## 10.4 Mann-Whitney Test: n1, n2 small, same shape

Ages of Statistics students: independent random samples from evening and day classes.

At the 5% significance level, are students in the evening classes older than those in the day classes?

| Eve | Day |
|---|---|
| 49 | 18 |
| 19 | 18 |
| 41 | 19 |
| 21 | 31 |
| 20 | 25 |
| 23 | 18 |
| 20 | 20 |
| 18 | |
| 19 | |

How do we solve this?

Which procedure do we use?

2 samples: Can we use

pooled-t? or non-pooled-t?

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| 49 | 18 |
| 19 | 18 |
| 41 | 19 |
| 21 | 31 |
| 20 | 25 |
| 23 | 18 |
| 20 | 20 |
| 18 | |
| 19 | |

How do we solve this?

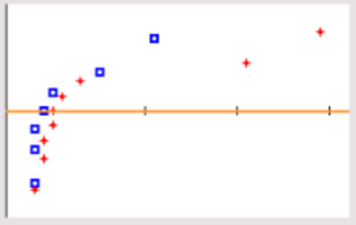
Which procedure do we use?

2 samples: Can we use

pooled-t? or non-pooled-t?

A: srs ✓ independent ✓ nd or ~~n> 30~~ ?

= $\sigma$'s or not



NPP not straight line.
NO! Not nd!

Cannot use pooled-t or non-pooled-t!

✓

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# 10.4 Mann-Whitney Test: n1, n2 small, same shape

GOALS: Perform a Mann-Whitney Test for 2 samples

1. Assumptions: data is not normally distributed and from small samples.
2. Use overall ranking to rank the data in the 2 data sets while keeping the sets separate.
3. Compute the test statistic, $M_T$, as the *sum of the ranks* of the first data set.
4. Use *critical values* as the criteria for hyp testing.
   Find the *right critical value* using **Table VI**.
   Find a *left critical value* using the right critical value.
6. Determining if $M_T$ is in the rejection region.

Study Ch. 10.4, # 105,106,107,111,112;
113-121, 2 of 123-127 [95-107,109 or 111,113]

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10.4 Mann-Whitney Test: n1, n2 small, same shape

## Mann- Whitney Test

p. 497

Small n

Assumptions: 1. SRS, 2. Independent samples
3. Populations have same shape

Use calc to check 1. not nd 2. same shape

Step 1: $H_0$: $\mu_1 = \mu_2$

$H_a$: $\mu_1 \neq \mu_2$ or $\mu_1 < \mu_2$ or $\mu_1 > \mu_2$

(Can also be used to test the median, $H_0$: $\eta_1 = \eta_2$ )

Step 2: Decide $\alpha$

$\eta_1 = \eta_2$

Step 3: Construct Table
* sort each sample separately
* find overall rank

| Sample 1 | overall Rank | Sample 2 | overall Rank |
|---|---|---|---|
| | | | |

Step 4: Find test statistic

$$M_T = \Sigma \text{ ranks, population } 1$$

Step 5: Find CV(s) **using Table VI and:**

$$M_{1-\alpha} = n_1(n_1+n_2+1) - M_\alpha$$

$$M_{1-\alpha/2} = n_1(n_1+n_2+1) - M_{\alpha/2}$$

or

get $M_R$ from Table VI

$$M_L = n_1(n_1+n_2+1) - M_R$$

Step 6: Decide whether to reject $H_0$ or not

Step 7: Verbal interpretation

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## 10.4 Mann-Whitney Test: n1, n2 small, same shape

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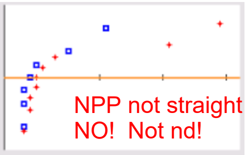

could also do box plot

| Eve | Day |
|---|---|
| 49 | 18 |
| 19 | 18 |
| 41 | 19 |
| 21 | 31 |
| 20 | 25 |
| 23 | 18 |
| 20 | 20 |
| 18 | |
| 19 | |

A: srs independent same shape

✓
✓
✓

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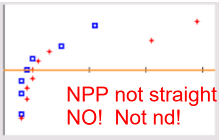

A: srs ✓ independent ✓ same shape ✓

### Use Mann-Whitney (MW)

Already on calc, so sort each and rewrite.

n1= 9, n2= 7 --> need list of 16 ranks.

Ages of Stats Students

| Eve | RANK | Day | RANK |
|---|---|---|---|
| 18 | 2.5 | 18 | 2.5 |
| 19 | 6 | 18 | 2.5 |
| 19 | 6 | 18 | 2.5 |
| 20 | 9 | 19 | 6 |
| 20 | 9 | 20 | 9 |
| 21 | 11 | 25 | 13 |
| 23 | 12 | 31 | 14 |
| 41 | 15 | | |
| 49 | 16 | | |

1
2
3
4 | 2.5
5
6
7 | 6
8
9
10 | 9
11
12
13
14
15
16

Note: 4 18's hold ranks 1 to 4, so each gets a rank of 2.5

Also: 3 19's hold ranks 5 to 7, so each gets a rank of 6

And: 3 20's hold ranks 8 to 10, so each gets a rank of 9

With no other duplicates, each gets the next rank in sequence, starting with 11, because 10 has already been used.

This column keeps track of which ranks have already been used. if there are no duplicate ranks, this is not needed.

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## 10.4 Mann-Whitney Test: n1, n2 small, same shape

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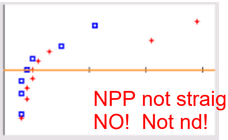
NPP not straight li
NO! Not nd!

A: srs ✓ independent ✓ same shape ✓

Use Mann-Whitney (MW)

Already on calc, so sort each and rewrite.

Ages of Stats Students

| Eve | RANK | Day | RANK |
|---|---|---|---|
| 18 | 2.5 | 18 | 2.5 |
| 19 | 6 | 18 | 2.5 |
| 19 | 6 | 18 | 2.5 |
| 20 | 9 | 19 | 6 |
| 20 | 9 | 20 | 9 |
| 21 | 11 | 25 | 13 |
| 23 | 12 | 31 | 14 |
| 41 | 15 | | |
| 49 | 16 | | |

$H_o$: $\mu1 = \mu2$ $H_a$: $\mu1 > \mu2$

Which sketch option?
(No known distribution, so just use the number line.)

n1= 9, n2= 7 --> need list of 16 rank

Note: 4 18's hold ranks 1 to 4, so each gets a rank of 2.5

Also: 3 19's hold ranks 5 to 7, so each gets a rank of 6

And: 3 20's hold ranks 8 to 10, so each gets a rank of 9

With no other duplicates, each gets the next rank in sequence, starting with 11, because 10 has already been used.

1
2 2.5
3
4
5
6 6
7
8
9 9
10
11
12
13
14
15
16

$M_T = \sum(\text{ranks})$

$= 86.5$

need to compare this to a critical value, $M_R$, from Table 4

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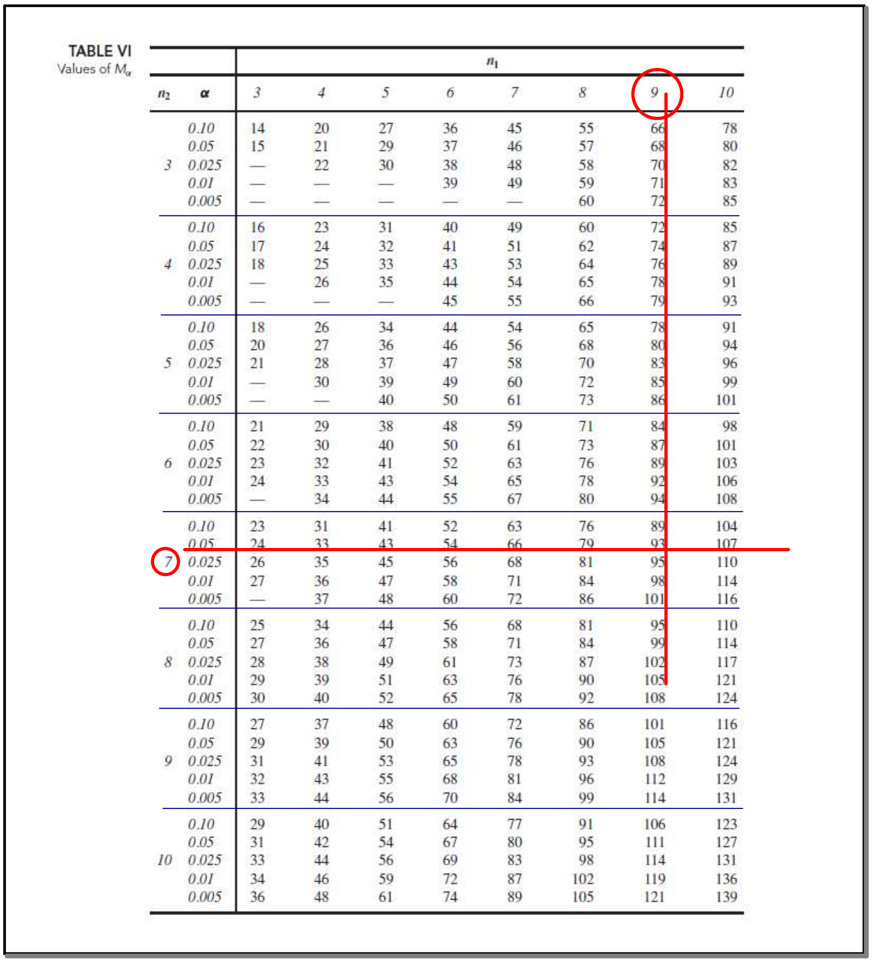


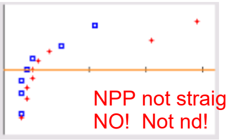
TABLE VI
Values of $M_\alpha$

| | | $n_1$ | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $n_2$ | α | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 3 | 0.10 | 14 | 20 | 27 | 36 | 45 | 55 | 66 | 78 |
| | 0.05 | 15 | 21 | 29 | 37 | 46 | 57 | 68 | 80 |
| | 0.025 | — | 22 | 30 | 38 | 48 | 58 | 70 | 82 |
| | 0.01 | — | — | — | 39 | 49 | 59 | 71 | 83 |
| | 0.005 | — | — | — | — | — | 60 | 72 | 85 |
| 4 | 0.10 | 16 | 23 | 31 | 40 | 49 | 60 | 72 | 85 |
| | 0.05 | 17 | 24 | 32 | 41 | 51 | 62 | 74 | 87 |
| | 0.025 | 18 | 25 | 33 | 43 | 53 | 64 | 76 | 89 |
| | 0.01 | — | 26 | 35 | 44 | 54 | 65 | 78 | 91 |
| | 0.005 | — | — | — | 45 | 55 | 66 | 79 | 93 |
| 5 | 0.10 | 18 | 26 | 34 | 44 | 54 | 65 | 78 | 91 |
| | 0.05 | 20 | 27 | 36 | 46 | 56 | 68 | 80 | 94 |
| | 0.025 | 21 | 28 | 37 | 47 | 58 | 70 | 83 | 96 |
| | 0.01 | — | 30 | 39 | 49 | 60 | 72 | 85 | 99 |
| | 0.005 | — | — | 40 | 50 | 61 | 73 | 86 | 101 |
| 6 | 0.10 | 21 | 29 | 38 | 48 | 59 | 71 | 84 | 98 |
| | 0.05 | 22 | 30 | 40 | 50 | 61 | 73 | 87 | 101 |
| | 0.025 | 23 | 32 | 41 | 52 | 63 | 76 | 89 | 103 |
| | 0.01 | 24 | 33 | 43 | 54 | 65 | 78 | 92 | 106 |
| | 0.005 | — | 34 | 44 | 55 | 67 | 80 | 94 | 108 |
| 7 | 0.10 | 23 | 31 | 41 | 52 | 63 | 76 | 89 | 104 |
| | 0.05 | 24 | 33 | 43 | 54 | 66 | 79 | 93 | 107 |
| | 0.025 | 26 | 35 | 45 | 56 | 68 | 81 | 95 | 110 |
| | 0.01 | 27 | 36 | 47 | 58 | 71 | 84 | 98 | 114 |
| | 0.005 | — | 37 | 48 | 60 | 72 | 86 | 101 | 116 |
| 8 | 0.10 | 25 | 34 | 44 | 56 | 68 | 81 | 95 | 110 |
| | 0.05 | 27 | 36 | 47 | 58 | 71 | 84 | 99 | 114 |
| | 0.025 | 28 | 38 | 49 | 61 | 73 | 87 | 102 | 117 |
| | 0.01 | 29 | 39 | 51 | 63 | 76 | 90 | 105 | 121 |
| | 0.005 | 30 | 40 | 52 | 65 | 78 | 92 | 108 | 124 |
| 9 | 0.10 | 27 | 37 | 48 | 60 | 72 | 86 | 101 | 116 |
| | 0.05 | 29 | 39 | 50 | 63 | 76 | 90 | 105 | 121 |
| | 0.025 | 31 | 41 | 53 | 65 | 78 | 93 | 108 | 124 |
| | 0.01 | 32 | 43 | 55 | 68 | 81 | 96 | 112 | 129 |
| | 0.005 | 33 | 44 | 56 | 70 | 84 | 99 | 114 | 131 |
| 10 | 0.10 | 29 | 40 | 51 | 64 | 77 | 91 | 106 | 123 |
| | 0.05 | 31 | 42 | 54 | 67 | 80 | 95 | 111 | 127 |
| | 0.025 | 33 | 44 | 56 | 69 | 83 | 98 | 114 | 131 |
| | 0.01 | 34 | 46 | 59 | 72 | 87 | 102 | 119 | 136 |
| | 0.005 | 36 | 48 | 61 | 74 | 89 | 105 | 121 | 139 |

## 10.4 Mann-Whitney Test: n1, n2 small, same shape

Ages of Statistics students: independent random samples from evening and day classes.

At the 5% significance level, are students in the evening classes older than those in the day classes?



NPP not straight li
NO! Not nd!

A: srs ✓ independent ✓ same shape ✓

Use Mann-Whitney (MW)

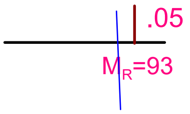
Already on calc, so sort each and rewrite.

n1= 9, n2= 7 --> need list of 16 rank

Ages of Stats Students

| Eve | RANK | Day | RANK |
|---|---|---|---|
| 18 | 2.5 | 18 | 2.5 |
| 19 | 6 | 18 | 2.5 |
| 19 | 6 | 18 | 2.5 |
| 20 | 9 | 19 | 6 |
| 20 | 9 | 20 | 9 |
| 21 | 11 | 25 | 13 |
| 23 | 12 | 31 | 14 |
| 41 | 15 | | |
| 49 | 16 | | |

$H_o$: $\mu 1 = \mu 2$ $H_a$: $\mu 1 > \mu 2$



$M_T$=86.5

Note: 4 18's hold ranks 1 to 4, so each gets a rank of 2.5

Also: 3 19's hold ranks 5 to 7, so each gets a rank of 6

And: 3 20's hold ranks 8 to 10, so each gets a rank of 9

With no other duplicates, each gets the next rank in sequence, starting with 11, because 10 has already been used.

1
2 2.5
3
4
5
6 6
7
8
9 9
10
11
12
13
14
15
16

$M_T = \sum(\text{ranks})$

$= 86.5$

need to compare this to a critical value, $M_R$, from Table 4

$M_T$=86.5 < $M_R$=93

Do NOT reject Ho.

Conclude that not sufficient evidence to claim that eve students are older.

or

Conclude that students in day and eve classes are the same age.

