16.1 - 16.3 One-Way ANOVA

GOALS:
1. Recognize that the F test statistic is a ratio
2. The F distribution is dependent on:
   - df for numerator, and df for denominator
3. ANOVA, or ANalysis Of VAriance, is an extension of a pooled-t test to 3 or more samples:
   "Are the means the same or different?"
4. Assumptions for ANOVA are the same as for the pooled-t test
5. ANOVA uses a right-tailed F test.
6. ANOVA compares variation between groups to the variation within groups to determine if the means of the groups are the same or different

Study Ch. 16.2#13-21, 25
Study Ch. 16.3# 33-41, 43(by formulas), 49-53(by function)

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Statistics Home Page

Consider how we compared means from 2 groups:

**Pooled-t Test**

Assumptions:
1. SRS, 2. Independent samples
3. ND or large samples
4. \( \sigma^2 = \sigma^2 \)

Step 1: 
- \( H_0: \mu_1 = \mu_2 \)
- \( H_a: \mu_1 \neq \mu_2 \) or \( \mu_1 < \mu_2 \) or \( \mu_1 > \mu_2 \)

Step 2: Decide \( \alpha \)

Step 3: Compute test statistic:
\[
t = \frac{(x_1 - x_2)}{\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}
\]
where
\[
s^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}
\]

Step 4: Find CV(s)
OR Find p-value

\[
df = n_1 + n_2 - 2 \quad \text{Table IV}
\]

Step 5: Decide whether to reject \( H_0 \) or not

Step 6: Verbal interpretation

eg: 4 sample sets, one variable (factor) - running times for movies of different ratings:

<table>
<thead>
<tr>
<th>Ratings</th>
<th>1 or 1.5,</th>
<th>2 or 2.5,</th>
<th>3 or 3.5,</th>
<th>4</th>
</tr>
</thead>
</table>

"At the 1% significance level, do the data provide sufficient evidence to conclude that a difference exists in mean running times among films in the four rating groups?"

Comparing means from more than 2 groups.
16.1 - 16.3 One-Way ANOVA

ANOVA: ANalysis Of VAriance vs. pooled - t

Some similarities, some differences

Similarities
1. Assumptions same.
2. Both use pooled s
   (groups have equal s)

Differences: In ANOVA,
1. Pooled s not obvious (hidden in MSE).
2. Compares variation among groups to variation within groups.
3. Uses F distribution instead of the t-distribution.

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Statistics Home Page  Class Notes  Homework
16.1 - 16.3 One-Way ANOVA Compare variation among groups to variation within groups.

A research study was conducted to evaluate a new antidepressant. Patients were randomly assigned to one of three groups: a placebo group, a low dose group, and a moderate dose group. After five weeks of treatment, the patients completed a self-report inventory. The higher the score, the more depressed the patient. At the 1% significance level, is the new medication effective?

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Researchers wanted to know if enrichment enhances learning. They randomly selected rats and assigned them to one of four conditions: Impoverished (housed alone), standard (cage with other rats), enriched (cage with other rats and toys), super enriched (cage with rats and toys changes on a periodic basis). After two months, the rats were tested on a maze and the number of trials to learn the maze recorded. At the 5% significance level, did the enrichment improve learning?

<table>
<thead>
<tr>
<th>Impoverished</th>
<th>Standard</th>
<th>Enriched</th>
<th>Super Enriched</th>
</tr>
</thead>
<tbody>
<tr>
<td>22</td>
<td>17</td>
<td>12</td>
<td>8</td>
</tr>
<tr>
<td>19</td>
<td>21</td>
<td>14</td>
<td>7</td>
</tr>
<tr>
<td>15</td>
<td>15</td>
<td>11</td>
<td>10</td>
</tr>
<tr>
<td>24</td>
<td>12</td>
<td>9</td>
<td>9</td>
</tr>
<tr>
<td>18</td>
<td>19</td>
<td>15</td>
<td>12</td>
</tr>
</tbody>
</table>

At the significance level of 0.05, the p-value for the F-statistic is 0.00017, which is less than 0.05. Therefore, we reject the null hypothesis and conclude that not all means are equal. So, enrichment levels seem to make a difference in learning.
16.1 - 16.3 One-Way ANOVA

Compare variation among groups to variation within groups.

