10.5 Paired *t* - test

GOALS:

1. Recognize problems with paired data.
2. Consider the difference of paired data, d.
3. Analyze normally distributed paired data using a single sample t-test and H_O: $\mu_1 = \mu_2$ or $d = 0$.

Study Ch. 10.5, # 139, 141,146, 149, 153, 155, 157
[127-135, 139-143]

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10.5 Paired *t* - test

What is similar about these problems that is different from the other 2-sample tests?

142. "additional sleep in hours obtained by 10 patients who used laevohysocyamine hydrobromide."
At the 5% s.l. ... to conclude that laevohysocyamine hydrobromide is effective in increasing sleep?

143. "weights in pounds of 17 anorexic young women before and after receiving a family therapy... ."
At the 5% s.l. ... does family therapy appear to be effective in helping anorexic women gain weight?

144. "11 tires were each measured for treadwear by 2 methods, one based on weight and the other on groove wear... ." At the 5% s.l. ... do the 2 measurement methods give different results?

146. 14 subjects drank 240 ml fortified orange juice per day. Concentrations of serum 25(OH)D at beginning and end of 12 weeks... . At the 1% s.l. ... does drinking fortified OJ increase serum 25(OH)D?

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10.5 Paired *t* - test

What is similar about these problems that is different from the other 2-sample tests?

# 142. "additional sleep in hours obtained by 10 patients who used laevohysocyamine hydrobromide." At the 5% s.l. ... to conclude that laevohysocyamine hydrobromide is effective in increasing sleep?	same patients - with and w/o drug
# 143. "weights in pounds of 17 anorexic young women before and after receiving a family therapy... ." At the 5% s.l. ... does family therapy appear to be effictive in helping anorexic women gain weight?	same women - with and w/o therapy
# 144. "11 tires were each measured for treadwear by 2 methods, one vased on weight and the other on groove wear... ." At the 5% s.l. ... do the 2 measurement methods give different results?	same tires - different type measurements
# 146. 14 subjects drank 240 ml fortified orange juice per day. Concentrations of serum 25(OH)D at beginning and end of 12 weeks... . At the 1% s.l. ... does drinking fortified OJ increase serum 25(OH)D?	same patients - before and after fortified OJ

By using the same individual:
remove differences between individuals
---> less variation, smaller sampling error
---> more likely to detect differences.

Same individual means that the data is dependent, NOT independent

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10.5 Paired *t* - test

By using the same individual:
remove differences between individuals
---> less variation, smaller sampling error
---> more likely to detect differences.

HOW?

Find differences for **each pair**,
d = sample1 - sample2

or: sample2 - sample1

find average $\bar{d}$, and s_d

then use **1 sample** *t*-test.

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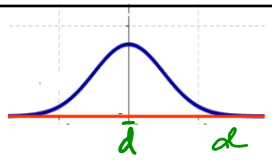
10.5 Paired *t* - test

Paired *t* - *Test*



Assumptions: 1. SR paired sample
2. Normal differences or large sample

Step 1: H_0: $\mu_1 = \mu_2$ $d = 0$

H_a: $\mu_1 \neq \mu_2$ or $\mu_1 < \mu_2$ or $\mu_1 > \mu_2$

$d \neq 0$ $d < 0$ $d > 0$

Step 2: Decide α

Step 3: Construct Table and calculate the paired differences

Sample 1	Sample 2	S1 - S2

d

Step 4: Find test statistic

$$t = \frac{\bar{d}}{s_d / \sqrt{n}}$$

Step 5: p-value from calculator

Step 6: Decide: reject H_0 or not?

Reject if $p < \alpha$

Step 7: Verbal interpretation

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10.5 Paired *t* - test

8 students were given a diagnostic test before studying the Pythagorean Theorum and again after completing the topic. Their scores on the tests are listed below.

Calculator Instructions

At the 5% s.l., do the data provide sufficient evidence to conclude that, on average, studying improved their understanding of the Pythagorean Theorum?

d = post - pre

	pre-Test	Post-Test	*d*
1	18	22	
2	16	17	
3	22	24	
4	17	20	
5	14	15	
6	18	18	
7	12	18	
8	17	16	

p. 538: Paire
Assumptions: 1. SR paire
2. Normal

Step 1: H_0: $\mu_1 = \mu_2$
H_a: $\mu_1 \neq \mu_2$ or μ

Step 3: Construct Table and calculate the paired differences

Step 4: Find test statistic

$t =$
s

Step 5: Find CV(s) **using df =** **and Table IV.**

Step 6: Decide: reject H_0
Reject if test statistic i
rejection region

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2-tailed, t=3.866, p=0.0031<0.05=a, rej null, mean d=3.755, n=11

10.5 Paired *t* - test

8 students were given a diagnostic test before studying the Pythagorean Theorum and again after completing the topic. Their scores on the tests are listed below.

Calculator Instructions

1. same person
2. NPP, d: ~ n.d.
Use *paired-t test*

At the 5% s.l., do the data provide sufficient evidence to conclude that, on average, studying improved their understanding of the Pythagorean Theorum?