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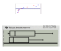
## 10.4 Mann-Whitney Test: n1, n2 small, same shape

A physician is interested in the effect of an anesthetic on reaction times. Two groups of individuals are randomly selected, 8 with anesthesia and 6 without. Subjects were given a simple visual stimulus and their reaction times were recorded.

| Anaethesia | Rank | No Anaethesia | Rank |
|---|---|---|---|
| 135 | | 138 | |
| 171 | | 230 | |
| 142 | | 142 | |
| 144 | | 156 | |
| 139 | | 138 | |
| 165 | | 191 | |
| 256 | | | |
| 244 | | | |



At the 5% significance level, does the anesthesia dull reaction time of patients?

Reaction times are not normally distributed in this experiment, and the sample sizes are small, so how do we proceed?

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a) 86, b) 48, c) 45, 99

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## 10.4 Mann-Whitney Test: n1, n2 small, same shape

A physician is interested in the effect of an anesthetic on reaction times. Two groups of individuals are randomly selected, 8 with anesthetic and 6 without. Subjects were given a simple visual stimulus and their reaction times were recorded.

0.05

Reaction times are not normally distributed in this experiment, and the sample sizes are small, so we cannot use a 2 sample t test.

How do we proceed?

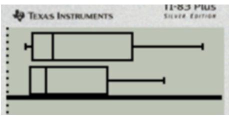
At the 5% significance level, does the anesthesia dull reaction time of patients?

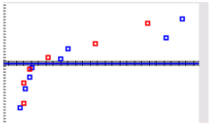
To dull reaction time is to take longer

| Anaethesia | Rank | No Anaethesia | Rank |
|---|---|---|---|
| 135 | | 138 | |
| 171 | | 230 | |
| 142 | | 142 | |
| 144 | | 156 | |
| 139 | | 138 | |
| 165 | | 191 | |
| 256 | | | |
| 244 | | | |

| |
|---|
| 1 |
| 2 |
| 3 |
| 4 |
| 5 |
| 6 |
| 7 |
| 8 |
| 9 |
| 10 |
| 11 |
| 12 |
| 13 |
| 14 |

Assumptions:
srs, independent samples,
same shape (right skewed)





--> MANN-WHITNEY

$H_0: \mu_A = \mu_N$

$H_a:$

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a) 86, b) 48, c) 45, 99

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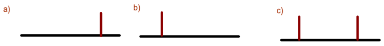

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## 10.4 Mann-Whitney Test: n1, n2 small, same shape

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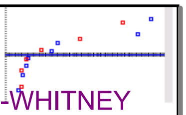
Reaction times are not normally distributed in this experiment, and the sample sizes are small, so we cannot use a 2 sample t test.
How do we proceed?

At the 5% significance level, does the anaethesia dull reaction time of patients?

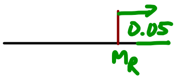
Assumptions:
srs, independent samples,
same shape (right skewed)



--> MANN-WHITNEY

$H_0: \mu_A = \mu_N$

$H_a: \mu_A > \mu_N$



| Anaethesia | Rank | No Anaethesia | Rank |
|---|---|---|---|
| 135 | | 138 | |
| 139 | | 138 | |
| 142 | | 142 | |
| 144 | | 156 | |
| 165 | | 191 | |
| 171 | | 230 | |
| 244 | | | |
| 256 | | | |

| |
|---|
| 1 |
| 2 |
| 3 |
| 4 |
| 5 |
| 6 |
| 7 |
| 8 |
| 9 |
| 10 |
| 11 |
| 12 |
| 13 |
| 14 |

135 139 142 144 165 171 244 256
138 138 142 156 191 230

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a) 86, b) 48, c) 45, 99

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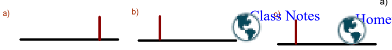

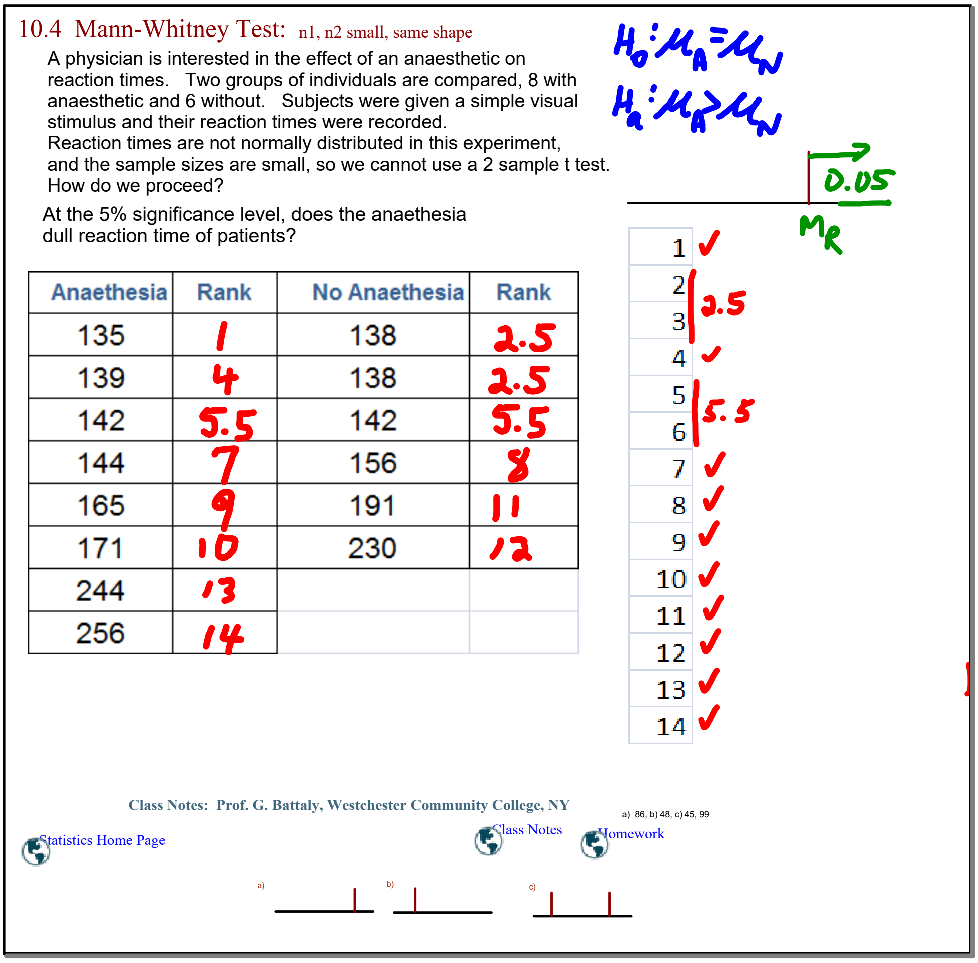

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At the 5% significance level, does the anaethesia dull reaction time of patients?

$H_0: \mu_A = \mu_N$

$H_a: \mu_A > \mu_N$

0.05

$M_R$

| Anaethesia | Rank | No Anaethesia | Rank |
|---|---|---|---|
| 135 | 1 | 138 | 2.5 |
| 139 | 4 | 138 | 2.5 |
| 142 | 5.5 | 142 | 5.5 |
| 144 | 7 | 156 | 8 |
| 165 | 9 | 191 | 11 |
| 171 | 10 | 230 | 12 |
| 244 | 13 | | |
| 256 | 14 | | |

| | |
|---|---|
| 1 | ✓ |
| 2 | 2.5 |
| 3 | |
| 4 | ✓ |
| 5 | 5.5 |
| 6 | |
| 7 | ✓ |
| 8 | ✓ |
| 9 | ✓ |
| 10 | ✓ |
| 11 | ✓ |
| 12 | ✓ |
| 13 | ✓ |
| 14 | ✓ |

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a) 86, b) 48, c) 45, 99

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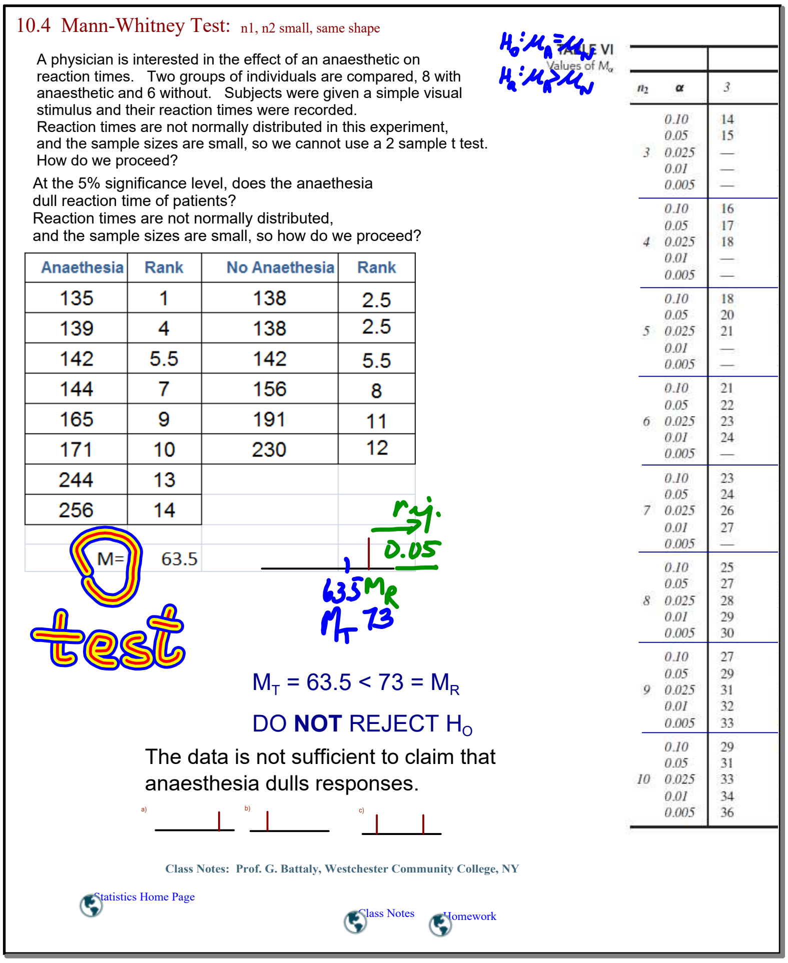
a) b) c)



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$H_0: \mu_A = \mu_N$

$H_a: \mu_A > \mu_N$

| Anaethesia | Rank | No Anaethesia | Rank |
|---|---|---|---|
| 135 | 1 | 138 | 2.5 |
| 139 | 4 | 138 | 2.5 |
| 142 | 5.5 | 142 | 5.5 |
| 144 | 7 | 156 | 8 |
| 165 | 9 | 191 | 11 |
| 171 | 10 | 230 | 12 |
| 244 | 13 | | |
| 256 | 14 | | |
| M= | 63.5 | | |

test

rej.

0.05

63.5 $M_R$

$M_T$ 73

$M_T = 63.5 < 73 = M_R$

DO **NOT** REJECT $H_O$

The data is not sufficient to claim that anaesthesia dulls responses.

a) b) c)

TABLE VI
Values of $M_\alpha$

| $n_2$ | $\alpha$ | 3 |
|---|---|---|
| 3 | 0.10 | 14 |
| | 0.05 | 15 |
| | 0.025 | — |
| | 0.01 | — |
| | 0.005 | — |
| 4 | 0.10 | 16 |
| | 0.05 | 17 |
| | 0.025 | 18 |
| | 0.01 | — |
| | 0.005 | — |
| 5 | 0.10 | 18 |
| | 0.05 | 20 |
| | 0.025 | 21 |
| | 0.01 | — |
| | 0.005 | — |
| 6 | 0.10 | 21 |
| | 0.05 | 22 |
| | 0.025 | 23 |
| | 0.01 | 24 |
| | 0.005 | — |
| 7 | 0.10 | 23 |
| | 0.05 | 24 |
| | 0.025 | 26 |
| | 0.01 | 27 |
| | 0.005 | — |
| 8 | 0.10 | 25 |
| | 0.05 | 27 |
| | 0.025 | 28 |
| | 0.01 | 29 |
| | 0.005 | 30 |
| 9 | 0.10 | 27 |
| | 0.05 | 29 |
| | 0.025 | 31 |
| | 0.01 | 32 |
| | 0.005 | 33 |
| 10 | 0.10 | 29 |
| | 0.05 | 31 |
| | 0.025 | 33 |
| | 0.01 | 34 |
| | 0.005 | 36 |

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10.4 Mann-Whitney Test: n1, n2 small, same shape

SKIP TO WORD PROBLEMS

## Finding Critical Values

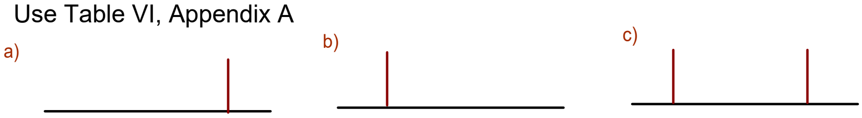
$M_L = n_1(n_1+n_2+1) - M_R$

(get $M_R$ from Table VI)

G: $n_1 = 8$, $n_2 = 9$, Significance Level = 0.05

F: Critical values for: a) Right-tailed, b) Left-tailed, c) Two-tailed test

Use Table VI, Appendix A



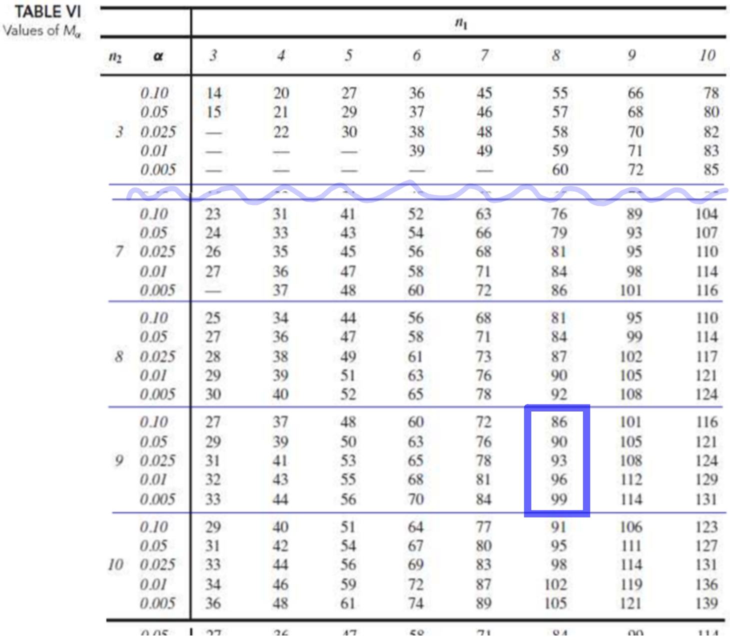
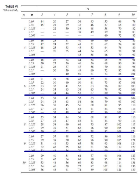