<table>
<thead>
<tr>
<th>Sample 1</th>
<th>Sample 2</th>
<th>Sample 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>9</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
<td>2</td>
</tr>
<tr>
<td>8</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>

SUM: 12, 35, 15
MEAN: 4, 7, 3

\[ n = 3 \]

Calculator: \( \rightarrow \) L1, L2, L3
Box-Plots for all using
STAT Plot (2nd Y=)
and zoom 9

From BoxPlots, do the means all look equal?
Note the variation within each sample set, then compare variation between (among) sample sets.

GOAL: Determine if the variation between (among) groups is sufficiently large compared to the variation within groups.
If yes, then can say that the means are not all the same.

sample 1
sample 2
sample 3

Do not seem to be the same.
Need quantitative analysis to show that they are different.
16.1 - 16.3 One-Way ANOVA

Need a way to compare variations mathematically:

Treatment: \( SSTR = \sum \left( \frac{T_i^2}{n_i} \right) - \left( \frac{\sum x^2}{n} \right) \)

Total: \( SST = \sum x^2 - \left( \frac{\sum x^2}{n} \right) \)

Error: \( SSE = SST - SSTR \)

---

### Treatment

<table>
<thead>
<tr>
<th>Sample 1 ( x^2 )</th>
<th>Sample 2 ( x^2 )</th>
<th>Sample 3 ( x^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>9</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>4</td>
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<td>3</td>
<td>7</td>
<td>2</td>
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<td>8</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>

**SUM**: 12 \( \sum = 35 \) \( \sum = 15 \)

**MEAN**: 4 \( \bar{x} = 7 \) \( \bar{x} = 3 \)

---

### Total

\( SSTR = \sum \left( \frac{T_i^2}{n_i} \right) - \left( \frac{\sum x^2}{n} \right) \)

\( SST = \sum x^2 - \left( \frac{\sum x^2}{n} \right) \)

\( SSE = SST - SSTR \)

---

### Comparison

<table>
<thead>
<tr>
<th>Source</th>
<th>df</th>
<th>SS</th>
<th>MS = SS/df</th>
<th>F-statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment</td>
<td>( k-1 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Error</td>
<td>( n-k )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>( n-1 )</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
16.1 - 16.3 One-Way ANOVA

\[ s = \sqrt{\frac{\sum x_i^2 - (\sum x_i)^2/n}{n-1}} \]

<table>
<thead>
<tr>
<th>Sample 1</th>
<th>Sample 2</th>
<th>Sample 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>6</td>
<td>9</td>
<td>2</td>
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<tr>
<td>3</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>6</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>[ \sum ]</td>
<td>12</td>
<td>54</td>
</tr>
<tr>
<td>Mean</td>
<td>4</td>
<td>7</td>
</tr>
<tr>
<td>[ \sum x ]</td>
<td>48</td>
<td>255</td>
</tr>
<tr>
<td>[ \sum x^2 ]</td>
<td>358</td>
<td>295.692</td>
</tr>
</tbody>
</table>

Treatment \[ SSTR = \sum \left( T_i^2/n_i \right) - \left( \sum x \right)^2/n \]
\[ SSTR = \left[ 12^2/3 + 35^2/5 + 15^2/5 \right] - 62^2/13 \]
\[ = 338 - 295.692 = 42.308 \]

Total \[ SST = \sum x^2 - \left( \sum x \right)^2/n \]
\[ SST = 358 - 295.692 = 62.308 \]

