16.1 - 16.3 One-Way ANOVA

Clinical trial to determine effectiveness of weight loss programs: a low calorie diet, a low fat diet, and a low carbohydrate diet. 20 people were randomly selected and randomly assigned to one of 4 groups, including the above 3 diets and a control group. After 8 weeks the number of pounds lost were:

Low Calorie	Low Fat	Low Carbo	Control
8	2	3	2
9	4	5	2
6	3	4	-1
7	5	2	0
4	1	3	3

Is there a statistically significant difference in the mean weight loss among the four diets? Use $\alpha = 0.05$.

How do we approach this problem?

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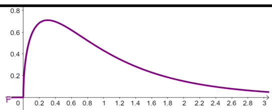
16.1 - 16.3 One-Way ANOVA



F distribution

GOALS:

1. Recognize that the F test statistic is a ratio
2. The F distribution is dependent on:
   df for numerator, and df for denominator
3. ANOVA, or **AN**alysis **O**f **VA**riance, is an extension of a **pooled-t** test to 3 or more samples:
   "Are the means the same or different?"
4. Assumptions for ANOVA are the same as those for the pooled-t test
5. ANOVA uses a **right-tailed** F test.
6. ANOVA compares variation between groups to the variation within groups to determine if the means of the groups are the same or different

Study Ch. 16.2#13-21, 25 (be aware that computations use variation)
Study Ch. 16.3# 33-41,43(by formulas),49-53(by function)

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16.1 - 16.3 One-Way ANOVA

Clinical trial to determine effectiveness of weight loss programs: a low calorie diet, a low fat diet, and a low carbohydrate diet. 20 people were randomly selected and randomly assigned to one of 4 groups, including the above 3 diets and a control group. After 8 weeks the number of pounds lost were:

Low Calorie	Low Fat	Low Carbo	Control
8	2	3	2
9	4	5	2
6	3	4	-1
7	5	2	0
3	1	3	3

Is there a statistically significant difference in the mean weight loss among the four diets?
Use $\alpha = 0.05$.

Consider how we compared means from **2** groups:

Since we have **4** groups, we need a different method.

Pooled - t Test

Assumptions: 1. SRS, 2. Independent samples 3. ND or large samples 4. = σ's

Step 1: H_0: $\mu_1 = \mu_2$
H_a: $\mu_1 \neq \mu_2$ or $\mu_1 < \mu_2$ or $\mu_1 > \mu_2$

Step 2: Decide α **df = $n_1 + n_2 - 2$**

Step 3: Compute test statistic: $t = \dfrac{(\bar{x}_1 - \bar{x}_2)}{s_p\sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}}$

where

$$s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$$

Step 4: Find p-value

Step 5: Decide whether to reject H_0 or not

Step 6: Verbal interpretation

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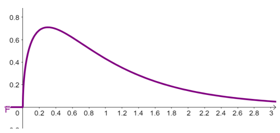
16.1 - 16.3 One-Way ANOVA

ANOVA: **AN**alysis **O**f **VA**riance vs. pooled - *t*

Some similarities, some differences

Similarities

1. Assumptions same.
2. Both use pooled s
   (groups have equal s)



F distribution

Differences: In ANOVA,

1. Pooled s not obvious (hidden in MSE).
2. Compares variation among groups to variation within groups.
3. Uses F distribution instead of the t-distribution.

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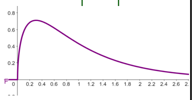
16.1 - 16.3 One-Way ANOVA

One-Way ANOVA p. 732

A	B	C
-	-	-
-	-	-
-	-	-
-	-	-

Assumptions: 1. SRS, 2. Independent samples
3. ND or large samples 4. = σ's



Step 1: H_0: $\mu_1 = \mu_2 = \mu_3 = \ldots = \mu_k$ Step 2: Decide α

H_a: NOT ALL EQUAL ← 1-tailed test

Step 3: Compute SST, SSTR, SSE

$$s = \sqrt{\frac{\Sigma x_i^2 - (\Sigma x_i)^2/n}{n-1}}$$

SST

Step 4: Construct ANOVA table to compute F statistic.