TABLE VI

Values of $M_\alpha$

| | | $n_1$ | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $n_2$ | α | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 3 | 0.10 | 14 | 20 | 27 | 36 | 45 | 55 | 66 | 78 |
| | 0.05 | 15 | 21 | 29 | 37 | 46 | 57 | 68 | 80 |
| | 0.025 | — | 22 | 30 | 38 | 48 | 58 | 70 | 82 |
| | 0.01 | — | — | — | 39 | 49 | 59 | 71 | 83 |
| | 0.005 | — | — | — | — | — | 60 | 72 | 85 |
| 7 | 0.10 | 23 | 31 | 41 | 52 | 63 | 76 | 89 | 104 |
| | 0.05 | 24 | 33 | 43 | 54 | 66 | 79 | 93 | 107 |
| | 0.025 | 26 | 35 | 45 | 56 | 68 | 81 | 95 | 110 |
| | 0.01 | 27 | 36 | 47 | 58 | 71 | 84 | 98 | 114 |
| | 0.005 | — | 37 | 48 | 60 | 72 | 86 | 101 | 116 |
| 8 | 0.10 | 25 | 34 | 44 | 56 | 68 | 81 | 95 | 110 |
| | 0.05 | 27 | 36 | 47 | 58 | 71 | 84 | 99 | 114 |
| | 0.025 | 28 | 38 | 49 | 61 | 73 | 87 | 102 | 117 |
| | 0.01 | 29 | 39 | 51 | 63 | 76 | 90 | 105 | 121 |
| | 0.005 | 30 | 40 | 52 | 65 | 78 | 92 | 108 | 124 |
| 9 | 0.10 | 27 | 37 | 48 | 60 | 72 | 86 | 101 | 116 |
| | 0.05 | 29 | 39 | 50 | 63 | 76 | 90 | 105 | 121 |
| | 0.025 | 31 | 41 | 53 | 65 | 78 | 93 | 108 | 124 |
| | 0.01 | 32 | 43 | 55 | 68 | 81 | 96 | 112 | 129 |
| | 0.005 | 33 | 44 | 56 | 70 | 84 | 99 | 114 | 131 |
| 10 | 0.10 | 29 | 40 | 51 | 64 | 77 | 91 | 106 | 123 |
| | 0.05 | 31 | 42 | 54 | 67 | 80 | 95 | 111 | 127 |
| | 0.025 | 33 | 44 | 56 | 69 | 83 | 98 | 114 | 131 |
| | 0.01 | 34 | 46 | 59 | 72 | 87 | 102 | 119 | 136 |
| | 0.005 | 36 | 48 | 61 | 74 | 89 | 105 | 121 | 139 |



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10.4 Mann-Whitney Test: n1, n2 small, same shape

$M_L = n_1(n_1+n_2+1) - M_R$

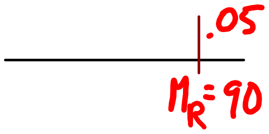
(get $M_R$ from Table VI)

G: $n_1 = 8$, $n_2 = 9$, Significance Level = 0.05

F: Critical values for: a) Right-tailed, b) Left-tailed, c) Two-tailed test

Use Table VI, Appendix A

a)



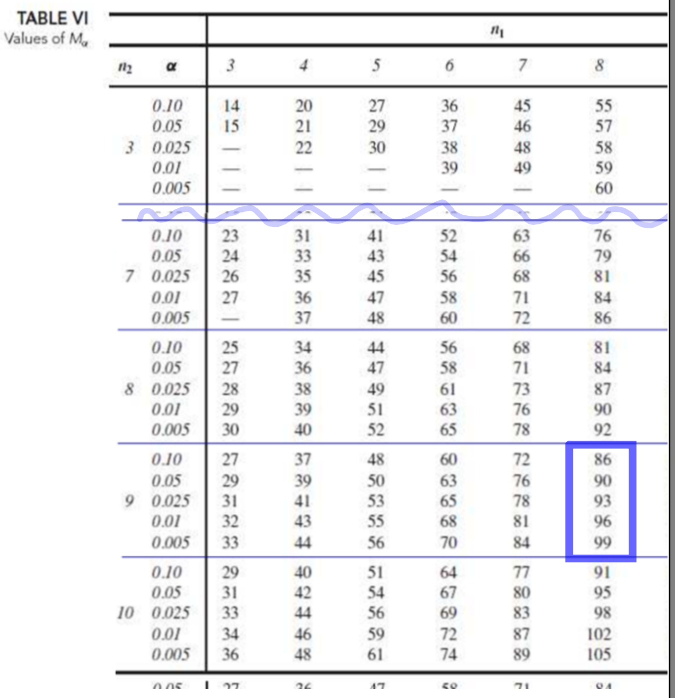


TABLE VI

Values of $M_\alpha$

| | | $n_1$ | | | | | |
|---|---|---|---|---|---|---|---|
| $n_2$ | α | 3 | 4 | 5 | 6 | 7 | 8 |
| 3 | 0.10 | 14 | 20 | 27 | 36 | 45 | 55 |
| | 0.05 | 15 | 21 | 29 | 37 | 46 | 57 |
| | 0.025 | — | 22 | 30 | 38 | 48 | 58 |
| | 0.01 | — | — | — | 39 | 49 | 59 |
| | 0.005 | — | — | — | — | — | 60 |
| 7 | 0.10 | 23 | 31 | 41 | 52 | 63 | 76 |
| | 0.05 | 24 | 33 | 43 | 54 | 66 | 79 |
| | 0.025 | 26 | 35 | 45 | 56 | 68 | 81 |
| | 0.01 | 27 | 36 | 47 | 58 | 71 | 84 |
| | 0.005 | — | 37 | 48 | 60 | 72 | 86 |
| 8 | 0.10 | 25 | 34 | 44 | 56 | 68 | 81 |
| | 0.05 | 27 | 36 | 47 | 58 | 71 | 84 |
| | 0.025 | 28 | 38 | 49 | 61 | 73 | 87 |
| | 0.01 | 29 | 39 | 51 | 63 | 76 | 90 |
| | 0.005 | 30 | 40 | 52 | 65 | 78 | 92 |
| 9 | 0.10 | 27 | 37 | 48 | 60 | 72 | 86 |
| | 0.05 | 29 | 39 | 50 | 63 | 76 | 90 |
| | 0.025 | 31 | 41 | 53 | 65 | 78 | 93 |
| | 0.01 | 32 | 43 | 55 | 68 | 81 | 96 |
| | 0.005 | 33 | 44 | 56 | 70 | 84 | 99 |
| 10 | 0.10 | 29 | 40 | 51 | 64 | 77 | 91 |
| | 0.05 | 31 | 42 | 54 | 67 | 80 | 95 |
| | 0.025 | 33 | 44 | 56 | 69 | 83 | 98 |
| | 0.01 | 34 | 46 | 59 | 72 | 87 | 102 |
| | 0.005 | 36 | 48 | 61 | 74 | 89 | 105 |

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a) 86, b) 48, c) 45, 99

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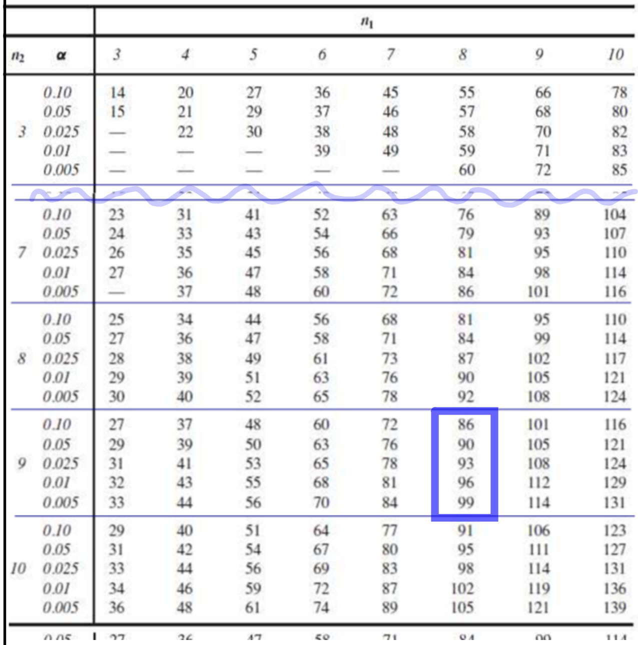
## 10.4 Mann-Whitney Test: n1, n2 small, same shape

G: $n_1 = 8$, $n_2 = 9$, Significance Level = 0.05
F: Critical values for: a) Right-tailed, b) Left-tailed, c) Two-tailed test
Use Table VI, Appendix A



| $n_2$ | α | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|
| | 0.10 | 14 | 20 | 27 | 36 | 45 | 55 | 66 | 78 |
| | 0.05 | 15 | 21 | 29 | 37 | 46 | 57 | 68 | 80 |
| 3 | 0.025 | — | 22 | 30 | 38 | 48 | 58 | 70 | 82 |
| | 0.01 | — | — | — | 39 | 49 | 59 | 71 | 83 |
| | 0.005 | — | — | — | — | — | 60 | 72 | 85 |
| | 0.10 | 23 | 31 | 41 | 52 | 63 | 76 | 89 | 104 |
| | 0.05 | 24 | 33 | 43 | 54 | 66 | 79 | 93 | 107 |
| 7 | 0.025 | 26 | 35 | 45 | 56 | 68 | 81 | 95 | 110 |
| | 0.01 | 27 | 36 | 47 | 58 | 71 | 84 | 98 | 114 |
| | 0.005 | — | 37 | 48 | 60 | 72 | 86 | 101 | 116 |
| | 0.10 | 25 | 34 | 44 | 56 | 68 | 81 | 95 | 110 |
| | 0.05 | 27 | 36 | 47 | 58 | 71 | 84 | 99 | 114 |
| 8 | 0.025 | 28 | 38 | 49 | 61 | 73 | 87 | 102 | 117 |
| | 0.01 | 29 | 39 | 51 | 63 | 76 | 90 | 105 | 121 |
| | 0.005 | 30 | 40 | 52 | 65 | 78 | 92 | 108 | 124 |
| | 0.10 | 27 | 37 | 48 | 60 | 72 | 86 | 101 | 116 |
| | 0.05 | 29 | 39 | 50 | 63 | 76 | 90 | 105 | 121 |
| 9 | 0.025 | 31 | 41 | 53 | 65 | 78 | 93 | 108 | 124 |
| | 0.01 | 32 | 43 | 55 | 68 | 81 | 96 | 112 | 129 |
| | 0.005 | 33 | 44 | 56 | 70 | 84 | 99 | 114 | 131 |
| | 0.10 | 29 | 40 | 51 | 64 | 77 | 91 | 106 | 123 |
| | 0.05 | 31 | 42 | 54 | 67 | 80 | 95 | 111 | 127 |
| 10 | 0.025 | 33 | 44 | 56 | 69 | 83 | 98 | 114 | 131 |
| | 0.01 | 34 | 46 | 59 | 72 | 87 | 102 | 119 | 136 |
| | 0.005 | 36 | 48 | 61 | 74 | 89 | 105 | 121 | 139 |

b)

.05

$M_L = 54$

$M_L = n_1(n_1+n_2+1) - M_R$
(get $M_R$ from Table VI)

$M_L = 8(8+9+1) - 90$
$= 144-90 = 54$

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a) 86, b) 48, c) 45, 99

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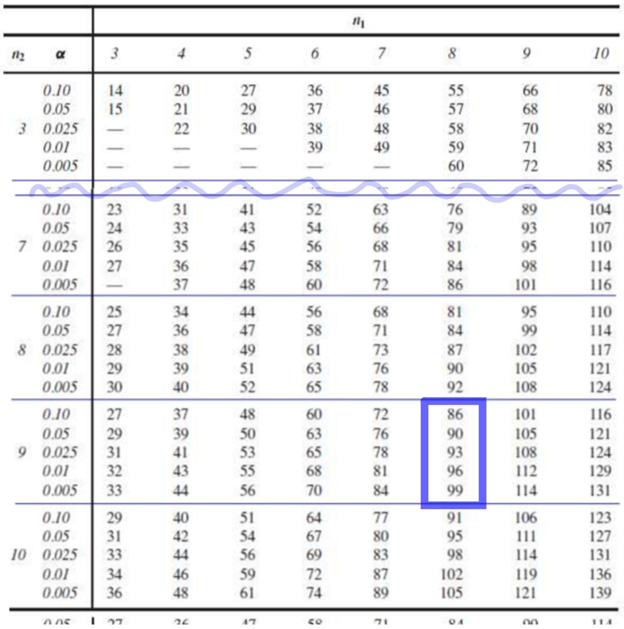
## 10.4 Mann-Whitney Test: n1, n2 small, same shape

G: $n_1 = 8$, $n_2 = 9$, Significance Level = 0.05
F: Critical values for: a) Right-tailed, b) Left-tailed, c) Two-tailed test
Use Table VI, Appendix A

c)

.025 .025

$M_L=51$ $M_R=93$



| $n_2$ | α | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|
| | 0.10 | 14 | 20 | 27 | 36 | 45 | 55 | 66 | 78 |
| | 0.05 | 15 | 21 | 29 | 37 | 46 | 57 | 68 | 80 |
| 3 | 0.025 | — | 22 | 30 | 38 | 48 | 58 | 70 | 82 |
| | 0.01 | — | — | — | 39 | 49 | 59 | 71 | 83 |
| | 0.005 | — | — | — | — | — | 60 | 72 | 85 |
| | 0.10 | 23 | 31 | 41 | 52 | 63 | 76 | 89 | 104 |
| | 0.05 | 24 | 33 | 43 | 54 | 66 | 79 | 93 | 107 |
| 7 | 0.025 | 26 | 35 | 45 | 56 | 68 | 81 | 95 | 110 |
| | 0.01 | 27 | 36 | 47 | 58 | 71 | 84 | 98 | 114 |
| | 0.005 | — | 37 | 48 | 60 | 72 | 86 | 101 | 116 |
| | 0.10 | 25 | 34 | 44 | 56 | 68 | 81 | 95 | 110 |
| | 0.05 | 27 | 36 | 47 | 58 | 71 | 84 | 99 | 114 |
| 8 | 0.025 | 28 | 38 | 49 | 61 | 73 | 87 | 102 | 117 |
| | 0.01 | 29 | 39 | 51 | 63 | 76 | 90 | 105 | 121 |
| | 0.005 | 30 | 40 | 52 | 65 | 78 | 92 | 108 | 124 |
| | 0.10 | 27 | 37 | 48 | 60 | 72 | 86 | 101 | 116 |
| | 0.05 | 29 | 39 | 50 | 63 | 76 | 90 | 105 | 121 |
| 9 | 0.025 | 31 | 41 | 53 | 65 | 78 | 93 | 108 | 124 |
| | 0.01 | 32 | 43 | 55 | 68 | 81 | 96 | 112 | 129 |
| | 0.005 | 33 | 44 | 56 | 70 | 84 | 99 | 114 | 131 |
| | 0.10 | 29 | 40 | 51 | 64 | 77 | 91 | 106 | 123 |
| | 0.05 | 31 | 42 | 54 | 67 | 80 | 95 | 111 | 127 |
| 10 | 0.025 | 33 | 44 | 56 | 69 | 83 | 98 | 114 | 131 |
| | 0.01 | 34 | 46 | 59 | 72 | 87 | 102 | 119 | 136 |
| | 0.005 | 36 | 48 | 61 | 74 | 89 | 105 | 121 | 139 |

$M_L = n_1(n_1+n_2+1) - M_R$
(get $M_R$ from Table VI)

$M_L = 8(8+9+1) - 93$
$= 144 - 93 = 51$

4

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a) 86, b) 48, c) 45, 99

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## 10.4 Mann-Whitney Test: n1, n2 small, same shape

Ranked, not continuous.

$M_L = n_1(n_1+n_2+1) - M_R$
(get $M_R$ from Table VI)

Six people were asked to taste Brand A coffee, and 6 different people asked to taste Brand B. They were then asked to rank how likely it was that they would purchase that Brand, using a scale of 1 to 10, with 10 being most likely.

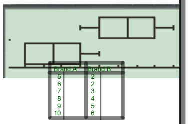
At the 5% significance level did they like both brands equally?

