10.1 Sampling Distribution of Differences between 2 independent sample means

## How is this different?

- G: \* random samples of 30 males and 30 females
  - \* tested frame of reference, pointed S, error recorded
  - \* table of pointing errors, in degrees, with:

		MALE			FEMALE							
13	68	60	22	30	14	78	18	32	80			
130	18	5	70	8	8	69	35	35	91			
39	3	9	58	20	20	111	111	12	68			
33	11	59	3	67	3	3	109	27	66			
10	38	5	167	26	138	128	36	8	176			
13	23	86	15	19	122	31	27	3	15			

$$\overline{X}_{M} = 37.6$$

$$\overline{X}_{E} = 55.8$$

$$S_{\rm M} = 38.5$$

$$\overline{X}_{M} = 37.6$$
  $\overline{X}_{F} = 55.8$   $S_{M} = 38.5$   $S_{F} = 48.3$ 

<- New is 2 sample test. Previous (below) is 1-sample test.



F: At the 1% significance level, do the data provide sufficient evidence to conclude that, on average, males have a better sense of direction and, in particular, a better frame of reference than females?

from 10.2

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- 10.1 Sampling Distribution of Differences between 2 Independent sample means
- 10.2 Two Population Means:  $= \sigma$ 's Pooled t-test
- 10.3 Two Population Means: σs NOT equal Non-Pooled t-test

## **GOALS:**

- 1. Consider how two samples can be compared to determine if they are the same or different or come from the same or different populations.
- 2. Consider the distribution of the difference of sample means, - the mean of the difference including:
  - the standard deviation of the difference
- 3. Use the Pooled-t Test to compare sample means when  $\sigma_1 = \sigma_2$
- 4. Use the Non-Pooled t Test to compare means when  $_{\sigma 1} \neq _{\sigma 2}$

Read Ch. 10.1, Study Key Fact 10.1

Study Ch. 10.2, # 33-43, 48, 49 Study Ch. 10.3, 67-70 all, 73-77(no CI), 81, 83

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## 10.1 Sampling Distribution of Differences between 2 independent sample means

How do we start to compare the males and females?

Know that mean for males is  $\sim$  nd for large samples.

Know that mean for females is  $\sim$  nd for large samples.

How do we compare them?

$$\overline{X}_{M} = 37.6$$
  $\overline{X}_{F} = 55.8$ 

We see that: mean for males < mean for females  $S_M = 38.5$   $S_F = 48.3$ 

or 
$$\overline{X}_{M} < \overline{X}_{F}$$

or 
$$\overline{X}_{M} - \overline{X}_{F} < 0$$

or or 
$$\overline{X}_1 = \overline{X}_2$$
  $\overline{X}_1 > \overline{X}_2$ 

We need to examine the distribution of

$$\overline{X}_M - \overline{X}_F$$

$$X_{1} = X_{2} \qquad X_{1} > X_{2}$$

$$\overline{X}_{1} - \overline{X}_{2} = 0 \qquad \overline{X}_{1} - \overline{X}_{2} > 0$$

$$\overline{X}_{1} - \overline{X}_{2} \qquad \overline{X}_{1} - \overline{X}_{2}$$

In general: the distribution of  $\overline{X}_1 - \overline{X}_2$ 

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## 10.1 Sampling Distribution of Differences between 2 independent sample means Chapter 7

the distribution of  $\overline{X}_1 - \overline{X}_2$ 

If x is  $\sim$  nd on each of the populations 1 and 2, then  $\overline{X}_1 - \overline{X}_2$  is  $\sim$ nd and:

$$\mu_{\overline{X}_1-\overline{X}_2} = \mu_1 - \mu_2$$

$$\sigma_{\overline{X}_1 - \overline{X}_2} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

## comparison

Mean of the Sample Mean

$$\mu_{\overline{x}}=\mu$$

Standard Deviation of the Sample Mean

$$\sigma_{\overline{x}} = \underline{\sigma}_{\sqrt{n}}$$

Standard Error (of the Mean)

$$\sigma_{R} = \frac{\sigma}{\sqrt{n}} = \frac{\sigma^{2}}{n}$$

$$\sigma = \sqrt{\sigma^{2}}$$

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

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10.1 Sampling Distribution of Differences between 2 independent sample means



When  $\sigma$  is known, can standardize to SNC,

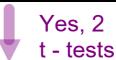
But rarely have a known  $\sigma$ , so will not consider a z test

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

$$z = \frac{(\overline{x_1} - \overline{x_2}) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$



