

10.2 Two Population Means: $=\sigma$'s *Pooled t-test*

GOALS:

1. Compare 2 sample means when the population standard deviations are believed to be the same but are not known.
2. Use the distribution of the difference between the sample means to evaluate the samples.
3. Arrive at a conclusion: are the means from the same population or are they different?

Study Ch. 10.2, # 33-43 (27-37), 48 (42), 49
[old 43 different]

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10.2 Two Population Means: $=\sigma$'s

Can standardize to SNC, but rarely have a known δ ,
so will not consider a z test (p.492)

$$Z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

Recap

Instead, either of two *t*-tests:

- $\sigma_1 = \sigma_2$ --> **pooled *t***
 s_1 and s_2 are estimates of same σ
- $\sigma_1 \neq \sigma_2$ --> **non-pooled *t***
 s_1 and s_2 are not known to estimate same σ

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10.2 Two Population Means: $=\sigma$'s

Instead, either of two *t*-tests:

- $\sigma_1 = \sigma_2 \rightarrow$ pooled *t*
 s_1 and s_2 are estimates of same σ

Using s_1 and s_2 as estimates of same σ , s_p is computed by weighting each sample s by the size of the sample it represents.

$$s_p = s_{pooled} = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$$

No need to memorize. Use textbook, **pooled-t test**, and copy formula down in your solution.

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10.2 Two Population Means: $=\sigma$'s

Pooled - *t* Test

Assumptions: 1. SRS, 2. Independent samples
 3. ND or large samples 4. $=\sigma$'s 😊

$\mu_1 - \mu_2 \neq 0$
 $\mu_1 - \mu_2 < 0$
 $\mu_1 - \mu_2 > 0$

Step 1: $H_0: \mu_1 = \mu_2$ $\rightarrow \mu_1 - \mu_2 = 0$
 $H_a: \mu_1 \neq \mu_2$ or $\mu_1 < \mu_2$ or $\mu_1 > \mu_2$

Step 2: Decide α and sketch

Step 3: Compute test statistic:

$$t_T = \frac{(\bar{x}_1 - \bar{x}_2)}{s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

$-(\mu_1 - \mu_2)$

compare to previous:

$$\frac{\bar{x} - \mu}{s/\sqrt{n}} = t_+$$

Is anything missing?

where

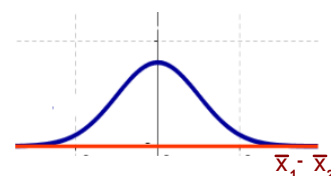
$$s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$$

$$df = n_1 + n_2 - 2$$

Step 4: Find P-value

Step 5: Decide whether to reject H_0 or not

Step 6: Verbal interpretation



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10.2 Two Population Means: σ 's

- G: * random samples of 30 males and 30 females
 * tested frame of reference, pointed S, error recorded
 * table of pointing errors, in degrees, with:

MALE					FEMALE				
13	68	60	22	30	14	78	18	32	80
130	18	5	70	8	8	69	35	35	91
39	3	9	58	20	20	111	111	12	68
33	11	59	3	67	3	3	109	27	66
10	38	5	167	26	138	128	36	8	176
13	23	86	15	19	122	31	27	3	15

$$\bar{X}_M = 37.6 \quad S_M = 38.5$$

$$\bar{X}_F = 55.8 \quad S_F = 48.3$$

F: At the 1% significance level, do the data provide sufficient evidence to conclude that, on average, males have a better sense of direction and, in particular, a better frame of reference than females?

Pooled-t Test

Assumptions: 1. SRS, 2. Independent samples
 3. ND or large samples 4. σ 's

Step 1: $H_0: \mu_1 = \mu_2$
 $H_a: \mu_1 \neq \mu_2$ or $\mu_1 < \mu_2$ or $\mu_1 > \mu_2$

Step 2: Decide α and sketch

Step 3: Compute test statistic: $t = \frac{(\bar{x}_1 - \bar{x}_2)}{s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$

where $s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$ $df = n_1 + n_2 - 2$

Step 4: Find P-value

Step 5: Decide whether to reject H_0 or not

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$$\bar{X}_M = 37.6 \quad S_M = 38.5$$

$$\bar{X}_F = 55.8 \quad S_F = 48.3$$

test $\sigma_M = \sigma_F?$

Assumptions: 1. SRS, 2. Independent sample
 3. ND or large samples 4. σ 's

Step 1: $H_0: \mu_1 = \mu_2$

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where $s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$

Step 4: Find P-value

OR Find CV

$df = n_1 + n_2 - 2$ Table IV

Step 5: Decide whether to reject H_0 or not

Step 6: Verbal interpretation

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10.2 Two Population Means: σ 's

Pooled or Non-pooled Variances: = ? or Not ?

- If **sample sizes are equal** and s_1 and s_2 are similar, assumption of equal population variance may be reasonable and the **pooled procedure** can be used.
- If **sample sizes are equal** and s_1 and s_2 are different, use **non-pooled procedure**.