1
2
3
4
5
6
7
8
9
10
11
12

**TABLE VI**
Values of $M_\alpha$

| $n_2$ | $\alpha$ |
|---|---|
| | 0.10 |
| | 0.05 |
| 3 | 0.02 |
| | 0.01 |
| | 0.00 |
| | 0.10 |
| | 0.05 |
| 4 | 0.02 |
| | 0.01 |
| | 0.00 |
| | 0.10 |
| | 0.05 |
| 5 | 0.02 |
| | 0.01 |
| | 0.00 |
| | 0.10 |
| | 0.05 |
| 6 | 0.02 |
| | 0.01 |
| | 0.00 |

same shape



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## 10.4 Mann-Whitney Test: n1, n2 small, same shape

$M_L = n_1(n_1+n_2+1) - M_R$
(get $M_R$ from Table VI)

Six people were asked to taste Brand A coffee, and 6 different people asked to taste Brand B. They were then asked to rank how likely it was that they would purchase that Brand, using a scale of 1 to 10, with 10 being most likely.

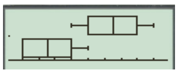
Ranked data, not continuous.

At the 5% significance level did they like both brands equally?

$H_O$: $\mu 1 = \mu 2$

$H_a$: $\mu 1 \neq \mu 2$

| Brand A | Brand B |
|---|---|
| 9 | 3 |
| 7 | 4 |
| 5 | 2 |
| 10 | 6 |
| 6 | 2 |
| 8 | 5 |



| Brand A | | Brand B | |
|---|---|---|---|
| 5 | 5.5 | 2 | 1.5 |
| 6 | 7.5 | 2 | 1.5 |
| 7 | 9 | 3 | 3 |
| 8 | 10 | 4 | 4 |
| 9 | 11 | 5 | 5.5 |
| 10 | 12 | 6 | 7.5 |

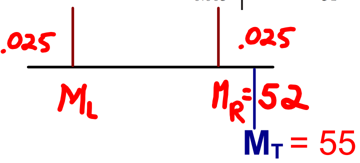
$M_T = 55$

1
2
3
4
5
6
7
8
9
10
11
12

**TABLE VI**
Values of $M_\alpha$

| $n_2$ | $\alpha$ | 3 | 4 |
|---|---|---|---|
| | 0.10 | 14 | 20 |
| | 0.05 | 15 | 21 |
| 3 | 0.025 | — | 22 |
| | 0.01 | — | — |
| | 0.005 | — | — |
| | 0.10 | 16 | 23 |
| | 0.05 | 17 | 24 |
| 4 | 0.025 | 18 | 25 |
| | 0.01 | — | 26 |
| | 0.005 | — | — |
| | 0.10 | 18 | 26 |
| | 0.05 | 20 | 27 |
| 5 | 0.025 | 21 | 28 |
| | 0.01 | — | 30 |
| | 0.005 | — | — |
| | 0.10 | 21 | 29 |
| | 0.05 | 22 | 30 |
| 6 | 0.025 | 23 | 32 |
| | 0.01 | 24 | 33 |
| | 0.005 | — | 34 |

$M_T = 55 > 52 = M_R$ Reject $H_O$



Conclude that the coffees were not equally liked. (Note: Have stronger evidence than 5% s.l.; actually have 0.5% significance, or 99.5% confidence.)

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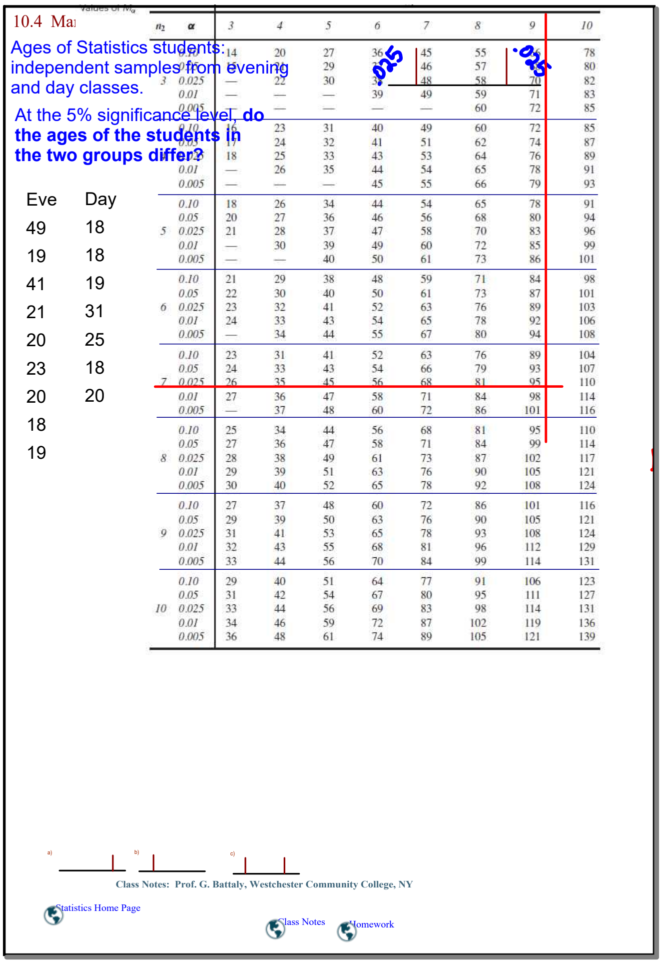

10.4 Ma

Ages of Statistics students: independent samples from evening and day classes.

At the 5% significance level, **do the ages of the students in the two groups differ?**

| Eve | Day |
|---|---|
| 49 | 18 |
| 19 | 18 |
| 41 | 19 |
| 21 | 31 |
| 20 | 25 |
| 23 | 18 |
| 20 | 20 |
| 18 | |
| 19 | |

Values of $M_\alpha$

| $n_2$ | $\alpha$ | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|
| | 0.10 | 14 | 20 | 27 | 36 | 45 | 55 | [illegible] | 78 |
| | 0.05 | [illegible] | [illegible] | 29 | [illegible] | 46 | 57 | [illegible] | 80 |
| 3 | 0.025 | — | 22 | 30 | [illegible] | 48 | 58 | 70 | 82 |
| | 0.01 | — | — | — | 39 | 49 | 59 | 71 | 83 |
| | 0.005 | — | — | — | — | — | 60 | 72 | 85 |
| | 0.10 | 16 | 23 | 31 | 40 | 49 | 60 | 72 | 85 |
| | 0.05 | 17 | 24 | 32 | 41 | 51 | 62 | 74 | 87 |
| | 0.025 | 18 | 25 | 33 | 43 | 53 | 64 | 76 | 89 |
| | 0.01 | — | 26 | 35 | 44 | 54 | 65 | 78 | 91 |
| | 0.005 | — | — | — | 45 | 55 | 66 | 79 | 93 |
| | 0.10 | 18 | 26 | 34 | 44 | 54 | 65 | 78 | 91 |
| | 0.05 | 20 | 27 | 36 | 46 | 56 | 68 | 80 | 94 |
| 5 | 0.025 | 21 | 28 | 37 | 47 | 58 | 70 | 83 | 96 |
| | 0.01 | — | 30 | 39 | 49 | 60 | 72 | 85 | 99 |
| | 0.005 | — | — | 40 | 50 | 61 | 73 | 86 | 101 |
| | 0.10 | 21 | 29 | 38 | 48 | 59 | 71 | 84 | 98 |
| | 0.05 | 22 | 30 | 40 | 50 | 61 | 73 | 87 | 101 |
| 6 | 0.025 | 23 | 32 | 41 | 52 | 63 | 76 | 89 | 103 |
| | 0.01 | 24 | 33 | 43 | 54 | 65 | 78 | 92 | 106 |
| | 0.005 | — | 34 | 44 | 55 | 67 | 80 | 94 | 108 |
| | 0.10 | 23 | 31 | 41 | 52 | 63 | 76 | 89 | 104 |
| | 0.05 | 24 | 33 | 43 | 54 | 66 | 79 | 93 | 107 |
| 7 | 0.025 | 26 | 35 | 45 | 56 | 68 | 81 | 95 | 110 |
| | 0.01 | 27 | 36 | 47 | 58 | 71 | 84 | 98 | 114 |
| | 0.005 | — | 37 | 48 | 60 | 72 | 86 | 101 | 116 |
| | 0.10 | 25 | 34 | 44 | 56 | 68 | 81 | 95 | 110 |
| | 0.05 | 27 | 36 | 47 | 58 | 71 | 84 | 99 | 114 |
| 8 | 0.025 | 28 | 38 | 49 | 61 | 73 | 87 | 102 | 117 |
| | 0.01 | 29 | 39 | 51 | 63 | 76 | 90 | 105 | 121 |
| | 0.005 | 30 | 40 | 52 | 65 | 78 | 92 | 108 | 124 |
| | 0.10 | 27 | 37 | 48 | 60 | 72 | 86 | 101 | 116 |
| | 0.05 | 29 | 39 | 50 | 63 | 76 | 90 | 105 | 121 |
| 9 | 0.025 | 31 | 41 | 53 | 65 | 78 | 93 | 108 | 124 |
| | 0.01 | 32 | 43 | 55 | 68 | 81 | 96 | 112 | 129 |
| | 0.005 | 33 | 44 | 56 | 70 | 84 | 99 | 114 | 131 |
| | 0.10 | 29 | 40 | 51 | 64 | 77 | 91 | 106 | 123 |
| | 0.05 | 31 | 42 | 54 | 67 | 80 | 95 | 111 | 127 |
| 10 | 0.025 | 33 | 44 | 56 | 69 | 83 | 98 | 114 | 131 |
| | 0.01 | 34 | 46 | 59 | 72 | 87 | 102 | 119 | 136 |
| | 0.005 | 36 | 48 | 61 | 74 | 89 | 105 | 121 | 139 |

.025 .025

a) b) c)

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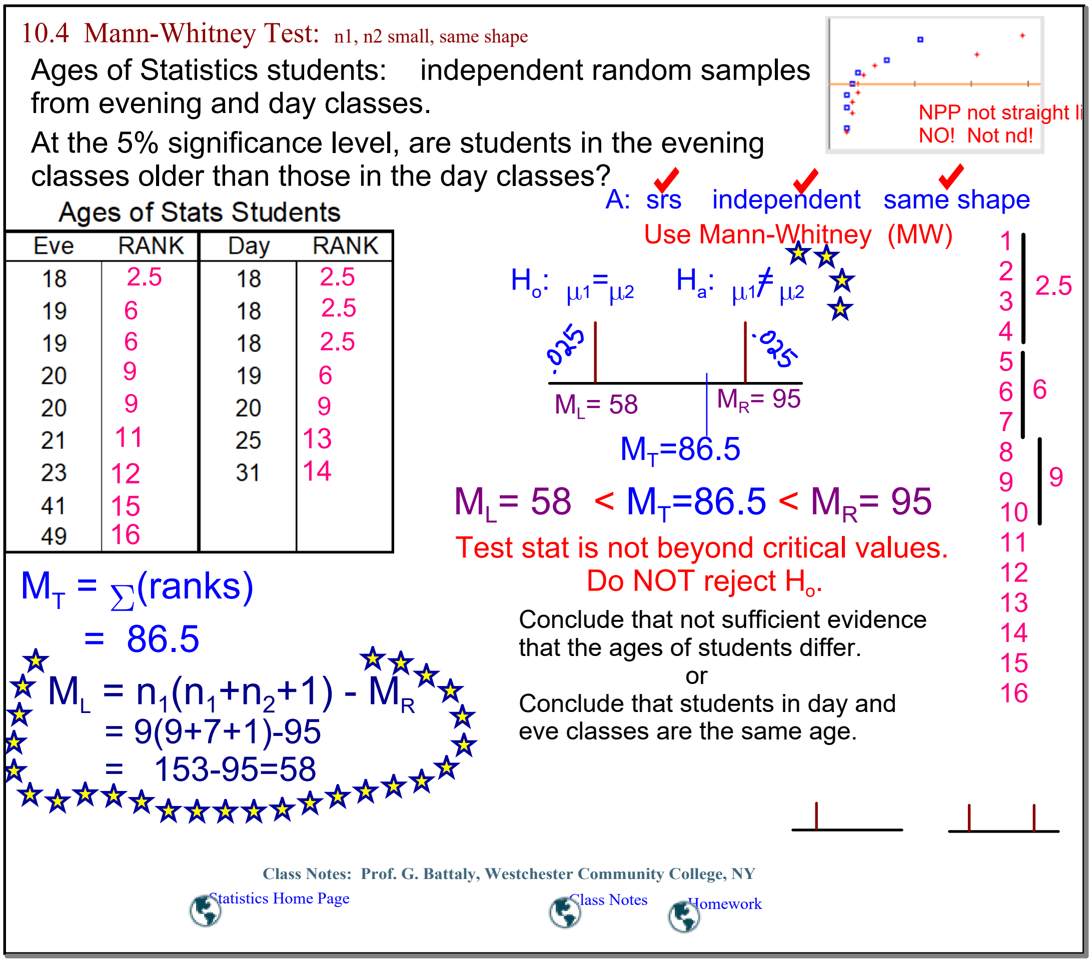

10.4 Mann-Whitney Test: n1, n2 small, same shape

Ages of Statistics students: independent random samples from evening and day classes.

At the 5% significance level, are students in the evening classes older than those in the day classes?

NPP not straight li

NO! Not nd!

Ages of Stats Students

| Eve | RANK | Day | RANK |
|---|---|---|---|
| 18 | 2.5 | 18 | 2.5 |
| 19 | 6 | 18 | 2.5 |
| 19 | 6 | 18 | 2.5 |
| 20 | 9 | 19 | 6 |
| 20 | 9 | 20 | 9 |
| 21 | 11 | 25 | 13 |
| 23 | 12 | 31 | 14 |
| 41 | 15 | | |
| 49 | 16 | | |

A: srs ✓ independent ✓ same shape ✓

Use Mann-Whitney (MW)

$H_o$: $\mu 1 = \mu 2$ $H_a$: $\mu 1 \neq \mu 2$

.025 .025

$M_L = 58$ $M_R = 95$

$M_T = 86.5$

$M_L = 58 < M_T = 86.5 < M_R = 95$

Test stat is not beyond critical values.

Do NOT reject $H_o$.

Conclude that not sufficient evidence that the ages of students differ.

or

Conclude that students in day and eve classes are the same age.

$$M_T = \sum(\text{ranks})$$
$$= 86.5$$

$$M_L = n_1(n_1+n_2+1) - M_R$$
$$= 9(9+7+1)-95$$
$$= 153-95=58$$

1
2 2.5
3
4
5
6 6
7
8
9 9
10
11
12
13
14
15
16

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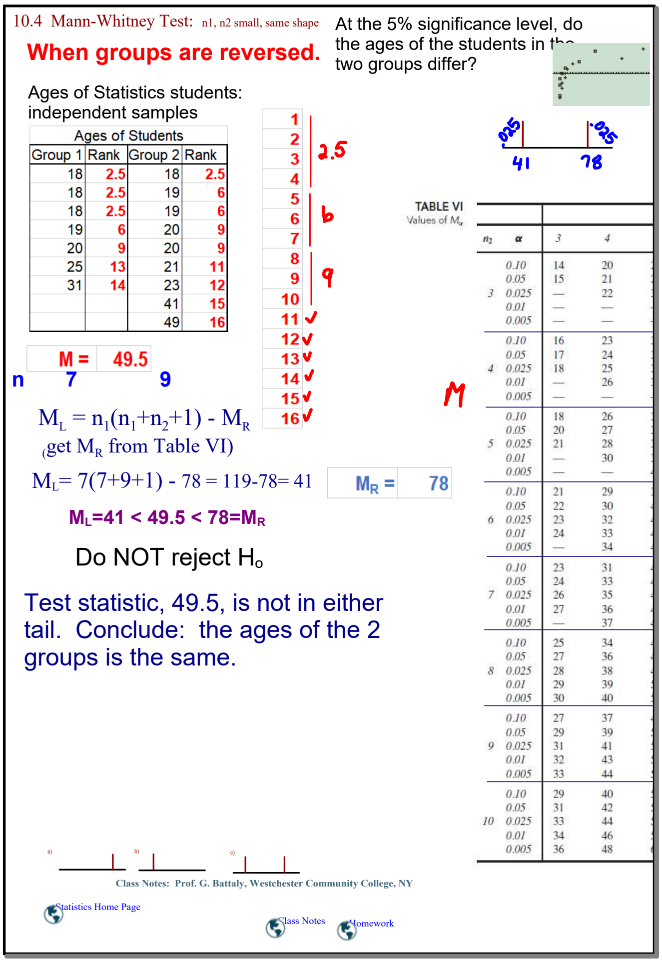

10.4 Mann-Whitney Test: n1, n2 small, same shape

**When groups are reversed.**

At the 5% significance level, do the ages of the students in the two groups differ?