Instead, either of two t-tests:  $\sigma_1$ ,  $\sigma_2$  unknown



•  $\sigma_1 = \sigma_2$  --> pooled t

 $s_1$  and  $s_2$  are estimates of same  $\sigma$ 

•  $\sigma_1 \neq \sigma_2$  --> non-pooled t $s_1$  and  $s_2$  are not known to estimate same  $\sigma$ 

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## 10.2 Two Population Means: = $\sigma$ 's

Instead, either of two *t*-tests:

•  $\sigma_1 = \sigma_2$  --> pooled t $s_1$  and  $s_2$  are estimates of same  $\sigma$ 

Using  $s_1$  and  $s_2$  as estimates of same  $\sigma$ ,  $s_p$  is computed by weighting each sample s by the size of the sample it represents.

$$s_p = s_{pooled} = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$$

$$df = n_1 + n_2 - 2$$

No need to memorize. Use textbook, *pooled-t test*, and copy formula down in your solution.

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## 10.2 Two Population Means: = $\sigma$ 's

# $\sigma_1 \neq \sigma_2$ Non-Pooled t Test

 $s_1$  and  $s_2$  are estimates of  $\sigma_1$  and  $\sigma_2$ , so there are actually 4 variables:  $s_1$ ,  $s_2$   $\overline{X}_1$   $\overline{X}_2$ 

This requires a different way to compute the **Degrees of Freedom** 

$$\Delta = \frac{\left[\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right]^2}{\frac{(s_1^2/n_1)^2}{n_1 - 1} + \frac{(s_2^2/n_2)^2}{n_2 - 1}}$$

will get from calculator,

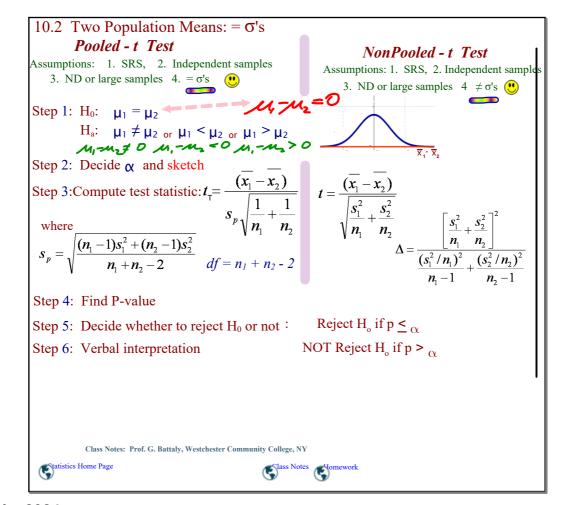
write df value for a problem.



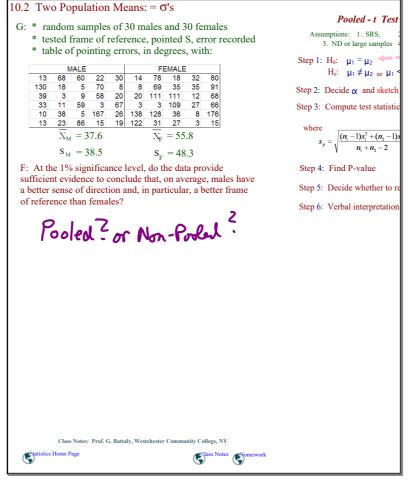
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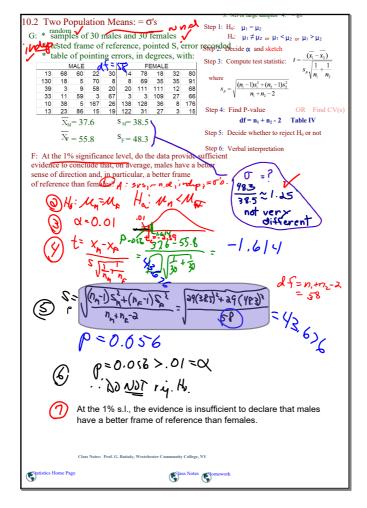


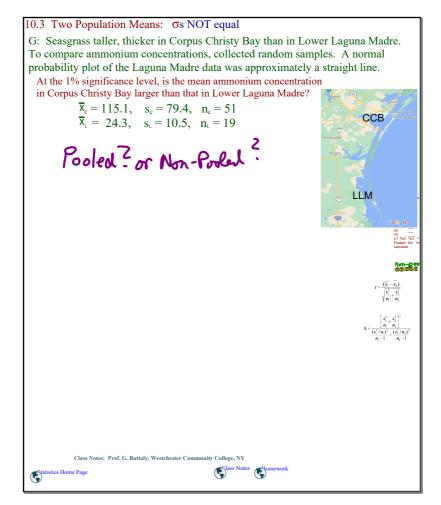


Two events are independent if the occurrence of one does not change the probability of the occurrence of the other.