Source	df	SS	MS=SS/df	F statistic
Treatment	k - 1	SSTR	MSTR=SSTR/(k-1)	F=MSTR/MSE
Error	n - k	SSE	MSE=SSE/(n-k)	
Total	n - 1	SST		

F = var.among / var.within

n = sample size, across all treatments

Step 5: Find p-value p = 1- Fcdf (0, Ftest, dfTR, dfE)

p-value is output in ANOVA function

Step 6: If p < α reject H_0

Step 7: Verbal interpretation

Source	df	SS	MS=SS/df	F=MSTR/MSE F statistic
Treatment k - 1				
Error n - k				
Total n - 1				

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16.1 - 16.3 One-Way ANOVA

Assumptions for One-Way ANOVA

Required: 1. SRS, 2. Independent samples

Robust, can be used when only approximately met:

3. ND or large samples
4. Equal σ's
   usually OK to use ANOVA if no sample standard deviation is more than twice as large as any other s

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16.1 - 16.3 One-Way ANOVA

Use the calculator.
Enter data into L1, L2, etc.
Use STAT/TESTS/ANOVA(L1,L2,...)

Clinical trial to determine effectiveness of weight loss programs: a low calorie diet, a low fat diet, and a low carbohydrate diet. 20 people were randomly selected and randomly assigned to one of 4 groups, including the above 3 diets and a control group. After 8 weeks the number of pounds lost were:

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8	2	3	2
9	4	5	2
6	3	4	-1
7	5	2	0
4	1	3	3

Is there a statistically significant difference in the mean weight loss among the four diets? Use $\alpha = 0.05$.

16.1 - 16.3 One-Way ANOVA

SKIP TO ANOVA in Practice

Clinical trial to determine effectiveness of weight loss programs: a low calorie diet, a low fat diet, and a low carbohydrate diet. 20 people were randomly selected and randomly assigned to one of 4 groups, including the above 3 diets and a control group. After 8 weeks the number of pounds lost were:

A: (1) 3 s.s. (2) indep.
(3) groups (n=5)
norm s=1.1 max 1.9
(4) σ's =
ANOVA

Low Calorie	Low Fat	Low Carbo	Control
8	2	3	2
9	4	5	2
6	3	4	-1
7	5	2	0
4	1	3	3

Is there a statistically significant difference in the mean weight loss among the four diets? Use $\alpha = 0.05$.

$H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4$
H_a: Not all =

Calculator: STAT/TESTS/ANOVA
ANOVA(L1,L2,L3,L4)

$p = 4.16 (10^{-4})$

	df	SS	MS	F_T
Treatment	3	82	27.333	27.333/2.55 = 10.719
Error	16	40.8	2.55	X
Total	19	122.8	X	X

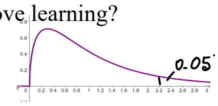
$s_p = 1.5969$

$p = 0.000416 < 0.05 = \alpha$ ∴ rej. H_0

Conclude: Yes. There is a statistically significant difference among the diets.

16.1 - 16.3 One-Way ANOVA Compare variation among groups to variation within groups.

Researchers wanted to know if enrichment enhances learning. They randomly selected rats and assigned them to one of four conditions: Impoverished (housed alone), standard (cage with other rats), enriched (cage with other rats and toys), and super enriched (cage with rats and toys changed on a periodic basis). After two months, the rats were tested on a maze and the number of trials to learn the maze recorded. At the 5% significance level, did the enrichment improve learning?



	Impoverished	Standard	Enriched	Super Enriched
	22	17	12	8
	19	21	14	7
	15	15	11	10
	24	12	9	9
	18	19	15	12
s	___	___	___	___

Temporarily bypass assumptions to look at the ANOVA table, which replaces a formula for the test statistic.

need = s

Source	df	SS	MS=SS/df	F=MSTR/MSE F statistic
Treatment	k - 1			
Error	n - k			
Total	n - 1			

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16.1 - 16.3 One-Way ANOVA

	Impoverished	Standard	Enriched	Super Enriched
	22	17	12	8
	19	21	14	7
	15	15	11	10
	24	12	9	9
	18	19	15	12
T	98	84	61	46

Treatment $SSTR = \sum (T_i^2 / n_i) - (\sum x)^2 / n$

Total $SST = \sum x^2 - (\sum x)^2 / n$

Error $SSE = SST - SSTR$

Instead, will use calculator

Source	df	SS	MS=SS/df	F statistic
Treatment	k - 1	SSTR	MSTR=SSTR/(k-1)	F=MSTR/MSE
Error	n - k	SSE	MSE=SSE/(n-k)	
Total	n - 1	SST		

show this on paper

Impoverished	Standard	Enriched	Super Enriched
22	17	12	8
19	21	14	7
15	15	11	10
24	12	9	9
18	19	15	12

Source	df	SS	MS=SS/df	F statistic
Treatment				
Error				
Total				

Source	df	SS	MS=SS/df	F statistic
Treatment	k - 1	SSTR	MSTR=SSTR/(k-1)	F=MSTR/MSE
Error	n - k	SSE	MSE=SSE/(n-k)	
Total	n - 1	SST		

16.1 - 16.3 One-Way ANOVA Compare variation among groups to variation within groups.