.025 .025
41 78

Ages of Statistics students: independent samples

| Ages of Students | | | |
|---|---|---|---|
| Group 1 | Rank | Group 2 | Rank |
| 18 | 2.5 | 18 | 2.5 |
| 18 | 2.5 | 19 | 6 |
| 18 | 2.5 | 19 | 6 |
| 19 | 6 | 20 | 9 |
| 20 | 9 | 20 | 9 |
| 25 | 13 | 21 | 11 |
| 31 | 14 | 23 | 12 |
| | | 41 | 15 |
| | | 49 | 16 |

1
2 2.5
3
4
5
6 b
7
8
9 9
10
11 ✓
12 ✓
13 ✓
14 ✓
15 ✓
16 ✓

M = 49.5

n 7 9

$M_L = n_1(n_1+n_2+1) - M_R$

(get $M_R$ from Table VI)

$M_L = 7(7+9+1) - 78 = 119-78 = 41$

$M_R =$ 78

$M_L=41 < 49.5 < 78=M_R$

Do NOT reject $H_o$

Test statistic, 49.5, is not in either tail. Conclude: the ages of the 2 groups is the same.

M

TABLE VI
Values of $M_\alpha$

| $n_2$ | $\alpha$ | 3 | 4 |
|---|---|---|---|
| | 0.10 | 14 | 20 |
| | 0.05 | 15 | 21 |
| 3 | 0.025 | — | 22 |
| | 0.01 | — | — |
| | 0.005 | — | — |
| | 0.10 | 16 | 23 |
| | 0.05 | 17 | 24 |
| 4 | 0.025 | 18 | 25 |
| | 0.01 | — | 26 |
| | 0.005 | — | — |
| | 0.10 | 18 | 26 |
| | 0.05 | 20 | 27 |
| 5 | 0.025 | 21 | 28 |
| | 0.01 | — | 30 |
| | 0.005 | — | — |
| | 0.10 | 21 | 29 |
| | 0.05 | 22 | 30 |
| 6 | 0.025 | 23 | 32 |
| | 0.01 | 24 | 33 |
| | 0.005 | — | 34 |
| | 0.10 | 23 | 31 |
| | 0.05 | 24 | 33 |
| 7 | 0.025 | 26 | 35 |
| | 0.01 | 27 | 36 |
| | 0.005 | — | 37 |
| | 0.10 | 25 | 34 |
| | 0.05 | 27 | 36 |
| 8 | 0.025 | 28 | 38 |
| | 0.01 | 29 | 39 |
| | 0.005 | 30 | 40 |
| | 0.10 | 27 | 37 |
| | 0.05 | 29 | 39 |
| 9 | 0.025 | 31 | 41 |
| | 0.01 | 32 | 43 |
| | 0.005 | 33 | 44 |
| | 0.10 | 29 | 40 |
| | 0.05 | 31 | 42 |
| 10 | 0.025 | 33 | 44 |
| | 0.01 | 34 | 46 |
| | 0.005 | 36 | 48 |

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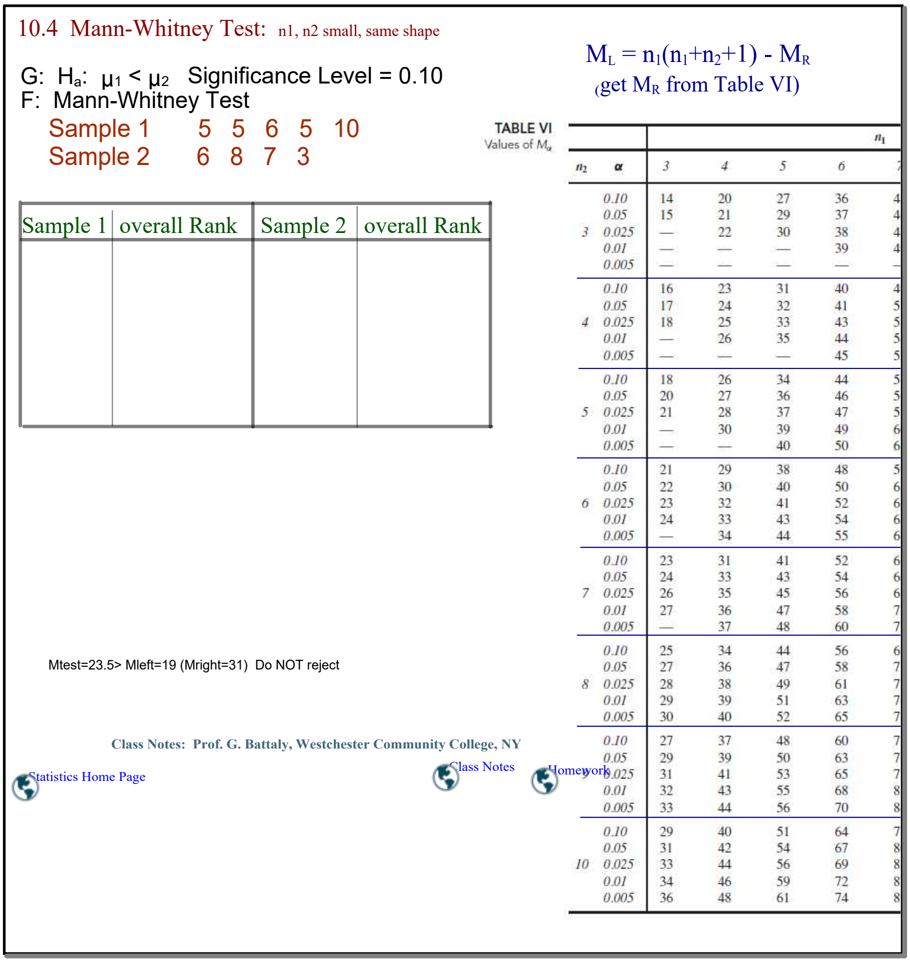

10.4 Mann-Whitney Test: n1, n2 small, same shape

G: $H_a$: $\mu_1 < \mu_2$ Significance Level = 0.10
F: Mann-Whitney Test

Sample 1 5 5 6 5 10
Sample 2 6 8 7 3

| Sample 1 | overall Rank | Sample 2 | overall Rank |
|---|---|---|---|
| | | | |

Mtest=23.5> Mleft=19 (Mright=31) Do NOT reject

$M_L = n_1(n_1+n_2+1) - M_R$

(get $M_R$ from Table VI)

TABLE VI
Values of $M_\alpha$

| | | $n_1$ | | | |
|---|---|---|---|---|---|
| $n_2$ | $\alpha$ | 3 | 4 | 5 | 6 |
| | 0.10 | 14 | 20 | 27 | 36 |
| | 0.05 | 15 | 21 | 29 | 37 |
| 3 | 0.025 | — | 22 | 30 | 38 |
| | 0.01 | — | — | — | 39 |
| | 0.005 | — | — | — | — |
| | 0.10 | 16 | 23 | 31 | 40 |
| | 0.05 | 17 | 24 | 32 | 41 |
| 4 | 0.025 | 18 | 25 | 33 | 43 |
| | 0.01 | — | 26 | 35 | 44 |
| | 0.005 | — | — | — | 45 |
| | 0.10 | 18 | 26 | 34 | 44 |
| | 0.05 | 20 | 27 | 36 | 46 |
| 5 | 0.025 | 21 | 28 | 37 | 47 |
| | 0.01 | — | 30 | 39 | 49 |
| | 0.005 | — | — | 40 | 50 |
| | 0.10 | 21 | 29 | 38 | 48 |
| | 0.05 | 22 | 30 | 40 | 50 |
| 6 | 0.025 | 23 | 32 | 41 | 52 |
| | 0.01 | 24 | 33 | 43 | 54 |
| | 0.005 | — | 34 | 44 | 55 |
| | 0.10 | 23 | 31 | 41 | 52 |
| | 0.05 | 24 | 33 | 43 | 54 |
| 7 | 0.025 | 26 | 35 | 45 | 56 |
| | 0.01 | 27 | 36 | 47 | 58 |
| | 0.005 | — | 37 | 48 | 60 |
| | 0.10 | 25 | 34 | 44 | 56 |
| | 0.05 | 27 | 36 | 47 | 58 |
| 8 | 0.025 | 28 | 38 | 49 | 61 |
| | 0.01 | 29 | 39 | 51 | 63 |
| | 0.005 | 30 | 40 | 52 | 65 |
| | 0.10 | 27 | 37 | 48 | 60 |
| | 0.05 | 29 | 39 | 50 | 63 |
| 9 | 0.025 | 31 | 41 | 53 | 65 |
| | 0.01 | 32 | 43 | 55 | 68 |
| | 0.005 | 33 | 44 | 56 | 70 |
| | 0.10 | 29 | 40 | 51 | 64 |
| | 0.05 | 31 | 42 | 54 | 67 |
| 10 | 0.025 | 33 | 44 | 56 | 69 |
| | 0.01 | 34 | 46 | 59 | 72 |
| | 0.005 | 36 | 48 | 61 | 74 |

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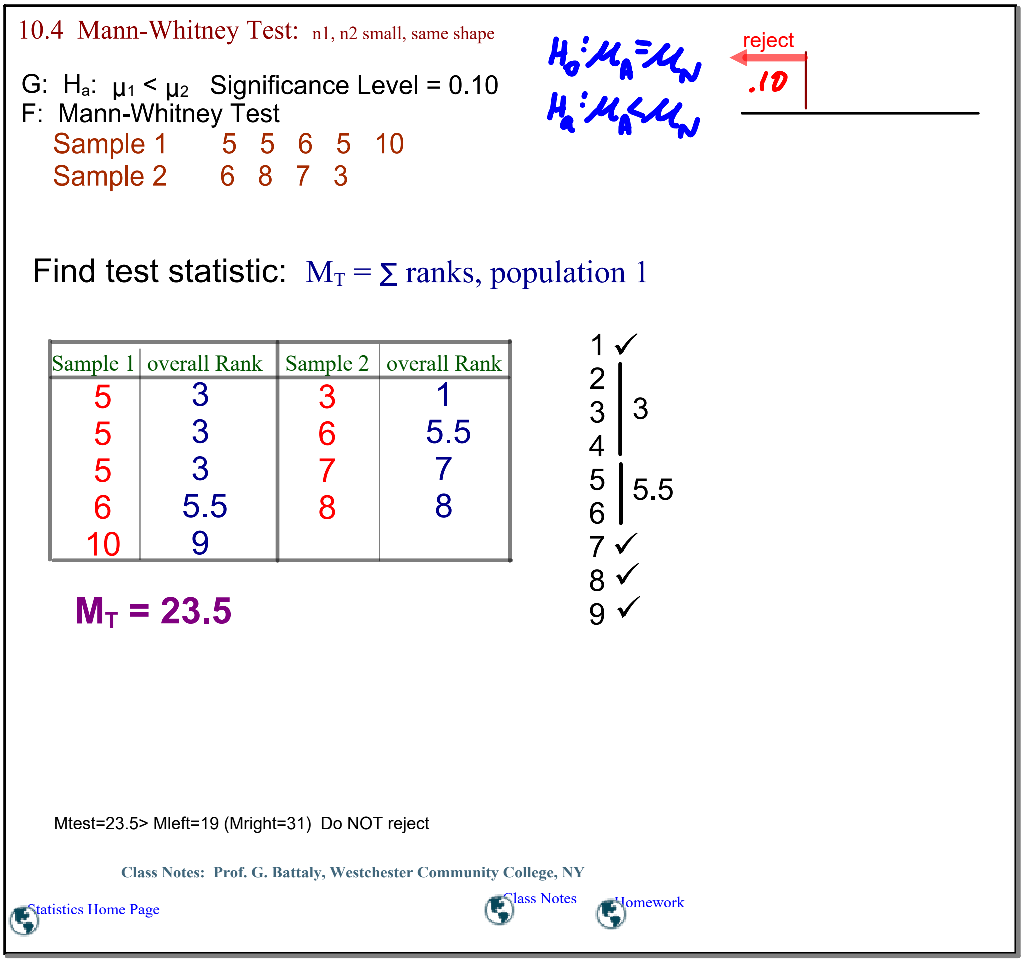

## 10.4 Mann-Whitney Test: n1, n2 small, same shape

G: $H_a$: $\mu_1 < \mu_2$ Significance Level = 0.10
F: Mann-Whitney Test
Sample 1 5 5 6 5 10
Sample 2 6 8 7 3

$H_0: \mu_A = \mu_N$
$H_a: \mu_A < \mu_N$

Find test statistic: $M_T = \sum$ ranks, population 1

| Sample 1 | overall Rank | Sample 2 | overall Rank |
|---|---|---|---|
| 5 | 3 | 3 | 1 |
| 5 | 3 | 6 | 5.5 |
| 5 | 3 | 7 | 7 |
| 6 | 5.5 | 8 | 8 |
| 10 | 9 | | |

1 ✓
2, 3, 4 | 3
5, 6 | 5.5
7 ✓
8 ✓
9 ✓

**$M_T$ = 23.5**

Mtest=23.5> Mleft=19 (Mright=31) Do NOT reject

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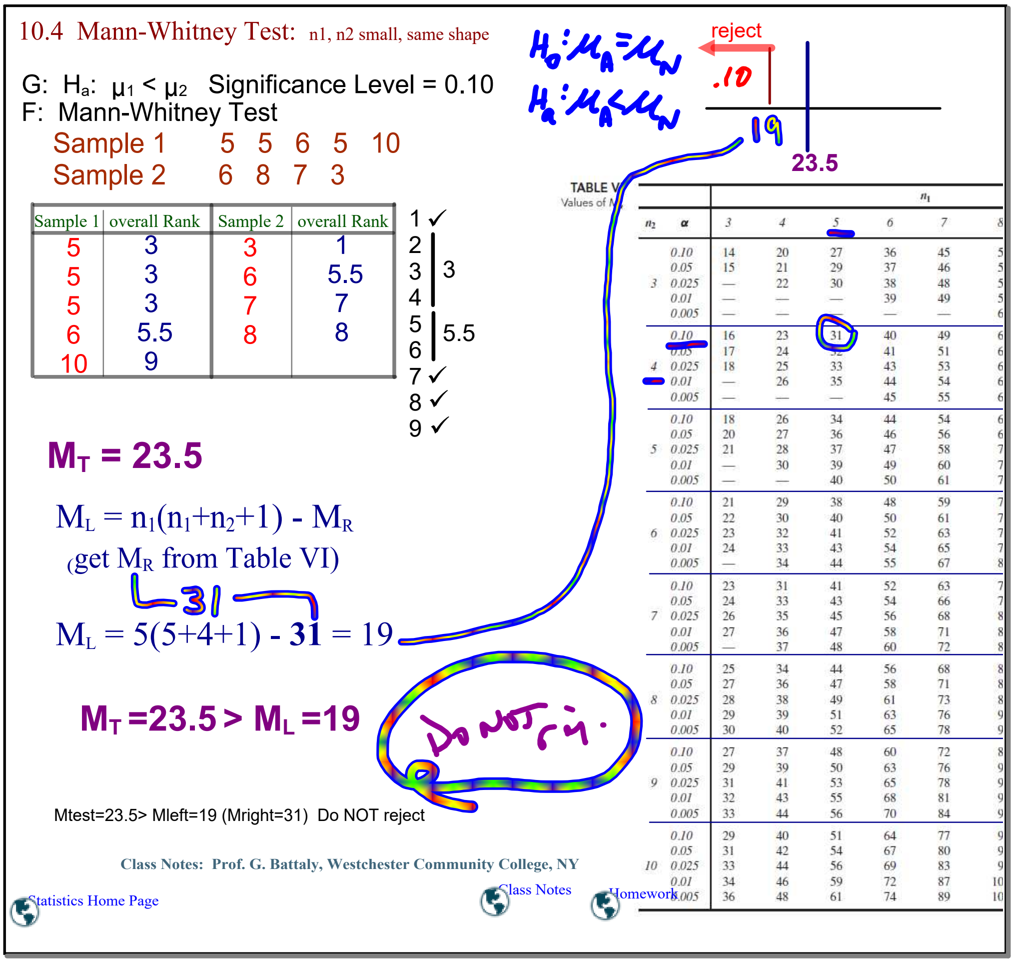

