6.1 Normal Distribution

GOALS:

1. Understand properties of:
   a) Density Curves
   b) Normal Curves
   c) Standard Normal Curve
2. Relate area under the curve to proportions of the population represented by the curve.

Study Ch. 6.1, # 5 -23, 39(33)

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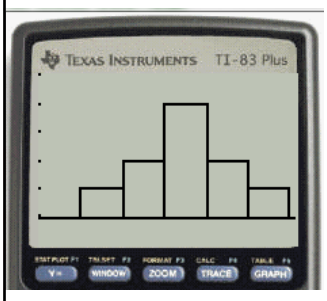
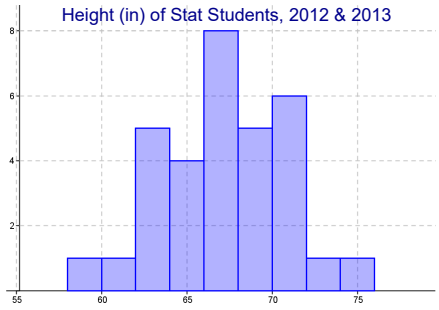
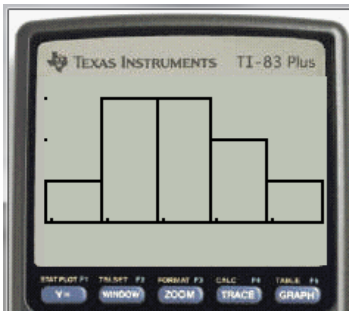
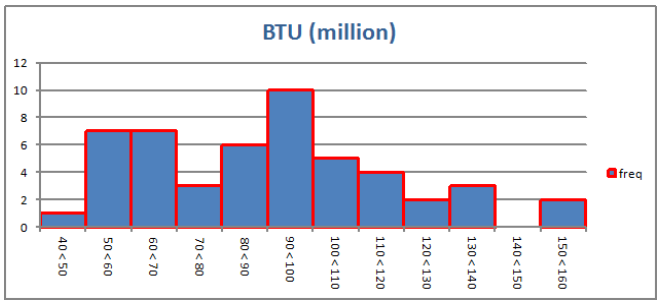
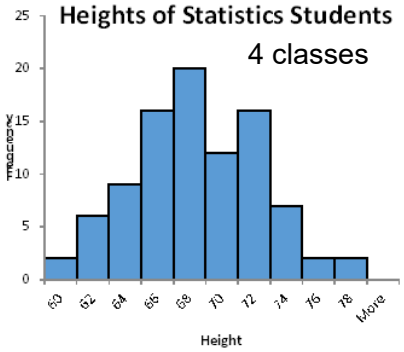
6.1 Normal Distribution various distributions











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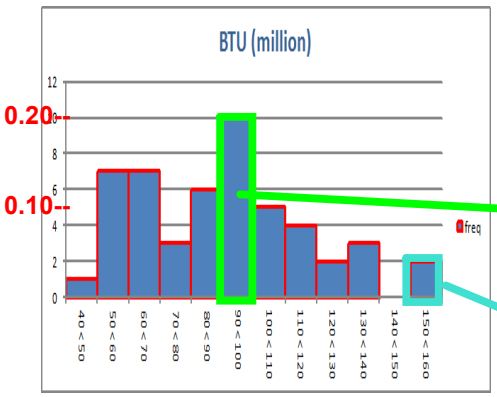
6.1 Normal Distribution

From section 2.3, Histograms:



BTU	tally	freq	rel f	midpt
40 < 50		1	0.02	45
50 < 60		7	0.14	55
60 < 70		7	0.14	65
70 < 80		3	0.06	75
80 < 90		6	0.12	85
90 < 100		10	0.20	95
100 < 110		5	0.10	105
110 < 120		4	0.08	115
120 < 130		2	0.04	125
130 < 140		3	0.06	135
140 < 150		0	0.00	145
150 < 160		2	0.04	155
			Σ = 1.00	
n=	50			