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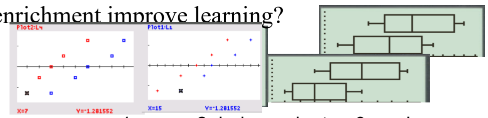


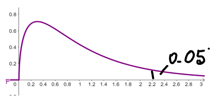
	Impoverished	Standard	Enriched	Super Enriched
	22	17	12	8
	19	21	14	7
	15	15	11	10
	24	12	9	9
	18	19	15	12
$\bar{x}$	19.6	16.8	12.2	9.2
s	3.51	3.49	2.39	1.92

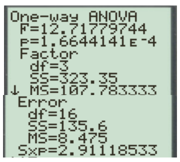
1. srs 2. independent 3. ~nd

4. similar σ's (2*1.92 = 3.84>3.51)

H_0: $\mu_1=\mu_2=\mu_3=\mu_4$ no difference

H_a: not all equal — are different





Source	df	SS	MS=SS/df	F=MSTR/MSE F statistic
Treatment (k - 1)	3	323.35	107.783	12.718
Error (n - k)	16	135.6	8.475	
Total (n - 1)	19	458.95		

s_p = 2.911 p = 0.00017 < 0.05 = α Reject H_0

Conclude: Not all means are equal. So, enrichment levels seem to make a difference in learning.

16.1 - 16.3 One-Way ANOVA Compare variation among groups to variation within groups.

A research study was conducted to evaluate a new antidepressant. Patients were randomly assigned to one of three groups: a placebo group, a low dose group, and a moderate dose group. After five weeks of treatment, the patients completed a self-report inventory. The higher the score, the more depressed the patient. At the 1% significance level, is the new medication effective?

Calculator: --> L1, L2, L3
NPP and Box-Plots for all using STAT Plot (2nd Y=) and zoom 9

	Placebo	Low	Moderate
	38	22	14
	47	19	26
	39	8	11
	25	23	18
	42	31	5
$\bar{x}$	38.2	20.6	14.8
s	8.17	8.32	7.85

From BoxPlots, do the means all look equal? Note the variation within each sample set, then compare variation between (among) sample sets.

Source	df	SS	MS=SS/df	F=MSTR/MSE F statistic
k - 1 Treatment				
n - k Error				
Total n - 1				

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16.1 - 16.3 One-Way ANOVA Compare variation among groups to variation within groups.

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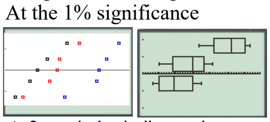
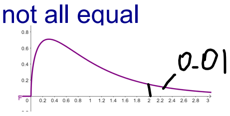
Calculator: --> L1, L2, L3
Box-Plots for all using STAT Plot (2nd Y=) and zoom 9



1. srs 2. independent 3. ~nd 4. similar variance

H_0: $\mu_1 = \mu_2 = \mu_3$ H_a: not all equal



	Placebo	Low	Moderate
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	42	31	5
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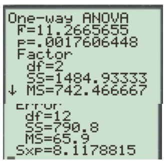
s	8.17	8.32	7.85



Source	df	SS	MS=SS/df	F=MSTR/MSE F statistic
k - 1 Treatment	2	1484.93	742.467	11.267
n - k Error	12	790.8	65.9	
Total n - 1	14	2275.73		

$s_p = 8.118$ $p = 0.0018 < 0.01 = \alpha$ Reject H_0

Conclude: Not all means are equal.

So, it is likely that the meds are effective. To be sure the meds are effective, need to determine if the doses are different from the placebo.

Subsequent pooled-t tests on pairs results in significant differences between placebo and low dose (p=0.005) and placebo and moderate (p=0.0009), but not between low dose and moderate (p=0.145). Note: If no difference in dosage, use low dose.

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16.1 - 16.3 One-Way ANOVA

Compare variation among groups to variation within groups.