## 10.4 Mann-Whitney Test: n1, n2 small, same shape

G: $H_a$: $\mu_1 < \mu_2$ Significance Level = 0.10
F: Mann-Whitney Test
Sample 1 5 5 6 5 10
Sample 2 6 8 7 3

$H_0: \mu_A = \mu_N$
$H_a: \mu_A < \mu_N$

| Sample 1 | overall Rank | Sample 2 | overall Rank |
|---|---|---|---|
| 5 | 3 | 3 | 1 |
| 5 | 3 | 6 | 5.5 |
| 5 | 3 | 7 | 7 |
| 6 | 5.5 | 8 | 8 |
| 10 | 9 | | |

1 ✓
2, 3, 4 | 3
5, 6 | 5.5
7 ✓
8 ✓
9 ✓

**$M_T$ = 23.5**

$M_L = n_1(n_1+n_2+1) - M_R$
(get $M_R$ from Table VI)

$M_L = 5(5+4+1) - \underbrace{\mathbf{31}}_{31} = 19$

**$M_T$ =23.5 > $M_L$ =19**

Do NOT rej.

TABLE V
Values of M

| $n_2$ | $\alpha$ | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|
| 3 | 0.10 | 14 | 20 | 27 | 36 | 45 |
| | 0.05 | 15 | 21 | 29 | 37 | 46 |
| | 0.025 | — | 22 | 30 | 38 | 48 |
| | 0.01 | — | — | — | 39 | 49 |
| | 0.005 | — | — | — | — | — |
| 4 | 0.10 | 16 | 23 | 31 | 40 | 49 |
| | 0.05 | 17 | 24 | 32 | 41 | 51 |
| | 0.025 | 18 | 25 | 33 | 43 | 53 |
| | 0.01 | — | 26 | 35 | 44 | 54 |
| | 0.005 | — | — | — | 45 | 55 |
| 5 | 0.10 | 18 | 26 | 34 | 44 | 54 |
| | 0.05 | 20 | 27 | 36 | 46 | 56 |
| | 0.025 | 21 | 28 | 37 | 47 | 58 |
| | 0.01 | — | 30 | 39 | 49 | 60 |
| | 0.005 | — | — | 40 | 50 | 61 |
| 6 | 0.10 | 21 | 29 | 38 | 48 | 59 |
| | 0.05 | 22 | 30 | 40 | 50 | 61 |
| | 0.025 | 23 | 32 | 41 | 52 | 63 |
| | 0.01 | 24 | 33 | 43 | 54 | 65 |
| | 0.005 | — | 34 | 44 | 55 | 67 |
| 7 | 0.10 | 23 | 31 | 41 | 52 | 63 |
| | 0.05 | 24 | 33 | 43 | 54 | 66 |
| | 0.025 | 26 | 35 | 45 | 56 | 68 |
| | 0.01 | 27 | 36 | 47 | 58 | 71 |
| | 0.005 | — | 37 | 48 | 60 | 72 |
| 8 | 0.10 | 25 | 34 | 44 | 56 | 68 |
| | 0.05 | 27 | 36 | 47 | 58 | 71 |
| | 0.025 | 28 | 38 | 49 | 61 | 73 |
| | 0.01 | 29 | 39 | 51 | 63 | 76 |
| | 0.005 | 30 | 40 | 52 | 65 | 78 |
| 9 | 0.10 | 27 | 37 | 48 | 60 | 72 |
| | 0.05 | 29 | 39 | 50 | 63 | 76 |
| | 0.025 | 31 | 41 | 53 | 65 | 78 |
| | 0.01 | 32 | 43 | 55 | 68 | 81 |
| | 0.005 | 33 | 44 | 56 | 70 | 84 |
| 10 | 0.10 | 29 | 40 | 51 | 64 | 77 |
| | 0.05 | 31 | 42 | 54 | 67 | 80 |
| | 0.025 | 33 | 44 | 56 | 69 | 83 |
| | 0.01 | 34 | 46 | 59 | 72 | 87 |
| | 0.005 | 36 | 48 | 61 | 74 | 89 |

Mtest=23.5> Mleft=19 (Mright=31) Do NOT reject

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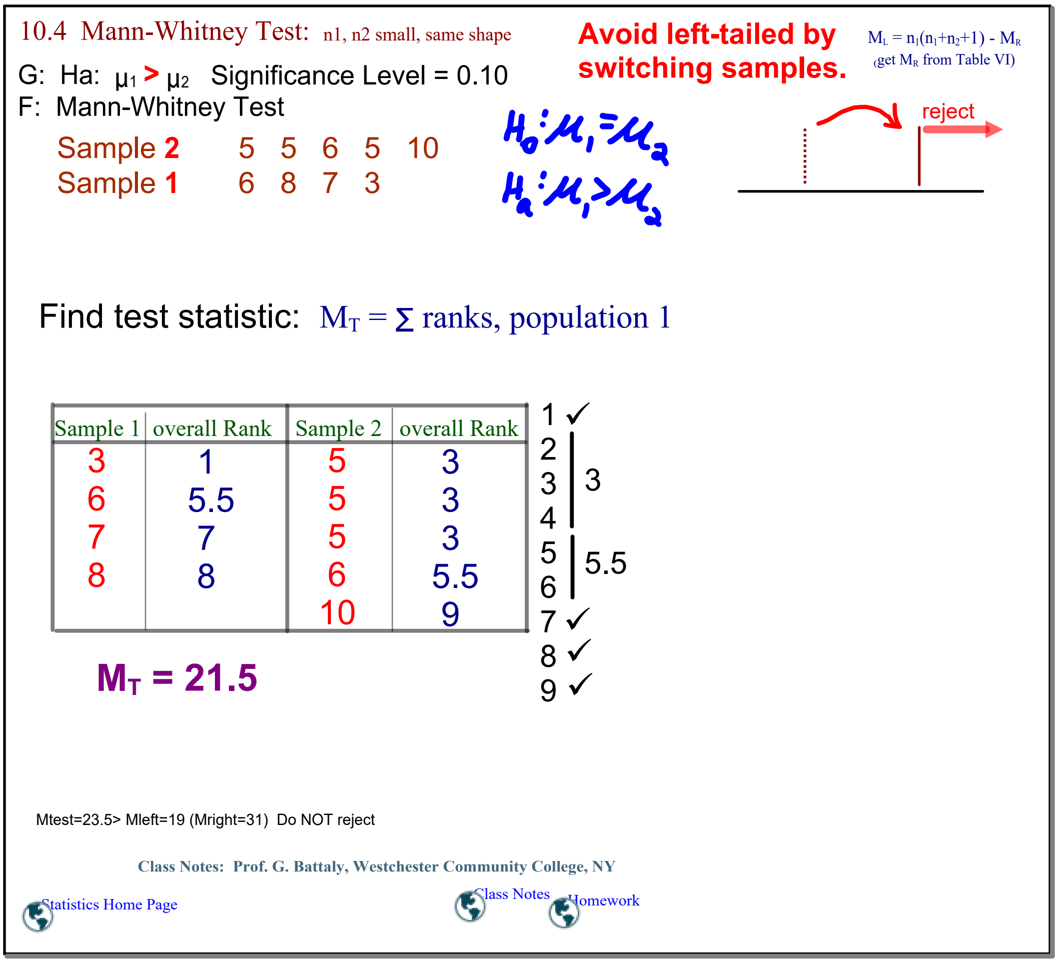

10.4 Mann-Whitney Test: n1, n2 small, same shape

**Avoid left-tailed by switching samples.**

$M_L = n_1(n_1+n_2+1) - M_R$ (get $M_R$ from Table VI)

G: Ha: $\mu_1 > \mu_2$ Significance Level = 0.10

F: Mann-Whitney Test

| | | | | | |
|---|---|---|---|---|---|
| Sample **2** | 5 | 5 | 6 | 5 | 10 |
| Sample **1** | 6 | 8 | 7 | 3 | |

$H_0: \mu_1 = \mu_2$

$H_a: \mu_1 > \mu_2$

Find test statistic: $M_T = \Sigma$ ranks, population 1

| Sample 1 | overall Rank | Sample 2 | overall Rank |
|---|---|---|---|
| 3 | 1 | 5 | 3 |
| 6 | 5.5 | 5 | 3 |
| 7 | 7 | 5 | 3 |
| 8 | 8 | 6 | 5.5 |
| | | 10 | 9 |

1 ✓
2
3 | 3
4
5 | 5.5
6
7 ✓
8 ✓
9 ✓

**$M_T = 21.5$**

Mtest=23.5> Mleft=19 (Mright=31) Do NOT reject

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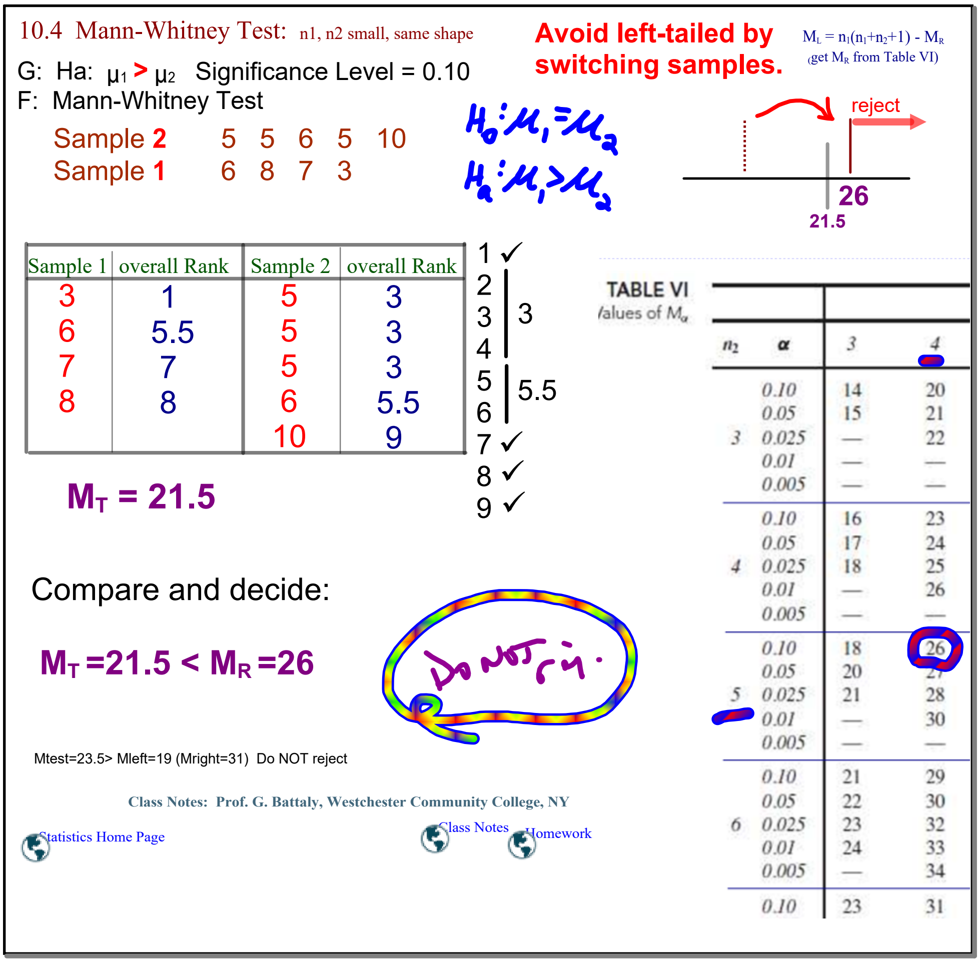

10.4 Mann-Whitney Test: n1, n2 small, same shape

**Avoid left-tailed by switching samples.**

$M_L = n_1(n_1+n_2+1) - M_R$ (get $M_R$ from Table VI)

G: Ha: $\mu_1 > \mu_2$ Significance Level = 0.10

F: Mann-Whitney Test

| | | | | | |
|---|---|---|---|---|---|
| Sample **2** | 5 | 5 | 6 | 5 | 10 |
| Sample **1** | 6 | 8 | 7 | 3 | |

$H_0: \mu_1 = \mu_2$

$H_a: \mu_1 > \mu_2$

| Sample 1 | overall Rank | Sample 2 | overall Rank |
|---|---|---|---|
| 3 | 1 | 5 | 3 |
| 6 | 5.5 | 5 | 3 |
| 7 | 7 | 5 | 3 |
| 8 | 8 | 6 | 5.5 |
| | | 10 | 9 |

1 ✓
2
3 | 3
4
5 | 5.5
6
7 ✓
8 ✓
9 ✓

**$M_T = 21.5$**

TABLE VI
Values of $M_\alpha$

| $n_2$ | $\alpha$ | 3 | 4 |
|---|---|---|---|
| | 0.10 | 14 | 20 |
| | 0.05 | 15 | 21 |
| 3 | 0.025 | — | 22 |
| | 0.01 | — | — |
| | 0.005 | — | — |
| | 0.10 | 16 | 23 |
| | 0.05 | 17 | 24 |
| 4 | 0.025 | 18 | 25 |
| | 0.01 | — | 26 |
| | 0.005 | — | — |
| | 0.10 | 18 | 26 |
| | 0.05 | 20 | 27 |
| 5 | 0.025 | 21 | 28 |
| | 0.01 | — | 30 |
| | 0.005 | — | — |
| | 0.10 | 21 | 29 |
| | 0.05 | 22 | 30 |
| 6 | 0.025 | 23 | 32 |
| | 0.01 | 24 | 33 |
| | 0.005 | — | 34 |
| | 0.10 | 23 | 31 |

Compare and decide:

**$M_T = 21.5 < M_R = 26$**

Do NOT rej.

Mtest=23.5> Mleft=19 (Mright=31) Do NOT reject

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## 10.4 Mann-Whitney Test: n1, n2 small, same shape

G: Independent random samples of picoplankton from North and South sections of San Francisco Bay.

F: At 5% significance level, are the concentrations of picoplankton different?