Consider relative frequency instead of actual frequency. Then each bar represents a proportion, and **the sum of all the bars represents a total area = 1**

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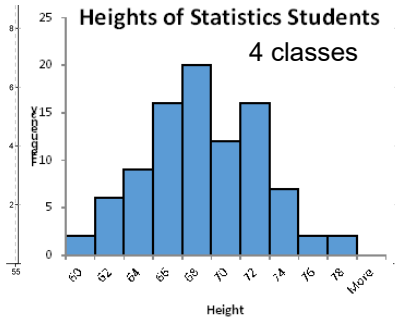
6.1 Normal Distribution

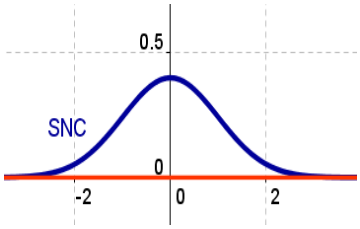
Measurements of natural characteristics - such as height, weight, etc. - follow a pattern of distribution with more individuals near the mean and few at the ends.



1. Consider relative frequency instead of actual frequency. Then the sum of the area of all the bars = 1

2. Consider a very large sample size or consider the population. Also consider a narrow width. Then the distribution can be represented by a curve.

Bell-shaped Curves



SNC - Standard Normal Curve

$\mu = 0, \quad \sigma = 1$

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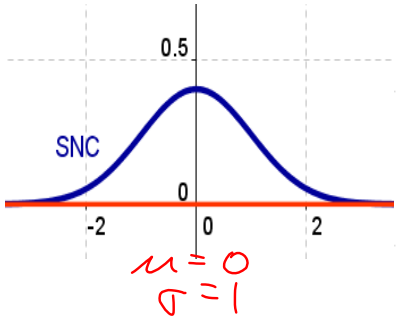
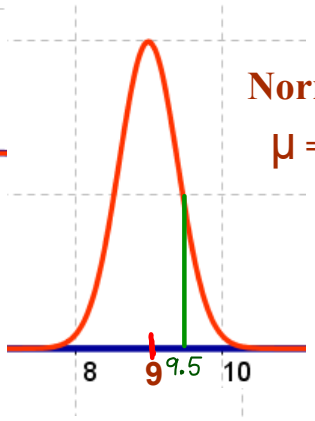
6.1 Normal Distribution

Bell-shaped Curves -
not all are SNC,
can be NC with other means and standard deviations

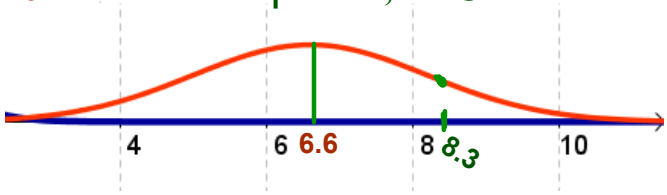




Normal Curve

$\mu = 9, \quad \sigma = 1/2$

$\mu = 6.6, \quad \sigma = 1.7$



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6.1 Normal Distribution

Normally Distributed Variable: Distribution has shape of normal curve

Approximately normally distributed: similar to but not exactly the same shape.

Dynamic Normal Curve

$$y = \frac{e^{-\frac{(x-\mu)^2}{2\sigma^2}}}{\sqrt{2\pi}\,\sigma}$$

SNC: μ=0, σ=1

$$y = \frac{e^{-\frac{x^2}{2}}}{\sqrt{2\pi}}$$

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6.1 Normal Distribution

Dynamic Normal Curve

Area of Normal Curve

Standard Normal Curve:

$$z = \frac{x - \mu}{\sigma}$$

SNC

$\mu = 0, \quad \sigma = 1$

connection between data & SNC:

1. convert x values to z values
2. then determine area and probability.

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6.1 Normal Distribution

Area of Normal Curve

Standard Normal Curve:

$$z = \frac{x - \mu}{\sigma}$$

$\mu = 0$

$\sigma = 1$

z - score is the number of standard deviations away from the mean of a specific item of data

If you earn a grade of 80 on Test #1, and statistics for the grades are $\bar{x} = 83$, $s = 10$, what is your z score? (Use statistics as estimates.)