Error \[ SSE = SST - SSTR \]
\[ SSE = 62.308 - 42.308 = 20 \]

\[ F = \frac{MSTR}{MSE} \]
\[ F = \frac{42.308/2}{21.154/2} = 10.577 \]

\[ s_{xp} = 1.414 \quad \text{pooled standard deviation} \]
### One-Way ANOVA

**Assumptions:**
1. SRS
2. Independent samples
3. ND or large samples
4. $\bar{\sigma} = \bar{\sigma}'$

**Step 1:**
- $H_0$: $\mu_1 = \mu_2 = \mu_3 = \cdots = \mu_k$
- $H_1$: NOT ALL EQUAL

**Step 2:** Decide $\alpha$

**Step 3:** Compute SST, SSTR, SSE

**Step 4:** Construct ANOVA table to compute F statistic.

**Step 5:** Find CV(s) OR Find p-value

$p = 1 - \text{Fcdf}(0, \text{Ftest}, \text{df}_{TR}, \text{df}_E)$

**Step 6:** Decide whether to reject $H_0$ or not

**Step 7:** Verbal interpretation

---

**Assumptions for One-Way ANOVA**

**Required:**
1. SRS
2. Independent samples

Robust, can be used when only approximately met:

3. ND or large samples
4. Equal $\sigma$'s

usually OK to use ANOVA if no sample standard deviation is more than twice as large as any other $s$

---

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16.1 - 16.3 One-Way ANOVA

To complete the hypothesis test:

1. $H_0$: $\mu_1 = \mu_2 = \mu_3 = \mu_4$
2. $\alpha = 0.05$
3. $p = 1 - \text{Fcdf}(0, F_{test}, df_{TR}, df_E) = 0.0034$
4. $q = 0.0034 < \alpha = 0.05$
5. $r > q$, $H_0$ reject
6. conclude that not all the means are equal

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Total

<table>
<thead>
<tr>
<th>Source</th>
<th>df</th>
<th>SS</th>
<th>MS=SS/df</th>
<th>F=MSR/MSR</th>
<th>F statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>k - 1</td>
<td>2</td>
<td>42.308</td>
<td>21.654</td>
<td>21.154/2.0=10.577</td>
<td></td>
</tr>
<tr>
<td>Error</td>
<td>10</td>
<td>62.308</td>
<td>2.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>12</td>
<td></td>
<td>2.0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Treatment $SSTR = \sum(x_i^2/n_i) - (\sum x)^2/n$ 
$SSTR = 12/3 = 35/5 + 15/5) = 62.308$ 
$SS = 358 - 295.692 = 62.308$ 
$SSE = SST - SSSTR$ 
$SST = 358 - 295.692 = 62.308$ 
$SSE = 62.308 - 42.308 = 20.000$ 
$F_{1,2} = 42.308/2 = 21.154$ 
$p = 1 - \text{Fcdf}(0, F_{test}, df_{TR}, df_E) = 0.0034$ 

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16.1 - 16.3 One-Way ANOVA

Movies are independently selected at random. The times for movies rated 1,...,4 are recorded. At the 1% s.l. is there a difference exists in mean running times among films in the four rating groups?

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>75</td>
<td>97</td>
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<td>95</td>
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<tr>
<td>84</td>
<td>105</td>
<td>97</td>
<td>93</td>
</tr>
<tr>
<td>86</td>
<td>119</td>
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<td>117</td>
</tr>
<tr>
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<td>87</td>
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<td>85</td>
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</table>

483 573 576 691

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<table>
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<tr>
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<tr>
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483 573 576 691

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16.1 - 16.3 One-Way ANOVA

Movies are independently selected at random. The times for movies rated 1...4 are recorded. At the 1% s.l. is there a difference in mean running times among films in the four rating groups?