	Sample 1	Sample 2	Sample 3
	6	9	4
	3	5	4
	3	7	2
		8	2
		6	3
SUM	12	35	15
MEAN	4	7	3

n=13

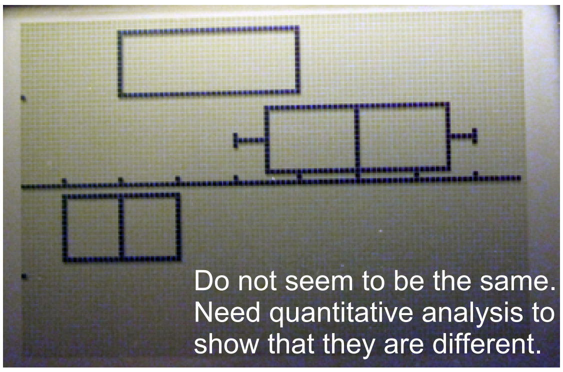
Calculator: --> L1, L2, L3
Box-Plots for all using
STAT Plot (2nd Y=)
and zoom 9

From BoxPlots, do the means all look equal?
Note the variation within each sample set, then
compare variation between (among) sample sets.

16.1 - 16.3 One-Way ANOVA

From BoxPlots, do the means all look equal?
Note the variation within each sample set, then
compare variation between (among) sample sets.

GOAL: Determine if the variation between (among) groups is sufficiently large compared to the variation within groups.
If yes, then can say that the means are not all the same.



sample 1

sample 2

sample 3

16.1 - 16.3 One-Way ANOVA

Need a way to compare variations mathematically:

Treatment $SSTR = \sum (T_i^2 / n_i) - (\sum x)^2 / n$

Total $SST = \sum x^2 - (\sum x)^2 / n$

Error $SSE = SST - SSTR$

Source	df	SS	MS=SS/df	F statistic
Treatment	k - 1	SSTR	MSTR=SSTR/(k-1)	F=MSTR/MSE
Error	n - k	SSE	MSE=SSE/(n-k)	
Total	n - 1	SST		

	x Sample 1	x^2	x Sample 2	x^2	x Sample 3	x^2
	6		9		4	
	3		5		4	
	3		7		2	
			8		2	
			6		3	
SUM T	12		35		15	
MEAN	4		7		3	

Treatment $SSTR = \sum (T_i^2 / n_i) - (\sum x)^2 / n$

Total $SST = \sum x^2 - (\sum x)^2 / n$

Error $SSE = SST - SSTR$

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16.1 - 16.3 One-Way ANOVA

Need a way to compare variations mathematically:

Treatment $SSTR = \sum (T_i^2 / n_i) - (\sum x)^2 / n$

Total $SST = \sum x^2 - (\sum x)^2 / n$

Error $SSE = SST - SSTR$

	x Sample 1	x^2	x Sample 2	x^2	x Sample 3	x^2		
	6	36	9	81	4	16		
	3	9	5	25	4	16		
	3	9	7	49	2	4		
			8	64	2	4		
			6	36	3	9	TOGETHER	
SUM T	12	54	35	255	15	49	62	358
MEAN	4		7		3			

Source	df	SS	MS=SS/df	F=MSTR/MSE F statistic
k - 1 Treatment				
n - k Error				
n - 1 Total				

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16.1 - 16.3 One-Way ANOVA

These computations are now done on the calculator.
Shown here if you want to try to work it out by hand.

$$s = \sqrt{\frac{\Sigma x_i^2 - (\Sigma x_i)^2/n}{n-1}}$$

	$n=3$ Sample 1		$n=5$ Sample 2		$n_3=5$ Sample 3			
	6	36	9	81	4	16		
	3	9	5	25	4	16		
	3	9	7	49	2	4		
			8	64	2	4		
			6	36	3	9	TOGETHER	
SUM	12	54	35	255	15	49	62 Σx	358 Σx^2
MEAN	4		7		3			
s	1.73		1.58		1			

Treatment $SSTR = \sum (T_i^2/n_i) - (\sum x)^2/n$

SSTR = [12²/3 + 35²/5 + 15²/5] - 62²/13
= 338 - 295.692 = **42.308**

Total $SST = \sum x^2 - (\sum x)^2/n$ $(62)^2/13$

SST = 358 - 295.692 = **62.308**

Error $SSE = SST - SSTR$

SSE = 62.308 - 42.308 = **20**

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16.1 - 16.3 One-Way ANOVA

Use the calculator.
Enter data into L1, L2, etc.
Use STAT/TESTS/ANOVA(L1,L2,...)