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6.1 Normal Distribution

Area of Normal Curve

Standard Normal Curve:

$$z = \frac{x - \mu}{\sigma}$$

$\mu = 0$

$\sigma = 1$

$$z = \frac{x-\mu}{\sigma}$$

$$z = \frac{x-\bar{x}}{s}$$

z - score is the number of standard deviations away from the mean of a specific item of data

$$= \frac{80-83}{10}$$

If you earn a grade of 80 on Test #1, and statistics for the grades are $\bar{x}$ = 83, s = 10, what is your z score? (Use statistics as estimates.)

$$= \frac{-3}{10}$$

$$z = -0.3$$

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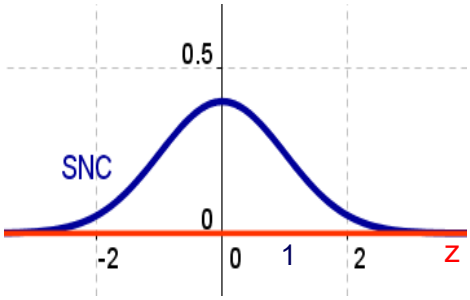
6.1 Normal Distribution



Total Area = 1

0 ≤ area of intervals under SNC ≤ 1

0 ≤ p ≤ 1

eg: area between z=0.5 and z=1.5

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6.1 Normal Distribution

Which has the wider spread?

$\mu = 1, \quad \sigma = 2$ | $\mu = 2, \quad \sigma = 1$

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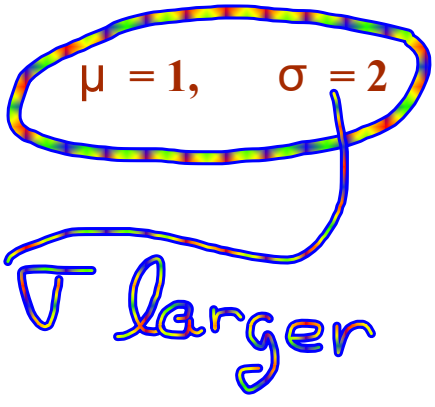

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6.1 Normal Distribution

Given: n.d.

curve 1: $\mu = -4$, $\sigma = 3$

curve 2: $\mu = -4$, $\sigma = 6$

True or False?

Same Shape?

Same Center?

n.d.: normal distribution

6.1 Normal Distribution

Given: n.d.

n.d.: normal distribution

curve 1: $\mu = -4$, $\sigma = 3$

curve 2: $\mu = -4$, $\sigma = 6$

True or False?

