9.5  *t*-test: one $\mu$, $\sigma$ unknown

**GOALS:**
1. Recognize the assumptions for a 1 mean t-test (srs, nd or large sample size, population stdev. NOT known).
2. Understand that the actual p-value (area in the tail past the test statistic) is not found on the t-table.
3. Use a calculator to find the p-value (part of t-test)
4. Test hypotheses for population means when population standard deviations are not known by applying the t-test.

Study Ch. 9.5, #101-113(89 - 101), 117(105), 119 (107)
9.5 \( t \)-test: one \( \mu \), \( \sigma \) unknown

What if \( \sigma \) is NOT Known?

From knowledge of CI's what would you expect?

Assumptions for \( t \) Test the Hypothesis Test for One \( \mu \):

1. Simple Random Sample (SRS)
2. Normal population or Large Sample
3. \( \sigma \) NOT Known

<table>
<thead>
<tr>
<th>z Test</th>
<th>t Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>( z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} )</td>
<td>( t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} )</td>
</tr>
<tr>
<td>z distribution</td>
<td>t distribution</td>
</tr>
<tr>
<td>estimates</td>
<td>( df = n - 1 )</td>
</tr>
</tbody>
</table>

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---

9.5 \( t \)-test: one \( \mu \), \( \sigma \) unknown

<table>
<thead>
<tr>
<th>Test</th>
<th>One ( \mu ), ( \sigma ) not known</th>
</tr>
</thead>
<tbody>
<tr>
<td>SRS, n.d. or large sample, ( \sigma ) not known</td>
<td></td>
</tr>
</tbody>
</table>

1. State the Null and Alternative Hypotheses: \( H_0, H_a \)

2. Decide the significance level, \( \alpha \), and sketch

3. Compute the test statistic: \( t \) for \( df = n - 1 \)

4. Find the P-value

5. Decision: Rej. \( H_0 \) if \( P \leq \alpha \)

6. Interpret results
9.5 \( t \)-test: one \( \mu \), \( \sigma \) unknown

G: srs, nd, \( \sigma \) not known.

Right-tailed test, n=11, \( t=1.246 \)

F: a) P - value  b) significance level for rej, not rej

*The actual p-value cannot be found on a t-table.
  t-table shows only 0.10, 0.05, 0.025, 0.01, 0.005

*The \( p \)-value can only be estimated from the t-table.

*Use a calculator to compute the \( p \)-value:

\[
\text{DISTR} / 6: \text{tcdf} / \quad \text{df} = n-1 = ______
\]

\[
tcdf(\text{lower bound}, \text{upper bound}, \text{df})
\]

\[
tcdf(______, 9,______) \quad p = ______
\]

\( (9 \text{ easier to type, same 4 digit result as 1EE99.} \)
\( \text{Note that t-distribution has larger tails than z-curve.} \)

Can also use STAT/TESTS/t-test if assumptions are met.

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\[
\text{G: right-tailed test, n=11, } t=1.246
\]

F: a) P - value  b) significance level for rej, not rej

DISTR / 6:tcdf /  \quad \text{df} = n-1 = 11-1 = 10

\[
tcdf(1.246,9,10)
\]

\[
p = 0.1206
\]

\( (9 \text{ easier to type, same 4 digit result as 1EE99.} \)
\( \text{Note that t-distribution has larger tails than z-curve.} \)

or:

STAT/TESTS/T-TEST

Use \( u_0 = 0, \ \bar{x} = t / \sqrt{n}, \) and \( s = 1 \)

Use \( u_0 = 0, \ \bar{x} = 1.246 / \sqrt{11}, \) and \( s = 1 \)

Select the alternative test and calculate

\[
p = 0.1206
\]

\( \alpha \) to reject?

\( \text{NOT REJECT at any } \alpha \)
9.5 t - test: one μ , σ unknown

G: srs, nd, σ not known.
G: Two-tailed test, n=8, t= 3.725
F: a) P - value  b) significance level for rej, not rej

DISTR / 6:tcdf /
tcdf(lower bound, upper bound, df)
tcdf(______, 9,____) = __________
p = 2(______) = __________
a) _______ < P = _______ < _______
b) α to reject?
   Reject at  α = _____, α = _____, α = _____
   Do NOT Reject at  α = _______

or:
STAT/TESTS/T-TEST
Use  u0 = 0,  = 3.725 /√8 , and s = 1
Select the alternative test and calculate 
p = .0074