Treatment $SSTR = \sum (T_i^2/n_i) - (\sum x)^2/n$

SSTR = [12²/3 + 35²/5 + 15²/5] - 62²/13
= 338 - 295.692 = **42.308**

Total $SST = \sum x^2 - (\sum x)^2/n$

SST = 358 - 295.692 = **62.308**

Error $SSE = SST - SSTR$

SSE = 62.308 - 42.308 = **20**

Source	df	SS	MS=SS/df	F=MSTR/MSE F statistic
Treatment k - 1	2	42.308	42.308/2=21.154	21.154/2.0=10.577
Error n - k	10	20.000	2.0	
Total n - 1	12	62.308		

s_{xp} = 1.414 *pooled standard deviation*

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16.1 - 16.3 One-Way ANOVA

One-Way ANOVA

Assumptions: 1. SRS, 2. Independent samples
3. ND or large samples 4. = δ's

Step 1: H_0: $\mu_1 = \mu_2 = \mu_3 = \ldots = \mu_k$
H_a: NOT ALL EQUAL

Step 2: Decide α
1-tailed test

Step 3: Compute SST, SSTR, SSE

Step 4: Construct ANOVA table to compute F statistic.

Source	df	SS	MS=SS/df	F statistic
Treatment	k - 1	SSTR	MSTR=SSTR/(k-1)	F=MSTR/MSE
Error	n - k	SSE	MSE=SSE/(n-k)	
Total	n - 1	SSTR		

var.among / var.within

Step 5: Find CV(s) OR Find p-value
p = 1- Fcdf (0, Ftest, dfTR, dfE)

Step 6: Decide whether to reject H_0 or not

Step 7: Verbal interpretation

16.1 - 16.3 One-Way ANOVA

***Assumptions for** One-Way ANOVA*

Required: 1. SRS, 2. Independent samples

Robust, can be used when only approximately met:

3. ND or large samples
4. Equal σ's
   usually OK to use ANOVA if no sample standard deviation is more than twice as large as any other s

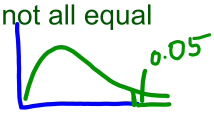
16.1 - 16.3 One-Way ANOVA

To complete the hypothesis test:

① H_0: $\mu_1 = \mu_2 = \mu_3 = \mu_4$

H_a: means are not all equal

② $\alpha = 0.05$



Treatment $SSTR = \sum(T_i^2/n_i) - (\sum x)^2/n$

SSTR = [12²/3 + 35²/5 + 15²/5] - 62²/13 = 338 - 295.692 = **42.308**

Error $SSE = SST - SSTR$

SSE = 62.308 - 42.308 = **20**

Total $SST = \sum x^2 - (\sum x)^2/n$

SST = 358 - 295.692 = **62.308**

③

Source	df	SS	MS=SS/df	F=MSTR/MSE F statistic
Treatment k - 1	2	42.308	42.308/2=21.154	21.154/2.0=10.577
Error n - k	10	20.000	2.0	
Total n - 1	12	62.308		

$10.577 = F_T > 4.10 = F_c$

$F_c = 4.10$

④ p = 1 - Fcdf(0,Ftest,dfTR,dfE) = 0.0034

$S_p = 1.4142$

⑤ $p = 0.0034 < \alpha = 0.05$

rej. H_0.

⑥ conclude that not all the means are equal; amount of enrichment seems to have an impact on the ability to solve a maze.

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16.1 - 16.3 One-Way ANOVA

Movies are independently selected at random. The times for movies rated 1,...4 are recorded.

At the 1% s.l. is there a difference in mean running times among films in the four rating groups?

Treatment $SSTR = \sum(T_i^2/n_i) - (\sum x)^2/n$

Error $SSE = SST - SSTR$

Total $SST = \sum x^2 - (\sum x)^2/n$

H_0: $\mu_1 = \mu_2 = \mu_3 = \mu_4$

H_a: means are not all equal

$\alpha = 0.01$

1	2	3	4
75	97	101	101
95	70	89	135
84	105	97	93
86	119	103	117
58	87	86	126
85	95	100	119
483	573	576	691

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16.1 - 16.3 One-Way ANOVA

Movies are independently selected at random. The times for movies rated 1,...4 are recorded.