-
- If **sample sizes are very different** and s_1 and s_2 are similar, and the larger sample size produced the larger standard deviation, the **pooled procedure** is acceptable because it will be conservative.
 - If **sample sizes are very different** and s_1 and s_2 are different, **do not use the pooled procedure**. The pooled test can be quite misleading unless sample standard deviations are similar, especially if the smaller standard deviation accompanies the larger sample size.

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$\bar{X}_M = 37.6$ $S_M = 38.5$
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Step 1: $H_0: \mu_1 = \mu_2$
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Step 4: Find P-value OR Find CV(s)
 $df = n_1 + n_2 - 2$ Table IV

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F: At the 1% significance level, do the data provide sufficient evidence to conclude that, on average, males have a better sense of direction and, in particular, a better frame of reference than females?

① $H_0: \mu_M = \mu_F$ $H_a: \mu_M < \mu_F$
 ② $\alpha = 0.01$
 ③ $t = \frac{\bar{x}_M - \bar{x}_F}{s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{37.6 - 55.8}{\sqrt{\frac{29(38.5)^2 + 29(48.3)^2}{58}}} = -1.614$
 ④ $p = 0.056$
 ⑤ $p = 0.056 > 0.01 = \alpha$
 \therefore Do not reject H_0
 ⑥ At the 1% s.l., males do not have a better frame of reference than females.

Handwritten notes: $\sigma = ?$
 $\frac{48.3}{38.5} \approx 1.25$
 not very different

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10.2 Two Population Means: = σ 's

G: independent SRS, n.d. F: Can use pooled t test?

$$\begin{array}{llll} \text{G: } \bar{x}_1 = 115.1, & s_1 = 79.4, & n_1 = 51 \\ \bar{x}_2 = 24.3, & s_2 = 10.5, & n_2 = 19 \end{array}$$

Pooled or Non-pooled Variances: = ? or Not ?

- If sample sizes are equal and s_1 and s_2 are similar, assumption of equal population variance may be reasonable and the **pooled procedure** can be used.
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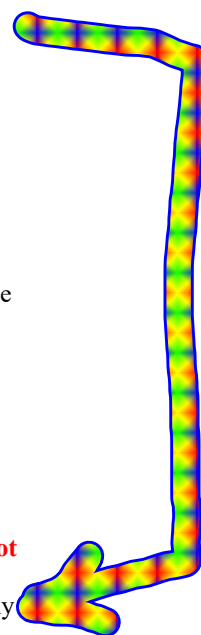
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10.2 Two Population Means: $= \sigma$'s

G: independent SRS, n.d. F: Can use pooled t test?

$$\begin{aligned} \text{G: } \bar{x}_1 &= 39.04, & s_1 &= 18.82, & n_1 &= 51 \\ \bar{x}_2 &= 49.92, & s_2 &= 18.97, & n_2 &= 53 \end{aligned}$$

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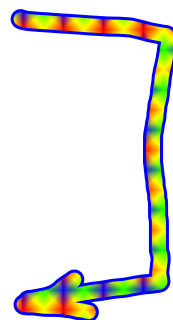
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10.2 Two Population Means: σ 's

Researchers are investigating how the amount of protein in the diet relates to weight gain. They randomly select 19 female rats, and consider their gain in weight between 28 and 84 days after birth. 12 were fed a high protein diet and 7 were fed a low protein diet. At the 95% confidence level, does the high protein diet relate to a higher weight gain?

High protein	134	146	104	119	124	161	107	83	113	129	97	123
Low protein	70	118	101	85	107	132	94					

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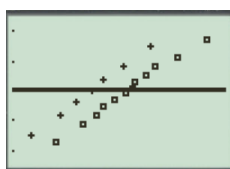
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Low protein	70	118	101	85	107	132	94					

Assumpt: srs, nd?, $\sigma_1 = \sigma_2$? NPP first, confirm nd and $\sigma_1 = \sigma_2$, then finish



~nd ✓

✓

$\sigma_1 = \sigma_2$

1-Var Stats $\bar{x}=120$ $\Sigma x=1440$ $\Sigma x^2=172832$ $Sx=21.38818705$ $\sigma x=20.47763007$ $n=12$	1-Var Stats $\bar{x}=101$ $\Sigma x=707$ $\Sigma x^2=73959$ $Sx=20.62361106$ $\sigma x=19.09375365$ $n=7$
High Protein	Low Protein

Can use pooled -t procedure.