9.5 t - test: one μ , σ unknown

G: srs, nd, σ not known.
G: Two-tailed test, n=8, t= 3.725
F: a) P - value  b) significance level for rej, not rej

calculator
DISTR / 6:tcdf /
tcdf(lower bound, upper bound, df)
tcdf(3.725,9,7) = .0037
p = 2(.0037) = 0.0074

table: compare given test t to t's from table

<table>
<thead>
<tr>
<th>df</th>
<th>1.00</th>
<th>2.00</th>
<th>2.306</th>
<th>2.576</th>
<th>3.000</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>1.415</td>
<td>1.665</td>
<td>2.365</td>
<td>2.998</td>
<td>3.499</td>
</tr>
</tbody>
</table>

a) 0.005 < P = 0.0074 < 0.01

b) α to reject?
   Reject at  α = .10, α = .05, α = .01
   Do NOT Reject at  α = .005

Calculator t-test automatically accounts for 2 tails
9.4 Hypothesis Tests: one $\mu, \sigma$ Known
G: srs, $\sigma$ not known.
G: $\bar{X} = 21, n=32, S = 4$  $H_0: \mu = 22, H_a: \mu < 22$
F: One mean $t$-test at $\alpha = 0.05$

Solving Word Problems
1. Read the problem. Try to identify the general type of problem. eg: CI, Hyp Test, specific value, etc.
2. Read the problem again, identifying what is given and what you need to find.
3. Use the Procedure Index to select a procedure.
4. Before beginning a procedure, determine if all assumptions are met.
5. If assumptions are not met, look for a different procedure. (Exception: if srs not met, then write “Assuming srs...”)  
6. If assumptions are met, follow the procedure including:
   - Draw a sketch to show $\alpha$ as left-tailed, 2-tailed, or right-tailed.
   - For Hypothesis Tests, include null and alternative hypotheses.
   - Include all equations, substitutions, and answers for the equations. (Indicate calculator or tables.)
   - Decide to reject null hypothesis or not. Explain why.
   - Write a verbal interpretation of your decision.
7. Check: Have you satisfied the to find above?
9.5 \( t \)-test: one mean, standard dev NOT known

G: srs, \( \sigma \) not known.

G: \( \bar{x} = 21, n=32, S = 4 \) \( H_0: \mu = 22, H_a: \mu < 22 \)

F: One mean \( t \)-test at \( \alpha = 0.05 \)

1. State the Null and Alternative Hypotheses: \( H_0, H_a \)

\( H_a: \mu < \mu_0 \)

2. Decide the significance level, \( \alpha \), and sketch

3. Compute the test statistic: \( t \)

4. Find the \( t \)-value

5. Decide: Reject if \( t < t_{\alpha} \)

6. Interpret results

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Class Notes  
Homework
A professor wants to know if her introductory statistics class has a good grasp of basic math. Six students are chosen at random from the class and given a math proficiency test. The professor wants the class to be able to score above 70 on the test. The six students get scores of 62, 92, 75, 68, 83, and 95. Can the professor have 90 percent confidence that the mean score for the class on the test would be above 70?

Solving Word Problems

1. Read the problem. Try to identify the general type of problem. eg: CI, Hyp Test, specific value, etc.
2. Read the problem again, identifying what is given and what you need to find.
3. Use the Procedure Index to select a procedure.
4. Before beginning a procedure, determine if all assumptions are met.
5. If assumptions are not met, look for a different procedure. (Exception: if srs not met, then write “Assuming srs... ”)
6. If assumptions are met, follow the procedure including:
   - Draw a sketch to show α as left-tailed, 2-tailed, or right-tailed.
   - For Hypothesis Tests, include null and alternative hypotheses.
   - Include all equations, substitutions, and answers for the equations. (Indicate calculator or tables.)
   - Decide to reject null hypothesis or not. Explain why.
   - Write a verbal interpretation of your decision.
7. Check: Have you satisfied the to find above?
9.5 t-test: one \( \mu, \sigma \) unknown

A professor wants to know if her introductory statistics class has a good grasp of basic math. Six students are chosen at random from the class and given a math proficiency test. The professor wants the class to be able to score above 70 on the test. The six students get scores of 62, 92, 75, 68, 83, and 95. Can the professor have 90 percent confidence that the mean score for the class on the test would be above 70?