At the 1% s.l. is there a difference in mean running times among films in the four rating groups? $\alpha = 0.01$

Treatment $SSTR = \sum (T_i^2 / n_i) - (\sum x)^2 / n$

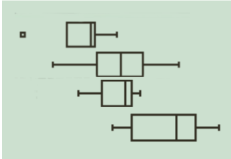
Error $SSE = SST - SSTR$

Total $SST = \sum x^2 - (\sum x)^2 / n$

H_0: $\mu_1 = \mu_2 = \mu_3 = \mu_4$
H_a: means are not all equal

1	2	3	4
75	97	101	101
95	70	89	135
84	105	97	93
86	119	104	117
58	87	80	126
85	95	100	119
483	573	571	691
s 12.7	16.5	9.0	15.6

Box Plots to get overview:

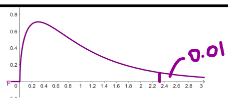


Calculator: --> L1, L2, L3,L4
Box-Plots for all using
STAT Plot (2nd Y=), zoom 9
STAT/TESTS/ANOVA
ANOVA(L1,L2,L3,L4)

16.1 - 16.3 One-Way ANOVA

① H_0: $\mu_1 = \mu_2 = \mu_3 = \mu_4$
H_a: means are not all equal

② $\alpha = 0.01$



③

1	2	3	4
75	97	101	101
95	70	89	135
84	105	97	93
86	119	103	117
58	87	86	126
85	95	100	119
483	573	576	691

⑤

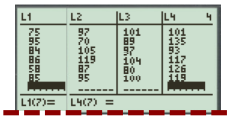
Source	df	SS	MS=SS/df	F=MSTR/MSE F statistic
Treatment	k - 1			
Error	n - k			
Total	n - 1			

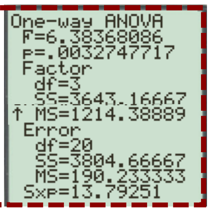
⑥ p =

⑦ p ? α

④

Calculator: --> L1, L2, L3,L4
STAT/TESTS/ANOVA
ANOVA(L1,L2,L3,L4)





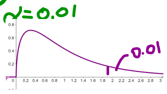
⑧ Rej ?

⑨ conclusion

16.1 - 16.3 One-Way ANOVA

① H_0: $\mu_1 = \mu_2 = \mu_3 = \mu_4$
H_a: means are not all equal

② $\alpha = 0.01$



Treatment $SSTR = \sum(T_i^2/n_i) - (\sum x)^2/n$
Error $SSE = SST - SSTR$
Total $SST = \sum x^2 - (\sum x)^2/n$

③

1	2	3	4
75	97	101	101
95	70	89	135
84	105	97	93
86	119	103	117
58	87	86	126
85	95	100	119
483	*573*	*576*	*691*

⑤

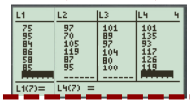
Source	df	SS	MS=SS/df	F=MSTR/MSE F statistic
Treatment (k - 1)	3	3643.17	1214.389	**6.384**
Error (n - k)	20	3804.67	190.233	
Total (n - 1)	23			$F_t = \frac{1214.389}{190.233}$

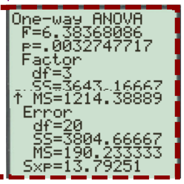
⑥ p = 0.00327
s_p = 13.793

⑦ p ? α
p=0.00327<0.01=α

⑧ Rej ?
REJECT H_o

④ Calculator: --> L1, L2, L3,L4
STAT/TESTS/ANOVA
ANOVA(L1,L2,L3,L4)





At the 1% s.l. is there a difference exists in mean running times among films in the four rating groups?

⑨ conclusion
Conclude:
99.7% confident
that running times for differently rated movies are not all equal, based on this data set.

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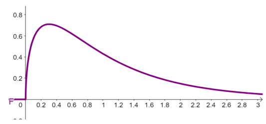
16.1 - 16.3 One-Way ANOVA

Data from independent, random samples.

H_0: $\mu_1 = \mu_2 = \mu_3$
H_a: means are not all equal



1	2	3
1	10	4
9	4	16
	8	10
	6	
	2	

Source	df	SS	MS=SS/df	F=MSTR/MSE F statistic
Treatment (k - 1)				
Error (n - k)				
Total (n - 1)				

How do we handle?
Use ANOVA if assumptions are met.
Are they?

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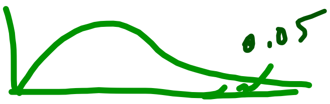
16.1 - 16.3 One-Way ANOVA

Data from independent, random samples.