$H_0: \eta_N = \eta_S$
$H_a: \eta_N \neq \eta_S$
$H_0: \mu_1 = \mu_2$
$H_a: \mu_1 \neq \mu_2$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| North | 16.2 | 11.2 | 24.8 | 36.4 | 15.0 | 23.6 | 12.1 |
| South | 9.8 | 18.7 | 26.0 | 7.4 | 15.0 | | |

.02

| North $n=7$ | Rank | South $n=5$ | Rank |
|---|---|---|---|
| | | | |

$M_L = n_1(n_1+n_2+1) - M_R$
(get $M_R$ from Table VI)

Σ ranks of
1st sample: $M_t$ = _____

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## 10.4 Mann-Whitney Test: n1, n2 small, same shape

G: Independent random samples of picoplankton from North and South sections of San Francisco Bay. (~ same shape)

F: At 5% significance level, are the concentrations of picoplankton different?

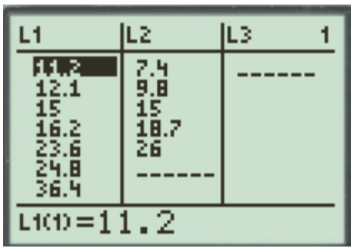
| | | | | | | | |
|---|---|---|---|---|---|---|---|
| North | 16.2 | 11.2 | 24.8 | 36.4 | 15.0 | 23.6 | 12.1 |
| South | 9.8 | 18.7 | 26.0 | 7.4 | 15.0 | | |

$H_0: \eta_N = \eta_S$ $\quad H_0: \mu_1 = \mu_2$

$H_a: \eta_N \neq \eta_S$ $\quad H_a: \mu_1 \neq \mu_2$

calc

| North $n=7$ | Rank | South $n=5$ | Rank |
|---|---|---|---|
| | | | |



```
L1      L2      L3     1
11.2    7.4     ------
12.1    9.8
15      15
16.2    18.7
23.6    26
24.8    ------
36.4
L1(1)=11.2
```

$M_L = n_1(n_1+n_2+1) - M_R$
(get $M_R$ from Table VI)

Σ ranks of
1st sample: $M_t$ = _____

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## 10.4 Mann-Whitney Test: n1, n2 small, same shape

7
5
12

G: Independent samples of picoplankton from North and South sections of San Francisco Bay.
F: At 5% significance level, are the concentrations of picoplankton different?

North 16.2 11.2 24.8 36.4 15.0 23.6 12.1
South 9.8 18.7 26.0 7.4 15.0

$H_0: \eta_N = \eta_S$ $H_0: \mu_1 = \mu_2$
$H_a: \eta_N \neq \eta_S$ $H_a: \mu_1 \neq \mu_2$

| North $n=7$ | Rank | South $n=5$ | Rank |
|---|---|---|---|
| 11.2 | | 7.4 | |
| 12.1 | | 9.8 | |
| 15 | | 15 | |
| 16.2 | | 18.7 | |
| 23.6 | | 26 | |
| 24.8 | | | |
| 36.4 | | | |

1
2
3
4
5
6
7
8
9
10
11
12

∑ ranks of
1st sample: $M_t$ = ______

$M_L = n_1(n_1+n_2+1) - M_R$
(get $M_R$ from Table VI)

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## 10.4 Mann-Whitney Test: n1, n2 small, same shape

7
5
12

G: Independent samples of picoplankton from North and South sections of San Francisco Bay.
F: At 5% significance level, are the concentrations of picoplankton different?

North 16.2 11.2 24.8 36.4 15.0 23.6 12.1
South 9.8 18.7 26.0 7.4 15.0

$H_0: \eta_N = \eta_S$ $H_0: \mu_1 = \mu_2$
$H_a: \eta_N \neq \eta_S$ $H_a: \mu_1 \neq \mu_2$

| North $n=7$ | Rank | South $n=5$ | Rank |
|---|---|---|---|
| 11.2 | 3 | 7.4 | 1 |
| 12.1 | 4 | 9.8 | 2 |
| 15 | 5.5 | 15 | 5.5 |
| 16.2 | 7 | 18.7 | 8 |
| 23.6 | 9 | 26 | 11 |
| 24.8 | 10 | | |
| 36.4 | 12 | | |

1 ✓
2 ✓
3 ✓
4 ✓
5 | 5.5
6 |
7 ✓
8 ✓
9 ✓
10 ✓
11 ✓
12 ✓

∑ ranks of
1st sample: $M_t$ = ______

$M_L = n_1(n_1+n_2+1) - M_R$
(get $M_R$ from Table VI)

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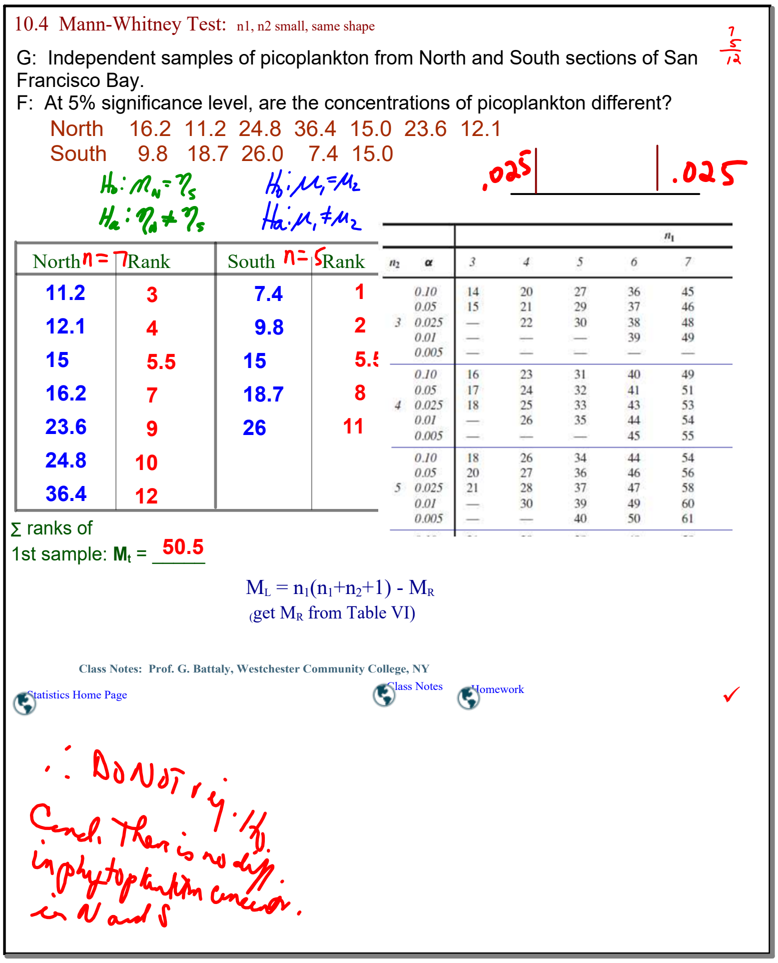

10.4 Mann-Whitney Test: n1, n2 small, same shape

$\frac{7}{\frac{5}{12}}$

G: Independent samples of picoplankton from North and South sections of San Francisco Bay.
F: At 5% significance level, are the concentrations of picoplankton different?

North 16.2 11.2 24.8 36.4 15.0 23.6 12.1
South 9.8 18.7 26.0 7.4 15.0

$H_0: \eta_N = \eta_S$ $H_0: \mu_1 = \mu_2$
$H_a: \eta_N \neq \eta_S$ $H_a: \mu_1 \neq \mu_2$

.025 | | .025

| North $n=7$ | Rank | South $n=5$ | Rank |
|---|---|---|---|
| 11.2 | 3 | 7.4 | 1 |
| 12.1 | 4 | 9.8 | 2 |
| 15 | 5.5 | 15 | 5.5 |
| 16.2 | 7 | 18.7 | 8 |
| 23.6 | 9 | 26 | 11 |
| 24.8 | 10 | | |
| 36.4 | 12 | | |

| $n_2$ | $\alpha$ | $n_1$ = 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|
| 3 | 0.10 | 14 | 20 | 27 | 36 | 45 |
| | 0.05 | 15 | 21 | 29 | 37 | 46 |
| | 0.025 | — | 22 | 30 | 38 | 48 |
| | 0.01 | — | — | — | 39 | 49 |
| | 0.005 | — | — | — | — | — |
| 4 | 0.10 | 16 | 23 | 31 | 40 | 49 |
| | 0.05 | 17 | 24 | 32 | 41 | 51 |
| | 0.025 | 18 | 25 | 33 | 43 | 53 |
| | 0.01 | — | 26 | 35 | 44 | 54 |
| | 0.005 | — | — | — | 45 | 55 |
| 5 | 0.10 | 18 | 26 | 34 | 44 | 54 |
| | 0.05 | 20 | 27 | 36 | 46 | 56 |
| | 0.025 | 21 | 28 | 37 | 47 | 58 |
| | 0.01 | — | 30 | 39 | 49 | 60 |
| | 0.005 | — | — | 40 | 50 | 61 |

Σ ranks of 1st sample: $M_t$ = 50.5

$M_L = n_1(n_1+n_2+1) - M_R$
(get $M_R$ from Table VI)

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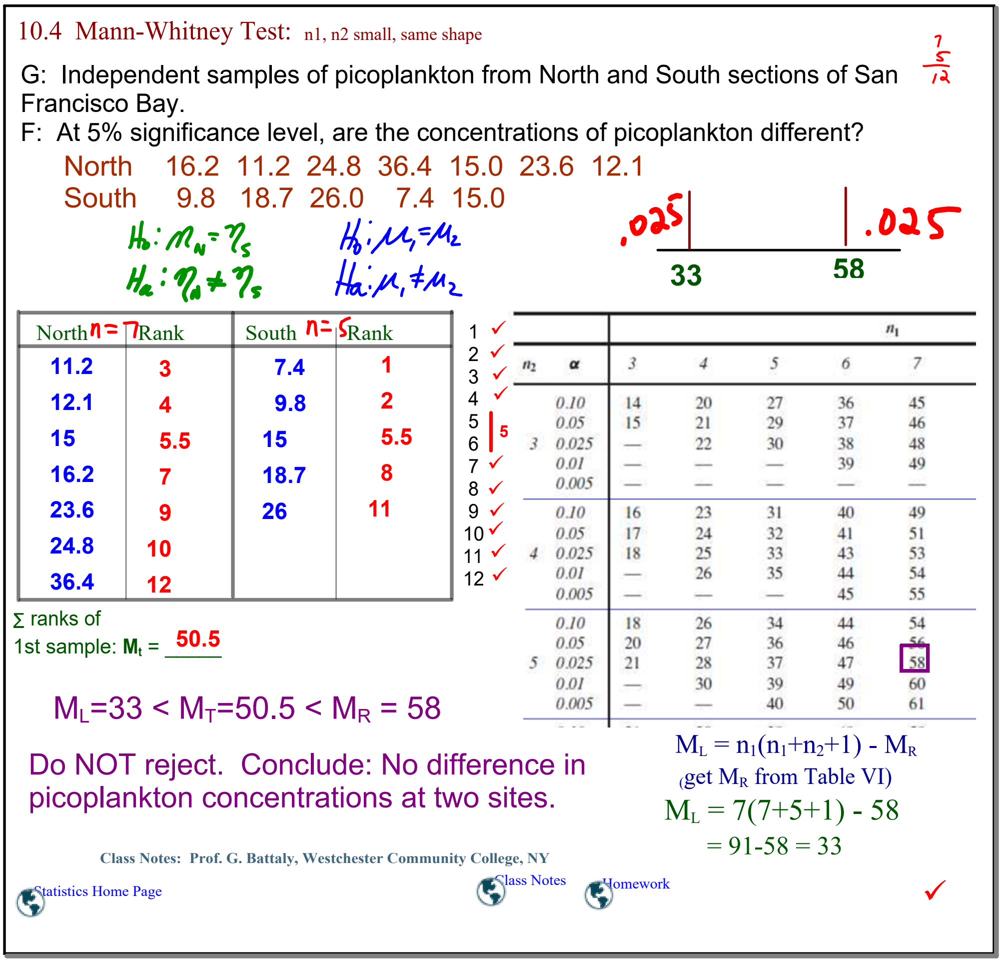
∴ Do NOT rej. $H_0$.
Concl. There is no diff. in phytoplankton concentr. in N and S.



10.4 Mann-Whitney Test: n1, n2 small, same shape

$\frac{7}{\frac{5}{12}}$

G: Independent samples of picoplankton from North and South sections of San Francisco Bay.
F: At 5% significance level, are the concentrations of picoplankton different?

North 16.2 11.2 24.8 36.4 15.0 23.6 12.1
South 9.8 18.7 26.0 7.4 15.0

$H_0: \eta_N = \eta_S$ $H_0: \mu_1 = \mu_2$
$H_a: \eta_N \neq \eta_S$ $H_a: \mu_1 \neq \mu_2$

.025 | | .025
33 58

| North $n=7$ | Rank | South $n=5$ | Rank |
|---|---|---|---|
| 11.2 | 3 | 7.4 | 1 |
| 12.1 | 4 | 9.8 | 2 |
| 15 | 5.5 | 15 | 5.5 |
| 16.2 | 7 | 18.7 | 8 |
| 23.6 | 9 | 26 | 11 |
| 24.8 | 10 | | |
| 36.4 | 12 | | |

1 ✓
2 ✓
3 ✓
4 ✓
5 | 5
6 |
7 ✓
8 ✓
9 ✓
10 ✓
11 ✓
12 ✓

| $n_2$ | $\alpha$ | $n_1$ = 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|
| 3 | 0.10 | 14 | 20 | 27 | 36 | 45 |
| | 0.05 | 15 | 21 | 29 | 37 | 46 |
| | 0.025 | — | 22 | 30 | 38 | 48 |
| | 0.01 | — | — | — | 39 | 49 |
| | 0.005 | — | — | — | — | — |
| 4 | 0.10 | 16 | 23 | 31 | 40 | 49 |
| | 0.05 | 17 | 24 | 32 | 41 | 51 |
| | 0.025 | 18 | 25 | 33 | 43 | 53 |
| | 0.01 | — | 26 | 35 | 44 | 54 |
| | 0.005 | — | — | — | 45 | 55 |
| 5 | 0.10 | 18 | 26 | 34 | 44 | 54 |
| | 0.05 | 20 | 27 | 36 | 46 | 56 |
| | 0.025 | 21 | 28 | 37 | 47 | $\boxed{58}$ |
| | 0.01 | — | 30 | 39 | 49 | 60 |
| | 0.005 | — | — | 40 | 50 | 61 |

Σ ranks of 1st sample: $M_t$ = 50.5

$M_L = 33 < M_T = 50.5 < M_R = 58$

Do NOT reject. Conclude: No difference in picoplankton concentrations at two sites.

$M_L = n_1(n_1+n_2+1) - M_R$
(get $M_R$ from Table VI)
$M_L = 7(7+5+1) - 58$
$= 91-58 = 33$

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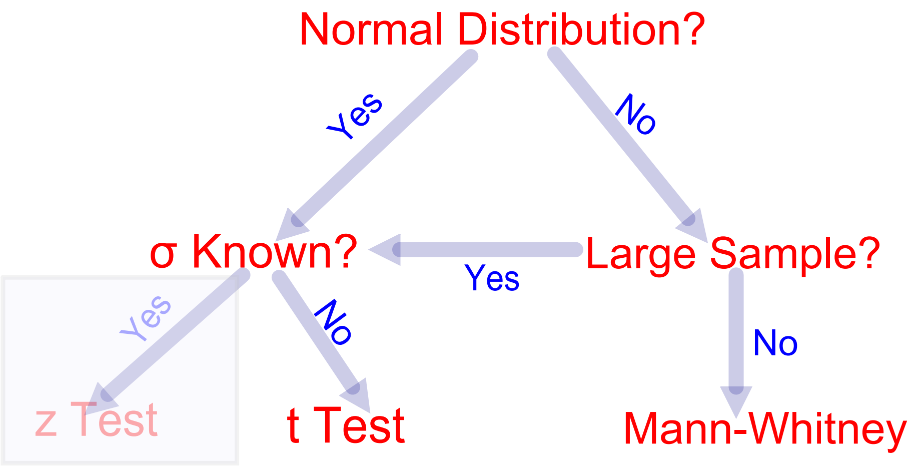
## 10.4 Mann-Whitney Test: n1, n2 small, same shape



Normal Distribution?