Same Shape? F

Same Center? T

both bell-shaped; but $\sigma = 6$ is flatter and wider

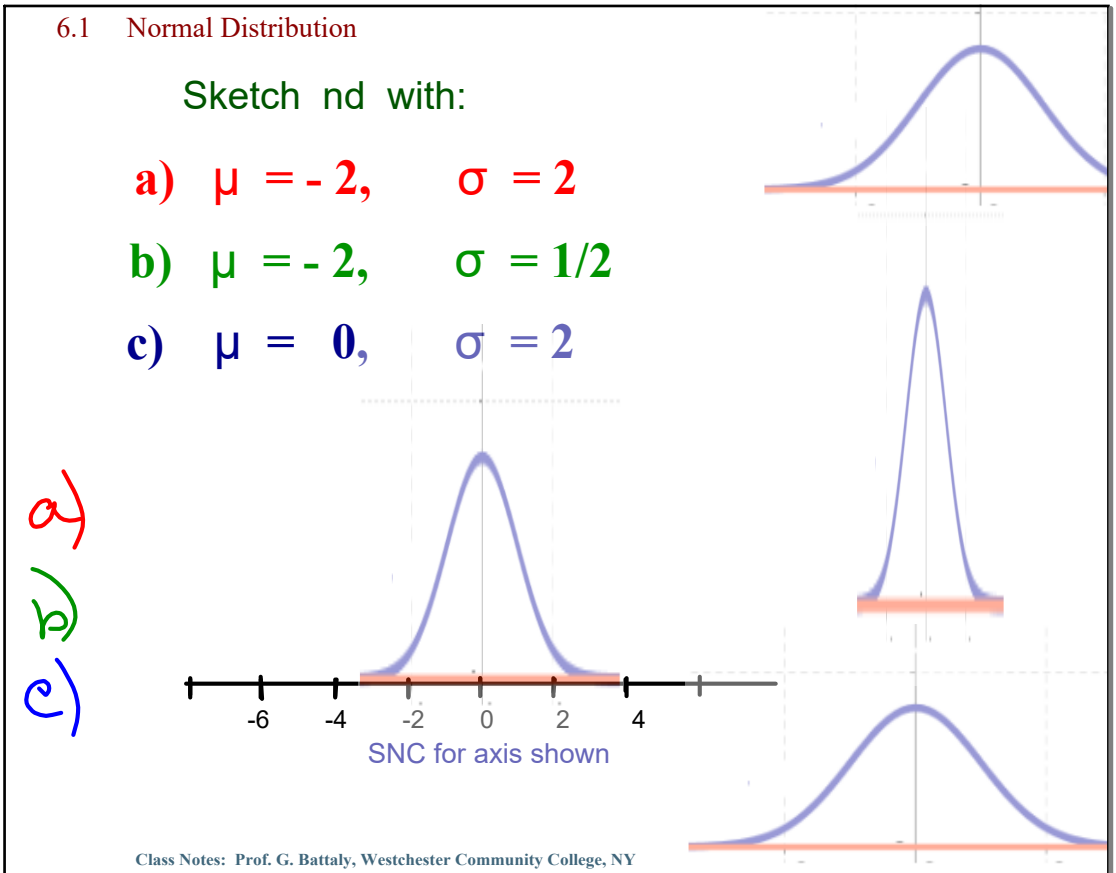
6.1 Normal Distribution

Sketch nd with:

a) $\mu = -2$, $\sigma = 2$

b) $\mu = -2$, $\sigma = 1/2$

c) $\mu = 0$, $\sigma = 2$



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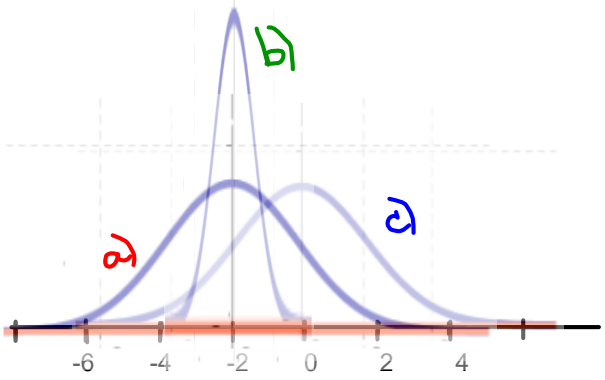

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6.1 Normal Distribution

G: A curve has area 0.425 to the left of 4 and area 0.585 to the right of 4. Could this curve be a density curve for some variable? Explain.

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6.1 Normal Distribution

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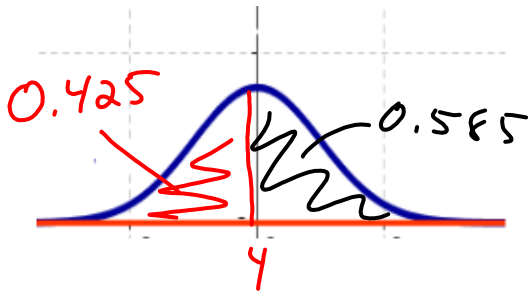

NO

$$\begin{array}{r} 0.425 \\ 0.585 \\ \hline 1.010 \end{array}$$

Sum of areas is NOT ≤ 1

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6.1 Normal Distribution

G: 33.6% of all possible observations of a variable exceed 8. Determine the area under the density curve that lies to the:
a) right of 8 b) left of 8

