8.2 Margin of Error, Sample Size (old 8.3)

GOALS:
1. Understand the Margin of Error as a measure of sampling error, and is directly proportional to the standard error.
2. Understand how the Margin of Error relates to the confidence interval for a population mean.
3. Recognize that the Margin of Error is decreased when sample sizes are increased.
4. Find sample sizes needed for a desired Margin of Error

Study 8.2, # 44(51), 49(53), 51(55), 53-56(57a-d), 61(59), 85(61)
( old 8.3 problems)

What if the interval that you have found is not small enough? What can you do?

eg: medicine - too large a dose dangerous
airplane parts - require precise fit, etc.

What can you do to narrow the interval?

\[
\bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}
\]
8.3 Margin of Error, Sample Size

What can you do to narrow the interval?

\[ \bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \]

Focus in the \[ \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \]

Can you change \( \sigma \)?

NO. \( \sigma \) is a population parameter. It is fixed for the population.

CAN:
1. decrease \( z \) by reducing CL - not usually desirable
2. increase \( n \) by increasing the sample size

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**Margin of Error**

\[ E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \]

Relationship between \( E \) and the **standard error**?

\[ E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \]

What is the **standard error**?
8.3 Margin of Error, Sample Size

**Margin of Error**

\[ E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \]

**What is the standard error?**

\[ \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} \]

**Relationship betw. \( E \) and the standard error?**

\[ E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \]

7.2 Mean and Standard Dev of Sample Mean

*from Ch.7*

**Mean of the Sample Mean**

\[ \mu_{\bar{x}} = \mu \]

**Standard Deviation of the Sample Mean**

\[ \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} \]

**Standard Error (of the Mean)**
8.3 Margin of Error, Sample Size

\[ \bar{x} \pm \frac{z_{\alpha/2} \cdot \sigma}{\sqrt{n}} \]

G: E = 0.047

a) F: length of CI
b) If \( \bar{x} = 0.205 \), F: CI
c) sketch

\[ a) 2(0.047) = 0.094 \]
\[ b) 0.205 \pm 0.047 \]
\[ (0.158, 0.252) \]
\[ 0.158 < \mu < 0.252 \]

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What about the sample size? How large should \( n \) be to get a desired Margin of Error

\[
E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}
\]

Solve the above equation for \( n \):

\[
E \sqrt{n} = z_{\alpha/2} \cdot \sigma
\]

\[
\sqrt{n} = \frac{z_{\alpha/2} \cdot \sigma}{E}
\]

\[
n = \left( \frac{z_{\alpha/2} \cdot \sigma}{E} \right)^2
\]

Use \( z_{\alpha/2} \) score for desired confidence with \( E \) for desired precision. Round up to next integer.

---

Assume the following:

G: \( n=27, \; \bar{x} = 22.46\%, \; \text{n.d.,} \; \sigma = 4.10\% \)

a) F: 95\% CI
b) F: \( E \)

c) F: explain \( E \) in terms of accuracy of estimate

d) F: \( n \) 
\( E = 1.55\% \) with 99\% Confidence

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Class Notes Homework
8.3 Margin of Error, Sample Size

G: \( n=27, \quad \bar{x} = 22.46\%, \quad \text{n.d., } \sigma = 4.10\% \)

a) \( F: \quad 95\% \text{ CI} \quad \bar{x} \pm 2.46 \frac{\sigma}{\sqrt{n}} \)

b) \( F: \quad E \)

\[
22.46 \pm 1.96 \left( \frac{4.10}{\sqrt{27}} \right)
\]

\[
20.91 \leq \mu \leq 23.91
\]

\[
E = 1.55\% \quad \text{body fat}
\]

c) \( F: \) explain \( E \) in terms of accuracy of estimate

For a sample size of \( n=27 \), we have 95% confidence that the population mean lies within the Margin of Error, 1.55%, of the sample mean of 22.46% body fat.

d) \( F: \quad n \ni E=1.55\% \text{ with 99\% Confidence} \)

\[
n = \left( \frac{2\alpha/2 \sigma}{E} \right)^2 \approx \left( \frac{2.576 (4.10)}{1.55} \right)^2 = 46.57 \quad \therefore n = 47
\]

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