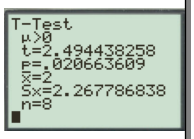
H_0: d=0
H_a: d>0

df = n-1 = 7

	pre-Test	Post-Test	*d = post - pre*
1	18	22	4
2	16	17	1
3	22	24	2
4	17	20	3
5	14	15	1
6	18	18	0
7	12	18	6
8	17	16	-1

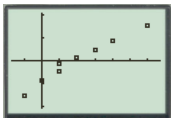
$$t_T = \frac{\bar{d}}{s_d / \sqrt{n}}$$

$$t_T = \frac{2.0}{2.27 / \sqrt{8}} = 2.494$$



p = 0.021 < 0.05 = α

Reject H_0



~n.d.

Conclude that have strong evidence that the study worked to improve student's scores.

Step 1: H
Step 3: C
Step 4: Fi
Step 5: Fi
Step 6: D

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2-tailed, t=3.866, p=0.0031<0.05=a, rej null, mean d=3.755, n=11

10.5 Paired *t* - test

Tires, randomly selected and n.d., measured for tread wear, 2 different methods:

Calculator Instructions

Weight	Groove	*d* W - G
30.5	28.7	
30.9	25.9	
31.9	23.3	
30.4	23.1	
27.3	23.7	
20.4	20.9	
24.5	16.1	
20.9	19.9	
18.9	15.2	
13.7	11.5	
11.4	11.2	

At the 5% s.l., do the data provide sufficient evidence to conclude that, on average, the two methods give different results?

p. 538: Paired *t* - Test

Assumptions: 1. SR paired sample
2. Normal differences or large sample

Step 1: H_0: $\mu_1 = \mu_2$ Step 2: Decide
H_a: $\mu_1 \neq \mu_2$ or $\mu_1 < \mu_2$ or $\mu_1 > \mu_2$

Step 3: Construct Table and calculate the paired differences

Sample 1	Sam

Step 4: Find test statistic

$$t = \frac{\bar{d}}{s_d / \sqrt{n}}$$

Step 5: Find CV(s) **using df = n - 1 and Table IV.**

Step 6: Decide: reject H_0 or not?
Reject if test statistic is in rejection region (tail).

Step 5: p
Step 6: Reject if

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2-tailed, t=3.866, p=0.0031<0.05=a, rej null, mean d=3.755, n=11

10.5 Paired *t* - test

Calculator Instructions

Tires measured for tread wear, 2 different methods:

Weight	Groove	W - G (*d*)
30.5	28.7	1.8
30.9	25.9	5
31.9	23.3	8.6
30.4	23.1	7.3
27.3	23.7	3.6
20.4	20.9	-0.5
24.5	16.1	8.4
20.9	19.9	1
18.9	15.2	3.7
13.7	11.5	2.2
11.4	11.2	0.2

At the 5% s.l., do the data provide sufficient evidence to conclude that, on average, the two methods give different results?

① $H_0: \mu_w = \mu_G$; $H_a: \mu_w \neq \mu_G$ (.025, .025)

② $\alpha = 0.05$ $df = n - 1 = 10$

③ $t = \frac{\bar{d}}{s_d/\sqrt{n}} = \frac{3.75}{3.321/\sqrt{11}} = 3.866$

④ $p = 0.0031$

⑤ $p ? \alpha$ $p = 0.0031 < \alpha = 0.05$ ∴ reg. H_0.

⑥ Conc. methods of tire wear measure are diff.

very strong evidence that the measurement methods are different

p. 538: Paired *t* - *Test*

Assumptions: 1. SR paired sample
2. Normal differences or large sample

Step 1: H_0: $\mu_1 = \mu_2$ H_a: $\mu_1 \neq \mu_2$ or $\mu_1 < \mu_2$ or $\mu_1 > \mu_2$

Step 2: Decide α

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Sample 1	Sample 2

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Step 5: p-value from calculator

Step 6: Decide: reject H_0 or not? Reject if $p < \alpha$

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2-tailed, t=3.866, p=0.0031<0.05=a, rej null, mean d=3.755, n=11

10.5 Paired *t* - test

Tobacco leaf viruses: 2 types

8 randomly selected leaves infected with both types: each on half of leaf

Is there a difference in the number of lesions from the 2 viruses?

Leaf	Virus 1	Virus 2	V1 - V2
1	31	18	
2	20	17	
3	18	14	
4	17	11	
5	9	10	
6	8	7	
7	10	5	
8	7	6	

normal?

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10.5 Paired t - test

Tobacco leaf viruses: 2 types
8 leaves infected with both types: each on half of leaf
Is there a difference in the number of lesions from the 2 viruses?

0

$H_0: \mu_1 = \mu_2$

$H_a: \mu_1 \neq \mu_2$

$d =$

Leaf	Virus 1	Virus 2	V1 - V2
1	31	18	13
2	20	17	3
3	18	14	4
4	17	11	6
5	9	10	-1
6	8	7	1
7	10	5	5
8	7	6	1

$$t = \frac{\bar{d}}{s_d/\sqrt{n}} = \frac{4}{4.309/\sqrt{8}}$$

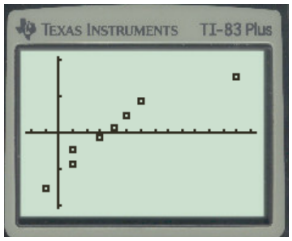
$t = 2.625$

$p = 0.034 < 0.05 = \alpha$

reject H_0 for $\alpha = 0.05$
not for $\alpha = 0.01$

Concl. that result of 10 based leaves to the two viruses is different

normal prob plot ~ normal



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