\[ H_0: \mu = 70 \]
\[ H_a: \mu > 70 \]

\[ \alpha = 0.10 \]
\[ \bar{x} = 82.8 \pm 79.17 - 70 = 1.705 \]
\[ t = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{79.17 - 70}{13.167/\sqrt{6}} \]
\[ P = 0.0744 \]
\[ P = 0.0744 < 0.10 = \alpha \]

Reject \( H_0 \) conclude: that prof can have \(~92.6\)% confidence that the mean >70

---

A health researcher read that a 200-pound male can burn an average of 524 calories per hour playing tennis. 37 males were randomly selected and the mean number of calories burned per hour playing squash was 534.8 with a standard deviation of 45.9 calories. At the 1% significance level, do squash players burn more calories per hour than tennis players?

Problem type: ____________________________.

G: ____________________________.

F: ____________________________

**MUST DO** all steps of procedure

Can use calculator

STAT / TESTS
2: T-Test
Inpt: Data
\( \mu_0: \) 70
List: L1
\( \mu: \) \( \mu_0 \), \( \mu_0 \), \( \mu_0 \)
Calculate Draw

---
9.5 *t*-test: one \( \mu \), \( \sigma \) unknown

A health researcher read that a 200-pound male can burn an average of 524 calories per hour playing tennis. 37 males were randomly selected and the mean number of calories burned per hour playing squash was 534.8 with a standard deviation of 45.9 calories. At the 1% significance level, do squash players burn more calories per hour than tennis players?

**Problem type:** _____test: z or t?__________.

**G:** \( \mu = 524; \) srs; n=37>30 ~n.d.; \( \bar{x} = 534.8, s = 45.9; \alpha=0.01 \)

\( \sigma \) not given --> Use *t*-test

**F:** Do squash players burn more calories than tennis players?

---

**MUST DO all steps of procedure**

<table>
<thead>
<tr>
<th>Test</th>
<th>Use ( \mu_0 ) or not known</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>State the Null and Alternative Hypotheses: ( H_0, H_1 )</td>
</tr>
<tr>
<td>2.</td>
<td>Decide the significance level, ( \alpha ), and sketch</td>
</tr>
<tr>
<td>3.</td>
<td>Compute the test statistic: ( t )</td>
</tr>
<tr>
<td>4.</td>
<td>Find the P-value</td>
</tr>
<tr>
<td>5.</td>
<td>Decision: Rej. ( H_0 ) if ( P \leq \alpha )</td>
</tr>
<tr>
<td>6.</td>
<td>Interpret results</td>
</tr>
</tbody>
</table>

**Can use calculator**

**STAT / TESTS**

2: T-Test

**Inpt:** Data

\( \mu_0: \) ________

**List:** ________

\( \mu: \neq \mu_0, < \mu_0, > \mu_0 \)

Calculate   Draw

**Conclude:** At 1% s.l. squash players to not burn more calories than tennis players.

---

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**Class Notes**

**Homework**
9.5 \( t \)-test: one \( \mu \), \( \sigma \) unknown

G: srs, nd, \( \sigma \) not known.

G: \( \bar{x} = 182.7 \) yds, \( n = 6 \), S = 2.7 yds  \( H_0: \mu = 180 \), \( H_a: \mu > 180 \)

F: a) One mean \( t \)-test at \( \alpha = 0.05 \)  b) at \( \alpha = 0.01 \)

\[ 180 \]
\[ 187 \]
\[ 181 \]
\[ 182 \]
\[ 185 \]
\[ 181 \]

MUST DO all steps of procedure

<table>
<thead>
<tr>
<th>Test</th>
<th>One ( \mu ), ( \sigma ) not known</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>State the Null and Alternative Hypotheses: ( H_0 ), ( H_a )</td>
</tr>
<tr>
<td>2</td>
<td>Decide the significance level, ( \alpha ), and sketch</td>
</tr>
<tr>
<td>3</td>
<td>Compute the test statistic: ( t )</td>
</tr>
<tr>
<td>4</td>
<td>Find the critical value</td>
</tr>
<tr>
<td>5</td>
<td>Decision: Rej. ( H_0 ) if ( t ) exceeds the critical value</td>
</tr>
<tr>
<td>6</td>
<td>Interpret results</td>
</tr>
</tbody>
</table>

Can use calculator

STAT / TESTS
2: T-Test
Inpt: Data
\( \mu_0: \) ________
List: ________
\( \mu: \) \( \neq \mu_0, < \mu_0, > \mu_0 \)
Calculate Draw

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