H_0: $\mu_1 = \mu_2 = \mu_3$

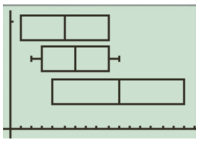
H_a: means are not all equal



1	2	3
1	10	4
9	4	16
	8	10
	6	
	2	

Source	df	SS	MS=SS/df	F=MSTR/MSE F statistic
Treatment (k - 1)				
Error (n - k)				
Total (n - 1)				

s 5.66 3.16 6



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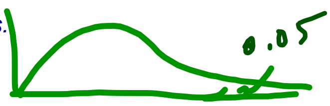
16.1 - 16.3 One-Way ANOVA

Data from independent, random samples.

H_0: $\mu_1 = \mu_2 = \mu_3$

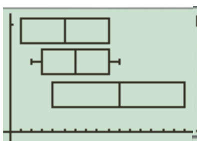
H_a: means are not all equal

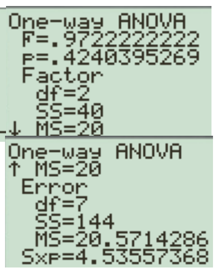


1	2	3
1	10	4
9	4	16
	8	10
	6	
	2	

Source	df	SS	MS=SS/df	F=MSTR/MSE F statistic
Treatment (k - 1)	2	40	20	0.972
Error (n - k)	7	144	20.571	
Total (n - 1)	9			

s 5.66 3.16 6





$p = 0.424$ $s_p = 4.536$

$p = 0.424 > 0.05 = \alpha$

Do NOT rj. H_0.

Conclude: The means are the same. There is insufficient evidence to show that the means are different.

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16.1 - 16.3 One-Way ANOVA

Complete the ANOVA table.

Source	df	SS	MS=SS/df	F=MSTR/MSE F statistic
k - 1 Treatment	4			
n - k Error	20		6.76	
Total n - 1		173.04		

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16.1 - 16.3 One-Way ANOVA

Complete the ANOVA table.

Source	df	SS	MS=SS/df	F=MSTR/MSE F statistic
k - 1 Treatment	4	37.84	9.46	1.40
n - k Error	20	135.2	6.76	
Total n - 1	24	173.04		

Source	df	SS	MS=SS/df	F=MSTR/MSE F statistic
k - 1 Treatment				
n - k Error				
Total n - 1				

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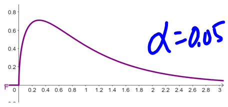
16.1 - 16.3 One-Way ANOVA

Data from independent, random samples.
Are the means the same?

H_0: $\mu_1 = \mu_2 = \mu_3$
H_a: means are not all equal



$\alpha = 0.05$

1	2	3	4	5
7	5	6	3	7
4	9	7	7	9
5	4	5	7	11
4		4	4	
		8	4	

Source	df	SS	MS=SS/df	F=MSTR/MSE F statistic
Treatment k - 1				
Error n - k				
Total n - 1				

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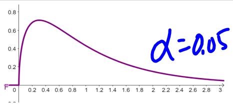
16.1 - 16.3 One-Way ANOVA

H_0: $\mu_1 = \mu_2 = \mu_3$
H_a: means are not all equal

Data from independent, random samples.



$\alpha = 0.05$

1	2	3	4	5
7	5	6	3	7
4	9	7	7	9
5	4	5	7	11
4		4	4	
		8	4	
1.41	2.65	1.58	1.87	2.00

Source	df	SS	MS=SS/df	F=MSTR/MSE F statistic
Treatment k - 1	4	36	9	2.596
Error n - k	15	52	3.47	
Total n - 1	19			

$s_p = 1.862$

$p = 0.079 > 0.05 = \alpha$

Do NOT reject H_o

Conclude: There is no difference in mean values.

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16.1 - 16.3 One-Way ANOVA

Source	df	SS	MS=SS/df	F statistic
Treatment	k - 1	SSTR	MSTR=SSTR/(k-1)	F=MSTR/MSE
Error	n - k	SSE	MSE=SSE/(n-k)	
Total	n - 1	SST		

HW: Complete the ANOVA table

Source	df	SS	MS=SS/df	F=MSTR/MSE F statistic
k - 1 Treatment		2.124	0.708	0.75
n - k Error	20			
n - 1 Total				

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16.1 - 16.3 One-Way ANOVA

Source	df	SS	MS=SS/df	F statistic
Treatment	k - 1	SSTR	MSTR=SSTR/(k-1)	F=MSTR/MSE
Error	n - k	SSE	MSE=SSE/(n-k)	
Total	n - 1	SST		