Yes

No

σ Known?

Yes

Large Sample?

Yes

No

No

z Test

t Test

Mann-Whitney

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Class Notes

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## 10.4 Mann-Whitney Test: n1, n2 small, same shape

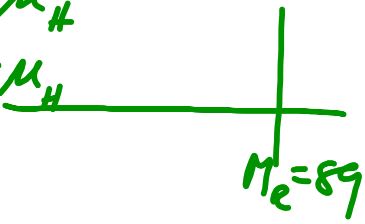
G: $\alpha = 0.10$ F: Does % of S exceed % of H

| S | H |
|---|---|
| 17 | 10.1 |
| 21 | 11.7 |
| 25 | 15.2 |
| 25 | 15.6 |
| 27 | 23.9 |
| 27 | 30.3 |
| 38 | 41.1 |
| 39 | |
| 40 | |

1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16

$H_0: \mu_S = \mu_H$

$H_a: \mu_S > \mu_H$



$M_e = 89$

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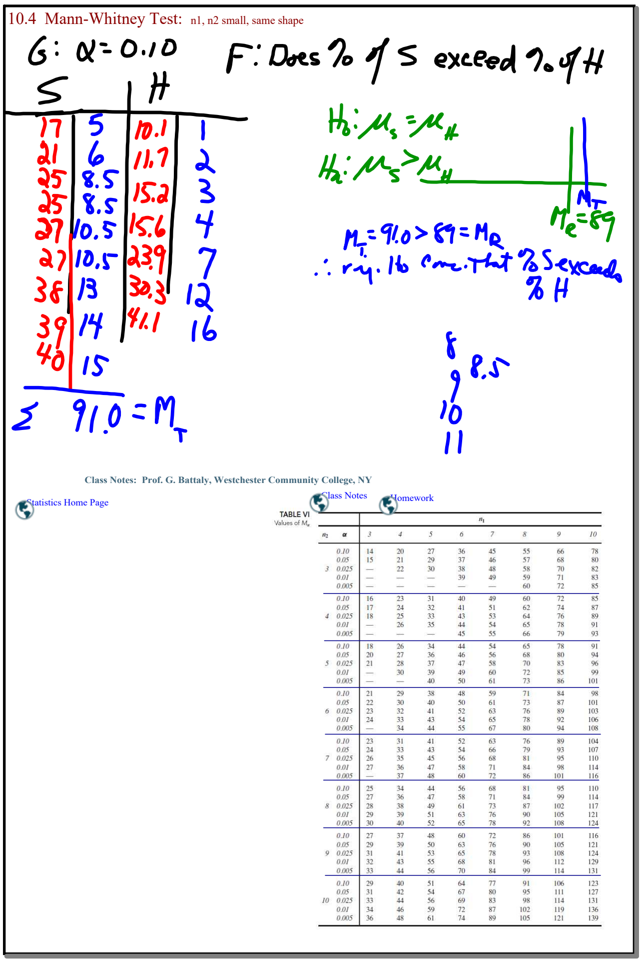

10.4 Mann-Whitney Test: n1, n2 small, same shape

G: $\alpha = 0.10$

F: Does % of S exceed % of H

| S | | H | |
|---|---|---|---|
| 17 | 5 | 10.1 | 1 |
| 21 | 6 | 11.7 | 2 |
| 25 | 8.5 | 15.2 | 3 |
| 25 | 8.5 | 15.6 | 4 |
| 27 | 10.5 | 23.9 | 7 |
| 27 | 10.5 | 30.3 | 12 |
| 38 | 13 | 41.1 | 16 |
| 39 | 14 | | |
| 40 | 15 | | |

$\Sigma$ $91.0 = M_T$

$H_0: \mu_S = \mu_H$

$H_a: \mu_S > \mu_H$

$M_T$

$M_R = 89$

$M_T = 91.0 > 89 = M_R$

$\therefore$ rej. $H_0$ conc. that % S exceeds % H

8
9 8.5
10
11

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TABLE VI
Values of $M_\alpha$

| | | $n_1$ | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $n_2$ | $\alpha$ | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 3 | 0.10 | 14 | 20 | 27 | 36 | 45 | 55 | 66 | 78 |
| | 0.05 | 15 | 21 | 29 | 37 | 46 | 57 | 68 | 80 |
| | 0.025 | — | 22 | 30 | 38 | 48 | 58 | 70 | 82 |
| | 0.01 | — | — | — | 39 | 49 | 59 | 71 | 83 |
| | 0.005 | — | — | — | — | — | 60 | 72 | 85 |
| 4 | 0.10 | 16 | 23 | 31 | 40 | 49 | 60 | 72 | 85 |
| | 0.05 | 17 | 24 | 32 | 41 | 51 | 62 | 74 | 87 |
| | 0.025 | 18 | 25 | 33 | 43 | 53 | 64 | 76 | 89 |
| | 0.01 | — | 26 | 35 | 44 | 54 | 65 | 78 | 91 |
| | 0.005 | — | — | — | 45 | 55 | 66 | 79 | 93 |
| 5 | 0.10 | 18 | 26 | 34 | 44 | 54 | 65 | 78 | 91 |
| | 0.05 | 20 | 27 | 36 | 46 | 56 | 68 | 80 | 94 |
| | 0.025 | 21 | 28 | 37 | 47 | 58 | 70 | 83 | 96 |
| | 0.01 | — | 30 | 39 | 49 | 60 | 72 | 85 | 99 |
| | 0.005 | — | — | 40 | 50 | 61 | 73 | 86 | 101 |
| 6 | 0.10 | 21 | 29 | 38 | 48 | 59 | 71 | 84 | 98 |
| | 0.05 | 22 | 30 | 40 | 50 | 61 | 73 | 87 | 101 |
| | 0.025 | 23 | 32 | 41 | 52 | 63 | 76 | 89 | 103 |
| | 0.01 | 24 | 33 | 43 | 54 | 65 | 78 | 92 | 106 |
| | 0.005 | — | 34 | 44 | 55 | 67 | 80 | 94 | 108 |
| 7 | 0.10 | 23 | 31 | 41 | 52 | 63 | 76 | 89 | 104 |
| | 0.05 | 24 | 33 | 43 | 54 | 66 | 79 | 93 | 107 |
| | 0.025 | 26 | 35 | 45 | 56 | 68 | 81 | 95 | 110 |
| | 0.01 | 27 | 36 | 47 | 58 | 71 | 84 | 98 | 114 |
| | 0.005 | — | 37 | 48 | 60 | 72 | 86 | 101 | 116 |
| 8 | 0.10 | 25 | 34 | 44 | 56 | 68 | 81 | 95 | 110 |
| | 0.05 | 27 | 36 | 47 | 58 | 71 | 84 | 99 | 114 |
| | 0.025 | 28 | 38 | 49 | 61 | 73 | 87 | 102 | 117 |
| | 0.01 | 29 | 39 | 51 | 63 | 76 | 90 | 105 | 121 |
| | 0.005 | 30 | 40 | 52 | 65 | 78 | 92 | 108 | 124 |
| 9 | 0.10 | 27 | 37 | 48 | 60 | 72 | 86 | 101 | 116 |
| | 0.05 | 29 | 39 | 50 | 63 | 76 | 90 | 105 | 121 |
| | 0.025 | 31 | 41 | 53 | 65 | 78 | 93 | 108 | 124 |
| | 0.01 | 32 | 43 | 55 | 68 | 81 | 96 | 112 | 129 |
| | 0.005 | 33 | 44 | 56 | 70 | 84 | 99 | 114 | 131 |
| 10 | 0.10 | 29 | 40 | 51 | 64 | 77 | 91 | 106 | 123 |
| | 0.05 | 31 | 42 | 54 | 67 | 80 | 95 | 111 | 127 |
| | 0.025 | 33 | 44 | 56 | 69 | 83 | 98 | 114 | 131 |
| | 0.01 | 34 | 46 | 59 | 72 | 87 | 102 | 119 | 136 |
| | 0.005 | 36 | 48 | 61 | 74 | 89 | 105 | 121 | 139 |

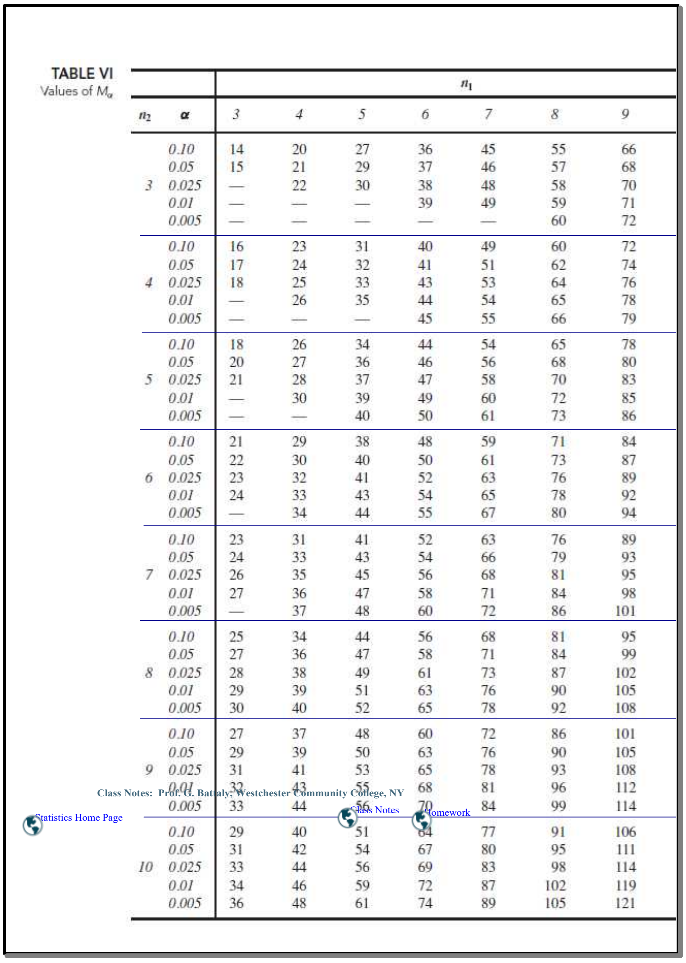

TABLE VI
Values of $M_\alpha$

| | | $n_1$ | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $n_2$ | $\alpha$ | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 3 | 0.10 | 14 | 20 | 27 | 36 | 45 | 55 | 66 |
| | 0.05 | 15 | 21 | 29 | 37 | 46 | 57 | 68 |
| | 0.025 | — | 22 | 30 | 38 | 48 | 58 | 70 |
| | 0.01 | — | — | — | 39 | 49 | 59 | 71 |
| | 0.005 | — | — | — | — | — | 60 | 72 |
| 4 | 0.10 | 16 | 23 | 31 | 40 | 49 | 60 | 72 |
| | 0.05 | 17 | 24 | 32 | 41 | 51 | 62 | 74 |
| | 0.025 | 18 | 25 | 33 | 43 | 53 | 64 | 76 |
| | 0.01 | — | 26 | 35 | 44 | 54 | 65 | 78 |
| | 0.005 | — | — | — | 45 | 55 | 66 | 79 |
| 5 | 0.10 | 18 | 26 | 34 | 44 | 54 | 65 | 78 |
| | 0.05 | 20 | 27 | 36 | 46 | 56 | 68 | 80 |
| | 0.025 | 21 | 28 | 37 | 47 | 58 | 70 | 83 |
| | 0.01 | — | 30 | 39 | 49 | 60 | 72 | 85 |
| | 0.005 | — | — | 40 | 50 | 61 | 73 | 86 |
| 6 | 0.10 | 21 | 29 | 38 | 48 | 59 | 71 | 84 |
| | 0.05 | 22 | 30 | 40 | 50 | 61 | 73 | 87 |
| | 0.025 | 23 | 32 | 41 | 52 | 63 | 76 | 89 |
| | 0.01 | 24 | 33 | 43 | 54 | 65 | 78 | 92 |
| | 0.005 | — | 34 | 44 | 55 | 67 | 80 | 94 |
| 7 | 0.10 | 23 | 31 | 41 | 52 | 63 | 76 | 89 |
| | 0.05 | 24 | 33 | 43 | 54 | 66 | 79 | 93 |
| | 0.025 | 26 | 35 | 45 | 56 | 68 | 81 | 95 |
| | 0.01 | 27 | 36 | 47 | 58 | 71 | 84 | 98 |
| | 0.005 | — | 37 | 48 | 60 | 72 | 86 | 101 |
| 8 | 0.10 | 25 | 34 | 44 | 56 | 68 | 81 | 95 |
| | 0.05 | 27 | 36 | 47 | 58 | 71 | 84 | 99 |
| | 0.025 | 28 | 38 | 49 | 61 | 73 | 87 | 102 |
| | 0.01 | 29 | 39 | 51 | 63 | 76 | 90 | 105 |
| | 0.005 | 30 | 40 | 52 | 65 | 78 | 92 | 108 |
| 9 | 0.10 | 27 | 37 | 48 | 60 | 72 | 86 | 101 |
| | 0.05 | 29 | 39 | 50 | 63 | 76 | 90 | 105 |
| | 0.025 | 31 | 41 | 53 | 65 | 78 | 93 | 108 |
| | 0.01 | 32 | 43 | 55 | 68 | 81 | 96 | 112 |
| | 0.005 | 33 | 44 | 56 | 70 | 84 | 99 | 114 |
| 10 | 0.10 | 29 | 40 | 51 | 64 | 77 | 91 | 106 |
| | 0.05 | 31 | 42 | 54 | 67 | 80 | 95 | 111 |
| | 0.025 | 33 | 44 | 56 | 69 | 83 | 98 | 114 |
| | 0.01 | 34 | 46 | 59 | 72 | 87 | 102 | 119 |
| | 0.005 | 36 | 48 | 61 | 74 | 89 | 105 | 121 |

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## 10.4 Mann-Whitney Test: n1, n2 small, same shape

At 10% s.l. $\alpha = 0.10$ is there a diff. in # Days of Ice on the 2 Lakes?

### Number of days of ice on lake

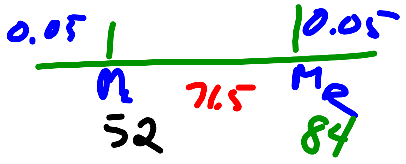
| Me | | Mo | |
|---|---|---|---|
| 47 | 1.5 | 47 | 1.5 |
| 53 | 4 | 52 | 3 |
| 74 | 5 | 83 | 6 |
| 87 | 7 | 91 | 8 |
| 102 | 10 | 96 | 9 |
| 108 | 13 | 107 | 11 |
| 115 | 15 | 108 | 13 |
| 119 | 16 | 108 | 13 |
| | $\boxed{71.5}$ | | |

# Days

1, 2 — 1.5
3 ✓
4 ✓
5 ✓
6 ✓
7 ✓
8 ✓
9 ✓
10 ✓
11 ✓
12, 13, 14 — 13
15 ✓
16 ✓

$H_0: \mu_1 = \mu_2$

$H_a: \mu_1 \neq \mu_2$



$M_T = 71.5$

$M_L = n_1(n_1+n_2+1) - M_R$
$= 8(8+8+1) - 84 =$
$136 - 84 = 52$

$52 < 71.5 < 84$

DO NOT rej.

Concl.: there is no diff. in # days of ice on the 2 lakes.