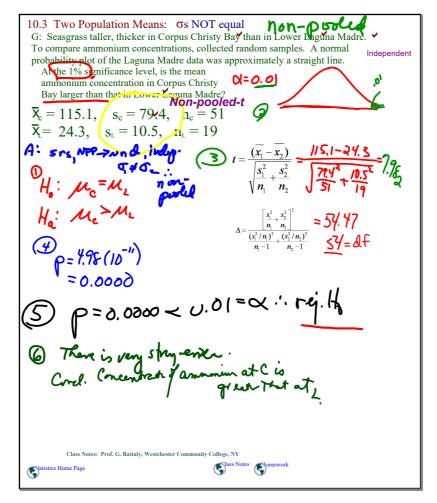


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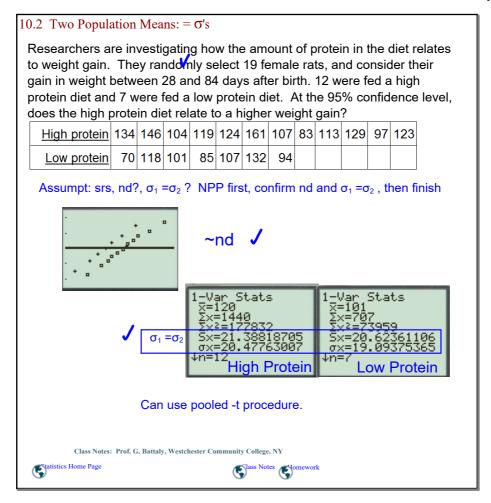
#### 10.2 Two Population Means: = $\sigma$ 's

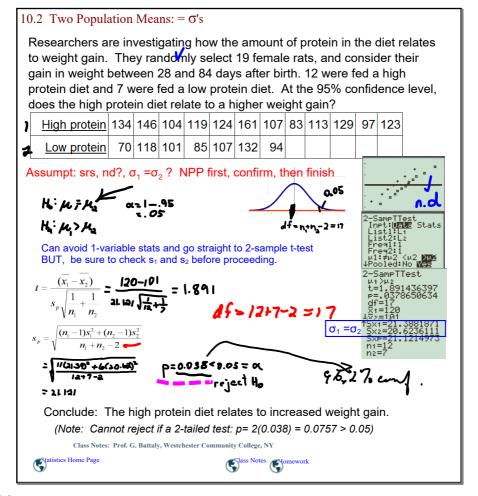
Researchers are investigating how the amount of protein in the diet relates to weight gain. They randomly select 19 female rats, and consider their gain in weight between 28 and 84 days after birth. 12 were fed a high protein diet and 7 were fed a low protein diet. At the 95% confidence level, does the high protein diet relate to a higher weight gain?

Low protein 70 118 101 85 107 132 94	High protein	134	146	104	119	124	161	107	83	113	129	97	123
	Low protein	70	118	101	85	107	132	94					

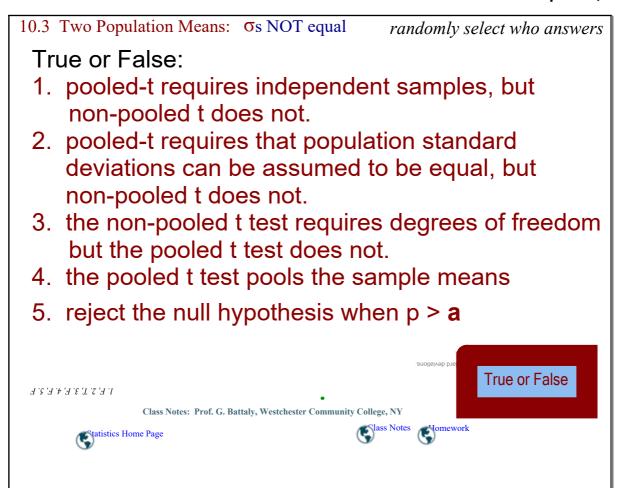
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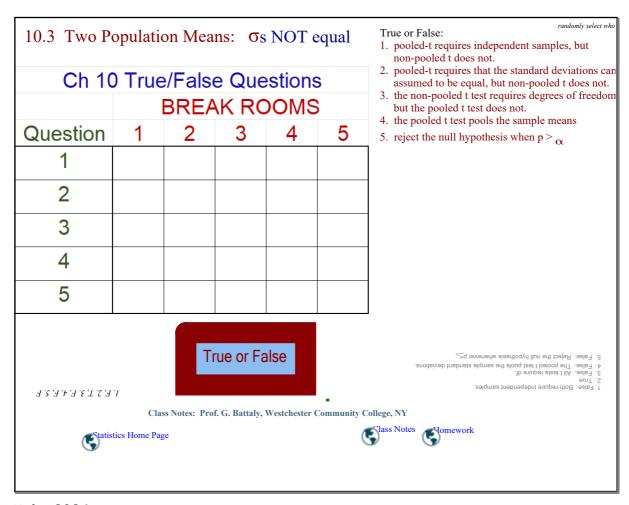
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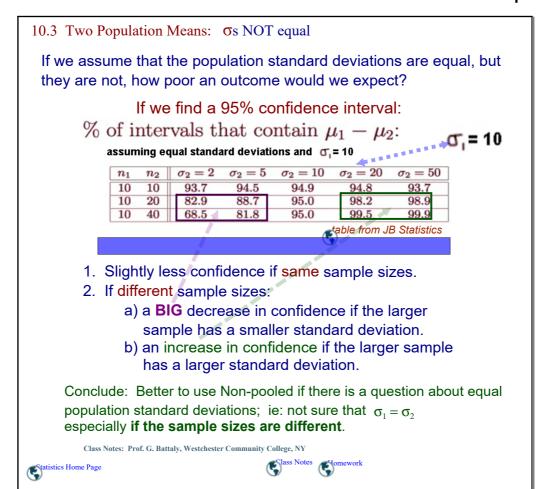