HW: Complete the ANOVA table

Source	df	SS	MS=SS/df	F=MSTR/MSE F statistic
k - 1 Treatment	3	2.124	0.708	0.75
n - k Error	20			
n - 1 Total				

1. MS=SS/df = 2.124/df = 0.708; df = 2.124 / 0.708 = 3

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16.1 - 16.3 One-Way ANOVA

Source	df	SS	MS=SS/df	F statistic
Treatment	k - 1	SSTR	MSTR=SSTR/(k-1)	F=MSTR/MSE
Error	n - k	SSE	MSE=SSE/(n-k)	
Total	n - 1	SST		

HW: Complete the ANOVA table

Source	df	SS	MS=SS/df	F=MSTR/MSE F statistic
k - 1 Treatment	3	2.124	0.708	0.75
n - k Error	20		0.944	
n - 1 Total				

1. MS=SS/df = 2.124/df = 0.708; df = 2.124 / 0.708 = 3

2. F=MSTR/MSE= 0.708/MSE= 0.75; MSE= 0.708/0.75 = 0.944

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16.1 - 16.3 One-Way ANOVA

Source	df	SS	MS=SS/df	F statistic
Treatment	k - 1	SSTR	MSTR=SSTR/(k-1)	F=MSTR/MSE
Error	n - k	SSE	MSE=SSE/(n-k)	
Total	n - 1	SST		

HW: Complete the ANOVA table

Source	df	SS	MS=SS/df	F=MSTR/MSE F statistic
k - 1 Treatment	3	2.124	0.708	0.75
n - k Error	20	18.880	0.944	
n - 1 Total	23	21.004		

1. MS=SS/df = 2.124/df = 0.708; df = 2.124 / 0.708 = 3

2. F=MSTR/MSE= 0.708/MSE= 0.75; MSE= 0.708/0.75 = 0.944

3. MS=SS/df = SS/20 = 0.944; SS = 0.944*20 = 18.880

4. SST = SSTR + SSE = 2.124 + 18.880 = 21.004

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16.1 - 16.3 One-Way ANOVA

✔*Addendum: F-table & finding critical F values on the calculator.*

This section removed. See 2023.

Study Ch. 16.1, F-Distribution
F-Distribution Table

geogebra

F-Distribution Table
- 8 pages long
- df numerator across top
- df denominator on sides, with
  - α = 0.10, 0.05, 0.025, 0.01, 0.005

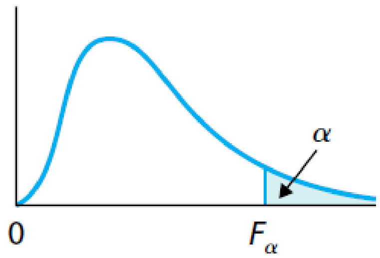
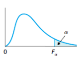


TABLE VIII
Values of F_α



dfd	α	dfn 1	2	3	4	5	6	7	8	9
1	0.10	39.86	49.50	53.59	55.83	57.24	58.20	58.91	59.44	59.86
	0.05	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54
	0.025	647.79	799.50	864.16	899.58	921.85	937.11	948.22	956.66	963.28
	0.01	4052.2	4999.5	5403.4	5624.6	5763.6	5859.0	5928.4	5981.1	6022.5
	0.005	16211	20000	21615	22500	23056	23437	23715	23925	24091
2	0.10	8.53	9.00	9.16	9.24	9.29	9.33	9.35	9.37	9.38
	0.05	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38
	0.025	38.51	39.00	39.17	39.25	39.30	39.33	39.36	39.37	39.39
	0.01	98.50	99.00	99.17	99.25	99.30	99.33	99.36	99.37	99.39
	0.005	198.50	199.00	199.17	199.25	199.30	199.33	199.36	199.37	199.39
3	0.10	5.54	5.46	5.39	5.34	5.31	5.28	5.27	5.25	5.24
	0.05	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81
	0.025	17.44	16.04	15.44	15.10	14.88	14.73	14.62	14.54	14.47
	0.01	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.35
	0.005	55.55	49.80	47.47	46.19	45.39	44.84	44.43	44.13	43.88
4	0.10	4.54	4.32	4.19	4.11	4.05	4.01	3.98	3.95	3.94
	0.05	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00
	0.025	12.22	10.65	9.98	9.60	9.36	9.20	9.07	8.98	8.90
	0.01	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66
	0.005	31.33	26.28	24.26	23.15	22.46	21.97	21.62	21.35	21.14

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Attachments

Statistical Tables.pdf