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10.2 Two Population Means: σ 's

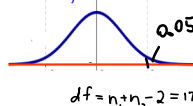
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Low protein	70	118	101	85	107	132	94					

Assumpt: srs, nd?, $\sigma_1 = \sigma_2$? NPP first, confirm, then finish

$$H_0: \mu_1 = \mu_2 \quad \alpha = 1 - .95 = .05$$

$$H_a: \mu_1 > \mu_2$$



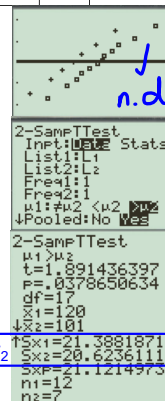
Can avoid 1-variable stats and go straight to 2-sample t-test
BUT, be sure to check s_1 and s_2 before proceeding.

$$t = \frac{(\bar{x}_1 - \bar{x}_2)}{s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{120 - 101}{21.121 \sqrt{\frac{1}{12} + \frac{1}{7}}} = 1.891$$

$$s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$$

$$= \sqrt{\frac{11(21.39)^2 + 6(20.60)^2}{12 + 7 - 2}} \quad p = 0.038 < 0.05 = \alpha$$

$$= 21.121 \quad \text{reject } H_0$$



Conclude: The high protein diet relates to increased weight gain.

(Note: Cannot reject if a 2-tailed test: $p = 2(0.038) = 0.0757 > 0.05$)

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10.2 Two Population Means: σ 's

practice for procedure

G: independent SRS, n.d. F: Use pooled t test: a) 2-tailed $\alpha=0.05$
b) 95% CI

$$G: \bar{x}_1 = 10, \quad s_1 = 4, \quad n_1 = 15$$

$$\bar{x}_2 = 12, \quad s_2 = 5, \quad n_2 = 15$$

Pooled - t Test

Assumptions: 1. SRS 2. Independent samples
3. ND or large samples 4. $\sigma_1 = \sigma_2$

Step 1: $H_0: \mu_1 = \mu_2$
 $H_a: \mu_1 \neq \mu_2$ or $\mu_1 < \mu_2$ or $\mu_1 > \mu_2$

Step 2: Decide α and sketch

Step 3: Compute test statistic: $t = \frac{(\bar{x}_1 - \bar{x}_2)}{s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$
where $s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$

Step 4: Find P-value OR Find CV(s)

df = $n_1 + n_2 - 2$ Table IV

Step 5: Decide whether to reject H_0 or not

Step 6: Verbal interpretation

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$t = -1.21$ $P = 0.237$ not reject

10.2 Two Population Means: $= \sigma$'s

practice for procedure

G: independent SRS, n.d. F: Use pooled t test: a) 2-tailed $\alpha=0.05$

b) 95% CI

G: $\bar{x}_1 = 10$, $s_1 = 4$, $n_1 = 15$
 $\bar{x}_2 = 12$, $s_2 = 5$, $n_2 = 15$

Calculator: CI for 2 means: Pooled-t Interval

STAT/TESTS

2-SampTInt

Stats/

mean1: 10

s1: 4

n1: 15

mean2: 12

s2: 5

n2: 15

C-level: 95

Pooled: yes

calculate

t=-1.21 P=0.237 not rel
 -5.39 to 1.39

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10.2 Two Population Means: $= \sigma$'s

practice for procedure

G: independent SRS, n.d. F: Use pooled t test: a) left-tailed $\alpha=0.05$

G: $\bar{x}_1 = 20$, $s_1 = 4$, $n_1 = 10$
 $\bar{x}_2 = 23$, $s_2 = 5$, $n_2 = 15$

t=-1.59 P=0.063 not rel
 -6.24 to 0.24

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10.2 Two Population Means: σ 's

practice for procedure

G: independent SRS, n.d. F: Use pooled t test: a) right-tailed $\alpha=0.05$

G: $\bar{x}_1 = 20, s_1 = 4, n_1 = 30$
 $\bar{x}_2 = 18, s_2 = 5, n_2 = 40$

t=1.80 P=0.038
 0.15 to 3.85

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10.2 Two Population Means: σ 's

p. 305 #37.

G: srs, independent samples of native species in two habitats

	Cropland	Wetland
mean	14.06	15.36
stdev	4.83	4.95
n	126	98

F: At the 5% significance level, is there a difference in the mean number of native species?

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srs, indep, n large $> \sim$ nd, $= \sigma$'s

$$\textcircled{1} H_0: \mu_1 = \mu_2 \quad H_a: \mu_1 \neq \mu_2$$

$$\textcircled{3} t = \frac{\bar{x}_1 - \bar{x}_2}{s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{14.06 - 15.36}{4.883 \sqrt{\frac{1}{126} + \frac{1}{98}}} = -1.977$$

$$s_p = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1 + n_2 - 2}} = \sqrt{\frac{125(4.83)^2 + 97(4.95)^2}{126 + 98 - 2}} = 4.883$$

$$\textcircled{4} df = 222 \quad p = 0.0493$$

$$\textcircled{5} p = 0.0493 < \alpha = 0.05$$

\therefore rej. H_0

$\textcircled{6}$ Conclude: there is a difference in the mean number of native species between cropland and wetlands.

```
2-SampTTest
μ1≠μ2
t=-1.976738143
p=.0493090103
df=222
x̄1=14.06
x̄2=15.36
```

```
2-SampTTest
μ1≠μ2
tSx1=4.83
tSx2=4.95
Sxp=4.88279522
n1=126
n2=98
```

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