$H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4$

$H_a$: means are not all equal

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<tr>
<td>4</td>
<td>86</td>
<td>119</td>
<td>104</td>
<td>117</td>
</tr>
</tbody>
</table>

$\bar{x} = 80.75$, $s = 12.7$

$T_{SSTR} = 691$, $T_{SSE} = 576$, $T_{SSR} = 115$, $T_{MSR} = 115 / 3 = 38.33$, $T_{MSE} = 576 / 9 = 64$,

$F = 38.33 / 64 = 0.62$

At the 1% s.l., there is no significant difference in mean running times among films in the four rating groups.

Calculators:

- L1, L2, L3, L4
- STAT/TESTS/ANOVA
- ANOVA(L1, L2, L3, L4)
16.1 - 16.3 One-Way ANOVA

Data from independent, random samples.

H₀: μ₁ = μ₂ = μ₃
Hₐ: means are not all equal

<table>
<thead>
<tr>
<th>Source</th>
<th>df</th>
<th>SS</th>
<th>MS=SS/df</th>
<th>F=MSTR/MSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>k-1</td>
<td>n-k</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Error</td>
<td>n-k</td>
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<tr>
<td>Total</td>
<td>n-1</td>
<td>463</td>
<td>3643.17</td>
<td>1214.389</td>
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</table>

<table>
<thead>
<tr>
<th>Treatment</th>
<th>k-1</th>
<th>Treatment</th>
<th>k-1</th>
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<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Total: 463

How do we handle?
Use ANOVA if assumptions are met.
Are they?

At the 1% s.l. is there a difference exists in mean running times among films in the four rating groups?

p = 0.00327

p=0.00327<0.01=α

Rej ?
REJECT H₀

Conclusion:
99.7% confident that running times for differently rated movies are not all equal, based on this data set.
16.1 - 16.3 One-Way ANOVA

Data from independent, random samples.

$H_0$: $\mu_1 = \mu_2 = \mu_3$

$H_a$: means are not all equal

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>4</td>
<td>16</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

s 5.66 3.16 6

Conclude: The means are the same.
There is insufficient evidence to show that the means are different.
Complete the ANOVA table.

<table>
<thead>
<tr>
<th>Source</th>
<th>df</th>
<th>SS</th>
<th>MS = SS/df</th>
<th>F = MSTR/MSE</th>
<th>F statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>k - 1 Treatment</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Error</td>
<td>20</td>
<td></td>
<td>6.76</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>24</td>
<td>173.04</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

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Statistics Home Page  Class Notes  Homework
### 16.1 - 16.3 One-Way ANOVA

Data from independent, random samples. Are the means the same?

**H\(_0\):** \(\mu_1 = \mu_2 = \mu_3\)  
**H\(_a\):** means are not all equal

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>5</td>
<td>6</td>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td>9</td>
<td>7</td>
<td>7</td>
<td>9</td>
</tr>
<tr>
<td>5</td>
<td>4</td>
<td>5</td>
<td>7</td>
<td>11</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>8</td>
</tr>
</tbody>
</table>

\[ s_p = 1.862 \]

\[ p = 0.079 > 0.05 = \alpha \]

**Do NOT reject** \(H_0\)

Conclude: There is no difference in mean values.
16.1 - 16.3 One-Way ANOVA

HW: Complete the ANOVA table

<table>
<thead>
<tr>
<th>Source</th>
<th>df</th>
<th>SS</th>
<th>MS=SS/df</th>
<th>F statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment</td>
<td>3</td>
<td>2.124</td>
<td>0.708</td>
<td>0.75</td>
</tr>
<tr>
<td>Error</td>
<td>20</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>n-1</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1. MS=SS/df = 2.124/df = 0.708; \( df = \frac{2.124}{0.708} = 3 \)
HW: Complete the ANOVA table