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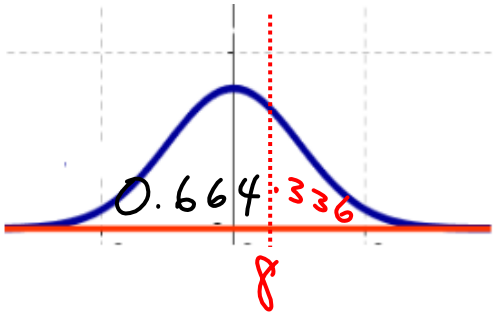

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6.1 Normal Distribution

cholesterol levels have:

G: μ = 206 mg/dL σ = 44.7 mg/dL

F: a) Sketch distribution of x.

b) z

c) Identify and sketch distribution of z.

d) % with cholesterol level between 150 mg/dL and 250 mg/dL = % of area under SNC betw _____ and _____

e) % with cholesterol level below 220 mg/dL = % area under SNC that lies to _____ of _____

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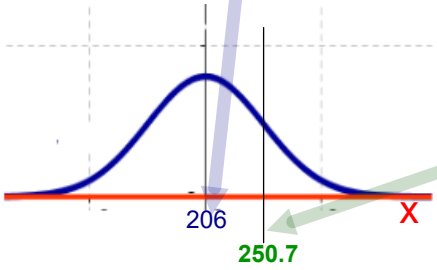
206 = μ
44.7 = σ
250.7



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6.1 Normal Distribution

cholesterol levels have:

G: μ = 206 mg/dL σ = 44.7 mg/dL

F: a) Sketch distribution of x.

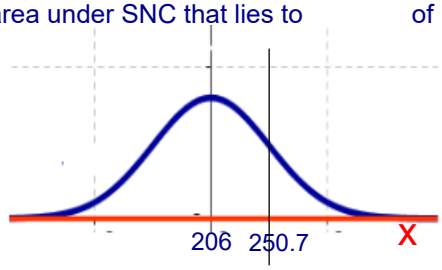
b) z convert x to z using z-score: $z = \frac{x-\mu}{\sigma} = \frac{x-206}{44.7}$

c) Identify and sketch distribution of z.

d) % with cholesterol level between 150 mg/dL and 250 mg/dL = area under SNC betw _____ and _____

e) % with cholesterol level below 220 mg/dL = area under SNC that lies to of _____

206 = μ
44.7 = σ
250.7



b) $z = \frac{\square - \square}{\square}$

$z = \frac{\square - \square}{\square}$

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6.1 Normal Distribution

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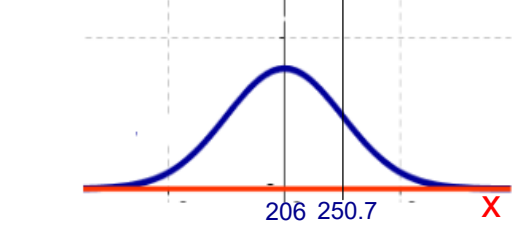
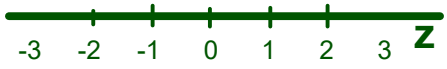
206
44.7
250.7

b) $z = \frac{x-\mu}{\sigma}$

$z = \frac{x-206}{44.7}$



c)



easiest to add line representing z with $\mu = 0$ and $\sigma = 1$

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6.1 Normal Distribution

cholesterol levels have:

SNC means need z values

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F: a) Sketch distribution of x. b) z c) Identify and sketch distribution of z.

