increasing the sample size

8.3 Margin of Error, Sample Size

Margin of Error

$$E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

Relationship betw. ***E***
and the ***standard error?***

$$E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

What is the standard error?

8.3 Margin of Error, Sample Size

Margin of Error

$$E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

Relationship betw. ***E*** and the ***standard error?***

$$E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

What is the standard error?

$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$

std. error of the mean

…from Ch.7…

7.2 Mean and Standard Dev of Sample Mean

Mean of the Sample Mean

$$\mu_{\bar{x}} = \mu$$

Standard Deviation of the Sample Mean

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

Standard Error (of the Mean)

8.3 Margin of Error, Sample Size

$$\bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

G: E = 0.047

a) F: length of CI b) If $\bar{x}$ = 0.205, F: CI

c) sketch (number line)

8.3 Margin of Error, Sample Size

$$\bar{x} \pm \boxed{z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}} \quad E$$

G: E = 0.047

a) F: length of CI b) If $\bar{x}$ = 0.205, F: CI

a) $2(0.047) = 0.094$ length of interval

b) 0.205 ± 0.047

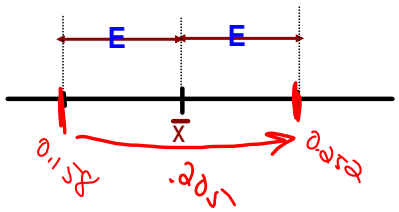
$(0.158, 0.252)$

$0.158 < \mu < 0.252$

to get bounds of the interval,
add and subtract the margin of error

simple arithemetic

c) sketch



8.3 Margin of Error, Sample Size

What about the sample size?
How large should *n* be to get a desired Margin of Error

$$E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

$E\sqrt{n} = z_{\alpha/2} \cdot \sigma$

$\sqrt{n} = \frac{z_{\alpha/2} \cdot \sigma}{E}$

$n = \left(\frac{z_{\alpha/2} \cdot \sigma}{E}\right)^2$

from algebra

Solve the above equation for *n*:

$$n = \left(\frac{z_{\alpha/2}\,\sigma}{E}\right)^2$$

Use $z_{\alpha/2}$ score for desired confidence with ***E*** for desired precision.
Round up to next integer.

8.3 Margin of Error, Sample Size

What about the sample size?
How large should *n* be to get a desired Margin of Error

$$E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

Solve the above equation for *n*:

$$n = \left(\frac{z_{\alpha/2}\,\sigma}{E}\right)^2$$

Use $z_{\alpha/2}$ score for desired confidence with ***E*** for desired precision.
Round up to next integer.

Class Notes: Prof. G. Battaly, Westchester Community College, NY
Statistics Home Page ©Gertrude Battaly, 2014 Class Notes Homework

NOTE: This **requires** finding the value of $z_{\alpha/2}$

Find $z_{\alpha/2}$
Use the calculator:
$z_{\alpha/2}$ = invNorm(α/2,0,1)

8.3 Margin of Error, Sample Size

Assm SRS

G: body fat; n=27, $\bar{x}$ = 22.46%, n.d., σ=4.10%

female graduate physical therapy students

a) F: 95% CI b) F: E

c) F: explain E in terms of accuracy of estimate

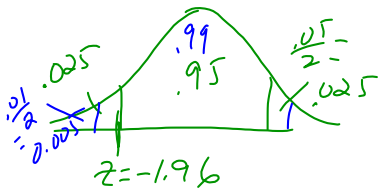
d) F: n ∋ E=1.55% with 99% Confidence

8.3 Margin of Error, Sample Size

G: n=27, $\bar{x}$ = 22.46%, n.d., σ=4.10%

a) F: 95% CI b) F: E

$$\bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$



Solve by finding $z_{\alpha/2}$

95% CI: CL=0.95, α=1-0.95=0.05; α/2=0.025;
$z_{\alpha/2}$=invNorm(0.025,0,1)=-1.96

22.46 ± 1.96(4.10)/√27
22.46 ± 1.55 — **E = 1.55%** body fat
(20.91,24.01)
20.91 ≤ μ ≤ 24.01

Solve by using zinterval

STAT/TESTS/ZINTERVAL, Stats/
σ=4.10/ x= 22.46 / n=27/C-level:.95
(20.913,24.007) 20.91 ≤ μ ≤ 24.01

E not obvious, find by subtracting right bound-left bound and dividing by 2:

$$\frac{24.007 - 20.913}{2} = 1.55 \text{ body fat}$$

8.3 Margin of Error, Sample Size

G: n=27, $\bar{x}$ = 22.46%, n.d., σ=4.10%

95% CI: CL=0.95,
α=1-0.95=0.05; α/2=0.025;
$z_{\alpha/2}$=invNorm(0.025,0,1)=-1.96

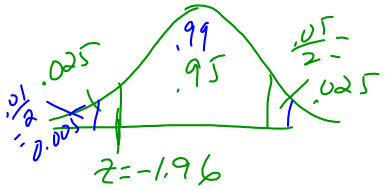
a) F: 95% CI

b) F: E

$$\bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

22.46 ± 1.96(4.10)/√27
22.46 ± 1.55
(20.91,24.01)
20.91 ≤ μ ≤ 24.01

E = 1.55% body fat



c) F: explain E in terms of accuracy of estimate

For a sample size of n=27, we have 95% confidence that the population mean lies within the Margin of Error, 1.55%, of the sample mean of 22.46% body fat.

Class Notes: Prof. G. Battaly, Westchester Community College, NY

Statistics Home Page ©Gertrude Battaly, 2014 Class Notes Homework

8.3 Margin of Error, Sample Size

G: n=27, $\bar{x}$ = 22.46%, n.d., σ=4.10%

95% CI: CL=0.95,
α=1-0.95=0.05; α/2=0.025;
$z_{\alpha/2}$=invNorm(0.025,0,1)=-1.96

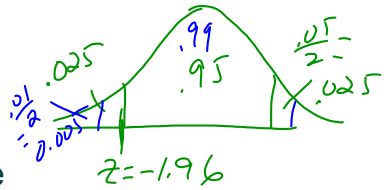
a) F: 95% CI

b) F: E

$$\bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

22.46 ± 1.96(4.10)/√27
22.46 ± 1.55
(20.91,24.01)
20.91 ≤ μ ≤ 24.01

E = 1.55% body fat



c) F: explain E in terms of accuracy of estimate

For a sample size of n=27, we have 95% confidence that the population mean lies within the Margin of Error, 1.55%, of the sample mean of 22.46% body fat.

same E, higher confidence

d) F: n ∋ E=1.55% with 99% Confidence

$$n = \left(\frac{z_{\alpha/2}\,\sigma}{E} \right)^2$$

Need $z_{\alpha/2}$

99% CI: CL=0.99,
α=1-0.99=0.01; α/2=0.005;
$z_{\alpha/2}$=invNorm(0.005,0,1)=**-2.58**

$$= \left[\frac{2.58\,(4.10)}{1.55} \right]^2 = 46.57 \therefore \; n = 47$$

need a sample of 47 to ensure a margin of error of 1.55% with 99% confidence

Class Notes: Prof. G. Battaly, Westchester Community College, NY

Statistics Home Page ©Gertrude Battaly, 2014 Class Notes Homework