<table>
<thead>
<tr>
<th>Source</th>
<th>df</th>
<th>SS</th>
<th>MS=SS/df</th>
<th>F=MSTR/MSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment</td>
<td>k - 1</td>
<td>2.124</td>
<td>0.708</td>
<td>0.75</td>
</tr>
<tr>
<td>Error</td>
<td>n - k</td>
<td>18.880</td>
<td>0.944</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>n - 1</td>
<td>21.004</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1. MS=SS/df = 2.124/df = 0.708; df = 2.124 / 0.708 = 3
2. F=MSTR/MSE= 0.708/MSE= 0.75; MSE= 0.708/0.75 = 0.944

3. MS=SS/df = SS/20 = 0.944; SS = 0.944*20 = 18.880
4. SST = SSTR + SSE = 2.124 + 18.880 = 21.004
16.1 - 16.3 One-Way ANOVA

Study Ch. 16.1, F-Distribution
F-Distribution Table

F-Distribution Table
8 pages long
df numerator across top
df denominator on sides, with
$\alpha = 0.10, 0.05, 0.025, 0.01, 0.005$

Addendum: F-table & finding critical F values on the calculator.

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16.1 - 16.3 One-Way ANOVA

F-Distribution on Calculator:
Fpdf (x, df numerator, df denominator)
Fcdf (lower, upper, df numerator, df denominator)
no invF to get Fα

To get Fα on the calculator, use a function that represents area in the tail.
1. Let Y1 = 1 - Fcdf (0,x,dfn,dfd)
dfn = df numerator, dfd = df denominator
for this graph, y is the area in the tail of the F function for each x,
where x is the actual Fα.
Let Y2 = 0.05 (or 0.025, etc) this is α, the s.l.
2. Use window: xMin -0.5 yMax 10
   yMin -0.1 yMax 1.1
3. 2nd/calc / 5 intersect
   Arrow up or down to select the graphs that intersect, select by hitting enter. Also hit enter at guess.
4. The x coordinate of the point of intersection is the critical value, Fα, for the given df (dfn,dfd)

Addendum: F-table & finding critical F values on the calculator.

Use this procedure if:
1. using critical value approach, and
2. no access to table
Not required if using a p-value approach

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16.1 - 16.3 One-Way ANOVA

G: df (6, 10)

F: a) $F_{0.05}$
   b) $F_{0.01}$
   c) $F_{0.025}$

Critical value approach

Addendum: F-table & finding critical $F$ values on the calculator.

To get $F_\alpha$ on the calculator, use a function that represents area in the tail.
1. Let $Y_1 = 1 - \text{Fcdf}(0,x,6,10)$
   dfn = df numerator, dfd= df denominator
   for this graph, $y$ is the area in the tail of the $F$ function for each $x$, where $x$ is the actual $F_\alpha$
   Let $Y_2 = 0.05$ (or 0.025, etc) this is $\alpha$, the s.l.
2. Use window: xMin -0.5 yMax 10
   yMin -0.1 yMax 1.1
3. 2nd/calc / 5 intersect
   Arrow up or down to select the graphs that intersect, select by hitting enter. Also hit enter at guess.
4. The x coordinate of the point of intersection is the critical value, $F_{\alpha}$, for the given df (6,10)
To get $F_\alpha$ on the calculator, use a function that represents area in the tail.

1. Let $Y_1 = 1 - \text{Fcdf} (0,x,6,10)$
   - dfn = df numerator, ddf = df denominator
   - for this graph, $y$ is the area in the tail of the $F$ function for each $x$,
     where $x$ is the actual $F_\alpha$
   - Let $Y_2 = 0.01$ (or 0.025, etc) this is $\alpha$, the s.l.

2. Use window: $x\text{Min} = -0.5$ $y\text{Max} = 10$
   - $y\text{Min} = -0.1$ $y\text{Max} = 1.1$

3. 2nd/calc / 5 intersect
   - Arrow up or down to select the graphs that intersect, select by hitting enter. Also hit enter at guess.

4. The $x$ coordinate of the point of intersection is the critical value, $F_{0.01}$ for the given df $(6,10)$