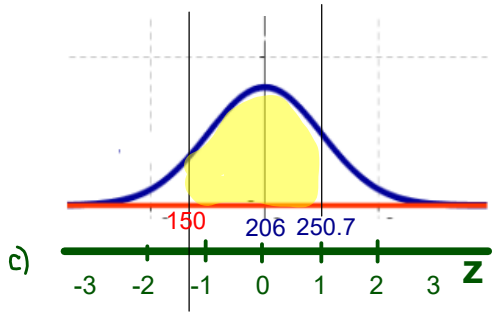
d) % with cholesterol level between 150 mg/dL and 250 mg/dL = % of area under SNC between _____ and _____

e) % with cholesterol level below 220 mg/dL = area under SNC that lies to _____ of _____

206
44.7
250.7

b) $z = \frac{x-\mu}{\sigma}$

$z = \frac{x-206}{44.7}$



$z = \frac{\square - \square}{\square}$

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6.1 Normal Distribution

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F: a) Sketch distribution of x. b) z c) Identify and sketch distribution of z.

d) % with cholesterol level between 150 mg/dL and 250 mg/dL = area under SNC between

$z = -1.25$ and $z = 0.98$

e) % with cholesterol level below 220 mg/dL = area under SNC that lies to _____ of _____

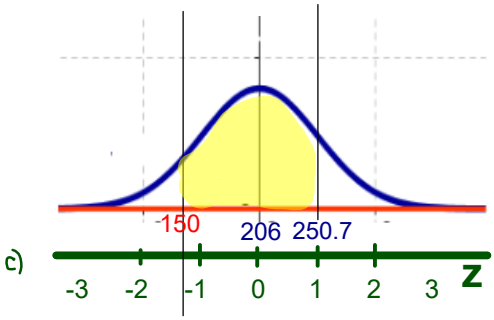
206
44.7
250.7

b) $z = \frac{x-\mu}{\sigma}$

$z = \frac{x-206}{44.7}$

d) $z = \frac{250-206}{44.7} = 0.98$

$z = \frac{150-206}{44.7} = -1.25$



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6.1 Normal Distribution

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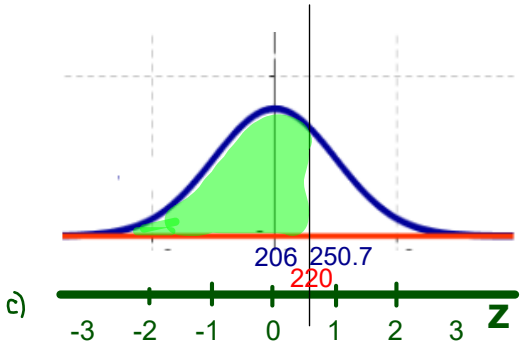
150 mg/dL and 250 mg/dL = area under SNC betw _____ and _____
e) % with cholesterol level below 220 mg/dL
= % of area under SNC that lies to _____ of _____



$z = \frac{\square - \square}{\square}$

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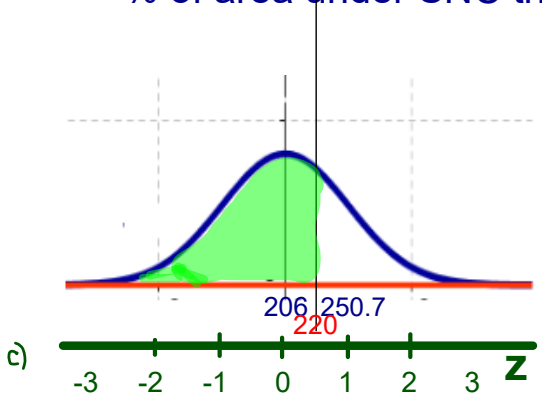
F: a) Sketch distribution of x. b) z c) Identify and sketch distribution of z.
d) % with cholesterol level between
150 mg/dL and 250 mg/dL = area under SNC betw _____ and _____
e) % with cholesterol level below 220 mg/dL
= % of area under SNC that lies to left of z=0.31



b) $z = \frac{x-\mu}{\sigma}$ $z = \frac{x-206}{44.7}$

d) $z = \frac{250-206}{44.7} \approx 0.98$ $z = \frac{150-206}{44.7} \approx -1.25$

$z = \frac{x-206}{44.7}$ left

$= \frac{220-206}{44.7}$

$= 0.31$

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