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### 10.2 Two Population Means: = $\sigma$ 's

Pooled or Non-pooled Variances: = ? or Not ?

- If sample sizes are equal and  $s_1$  and  $s_2$  are similar, assumption of equal population variance may be reasonable and the pooled procedure can be used.
- If sample sizes are equal and  $s_1$  and  $s_2$  are different, use non-pooled procedure.

\_\_\_\_\_

- If sample sizes are very different and  $s_1$  and  $s_2$  are similar, and the larger sample size produced the larger standard deviation, the pooled procedure is acceptable because it will be conservative.
- If sample sizes are very different and  $s_1$  and  $s_2$  are different, do not use the pooled procedure. The pooled test can be quite misleading unless sample standard deviations are similar, especially if the smaller standard deviation accompanies the larger sample size.

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#### 10.3 Two Population Means: σs NOT equal

G: Efforts to reduce disabilities of bus drivers by improving bus routes (intervention) to alleviate stress. Heart rates of drivers on improved routes were compared to that of drivers on normal routes (control).

G: Independent - diff. drivers, srs, nd

At the 5% significance level, does the intervention reduce the mean heart rate in bus drivers?

$$\bar{\mathbf{x}}_i = 67.90, \quad \mathbf{s}_i = 5.49, \quad \mathbf{n}_i = 10$$
  
 $\bar{\mathbf{x}}_c = 66.81, \quad \mathbf{s}_c = 9.04, \quad \mathbf{n}_c = 31$ 

But, 31/10=3.1

Approach:
What kind of problem is this? "At 1% s.l..." -> hypothesis tes:
Which procedure?

2 samples -> either pooled-t or non-pooled-t (at this point)

Need  $\sigma_1 = \sigma_2$  for pooled-t but  $n_c > n_t$  and  $s_c > s_b$  so pooled would be conservative; use NON-pooled







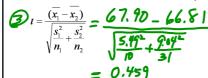
#### 10.3 Two Population Means: σs NOT equal

G: Efforts to reduce disabilities of bus drivers by improving bus routes (intervention) to alleviate stress. Heart rates of drivers on improved routes were compared to that of drivers on normal routes (control). (Independent - diff. drivers, srs, nd)

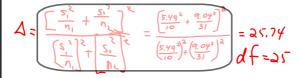
At the 5% significance level, does the intervention reduce the mean heart rate  $\overline{x}_i = 67.90$ ,  $s_i = 5.49$ ,  $n_i = 10$  in bus drivers?  $\overline{x}_c = 66.81$ ,  $s_c = 9.04$ ,  $n_c = 31$ 

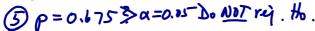
DH: M=Mc Ha: M<Mc

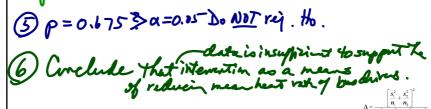
















ATTENDANCE QUESTION:

G: Indep. random samples: 126 cropland, 98 grassland, to compare the number of native species. n.d.

Samples Mean Stdev n Cropland 14.1 4.83 126 Grassland 15.3 4.95 98

F: At 5% s.l. does a difference exist in the mean number of native species?

Which procedure should you use to answer the question?

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 $HypTestingFunPuzzle\_2 means. notebook$