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Chapter 10 Hypothesis Tests – Comparing 2 Means

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For my students during the COVID crises, 2020

Use with a TI-84 calculator and
Weiss, N.S. *Introductory Statistics*, 10th ed.

Hypothesis Tests – Comparing 2 Means

We frequently need to compare samples from two different populations, or even samples from the same population in different years. In this chapter, we consider different ways to evaluate these comparisons.

1. Pooled-t Test in Section 2
2. Non-pooled t Test in Section 3
3. Mann-Whitney Test in Section 4
4. Paired-t Test in Section 5

Chapter 10 Hypothesis Tests – Comparing 2 Means

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We frequently need to compare samples from two different populations, or even samples from the same population in different years. In this chapter, we consider different ways to evaluate these comparisons.

1. Pooled-t Test in Section 2
2. Non-pooled t Test in Section 3
3. Mann-Whitney Test in Section 4
4. Paired-t Test in Section 5

Chapter 10, Section 1: Difference of 2 Means

Hypothesis Tests – Difference of 2 Means

We frequently need to compare samples from two different populations, or even samples from the same population in different years. To do this comparison we consider the distribution of the difference between two means,

Objectives:

Students will:

1. Consider how two samples can be compared to determine if they are the same or different or come from the same or different populations.

2. Consider the distribution of the difference of sample means, including:

- the mean of the difference
- the standard deviation of the difference

[Note: Instead of the distribution of sample means, we look at the distribution of the difference in sample means.]

3. Use the difference of two sample means for two similar, but different procedures:

- Pooled-t Test
- Non-pooled t-Test

Distribution of $\bar{X}_1 - \bar{X}_2$

1. If x is approximately normally distributed on each of the populations 1 and 2, then the difference of sample means is approximately normally distributed, and:

$$a) \quad \mu_{\bar{x}_1 - \bar{x}_2} = \mu_1 - \mu_2$$

$$b) \quad \sigma_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

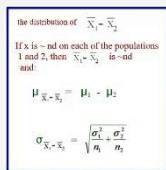
2. When σ is known, we can standardize to the Standard Normal Curve. But σ is rarely known. So, we will not conduct any 2 sample z-tests.

3. When σ_1 and σ_2 are not known, we use s_1 and s_2 to estimate them, and use the following t-tests

a) If $\sigma_1 = \sigma_2$ we use the **Pooled-t Test**
and s_1 and s_2 estimate the same σ

a) If $\sigma_1 \neq \sigma_2$ we use the **Non-Pooled t-Test**
and s_1 and s_2 estimate different σ 's

VIDEO: [Hypothesis Testing – 2 Means](http://www.battaly.com/stat/classnotes/video/10.2_pooled-t_zoom_04132020.mp4) (First 16 min in 1 hr video) Apr 2020
http://www.battaly.com/stat/classnotes/video/10.2_pooled-t_zoom_04132020.mp4



the distribution of $\bar{x}_1 - \bar{x}_2$

If x_1 is random on each of the populations 1 and 2, then $\bar{x}_1 - \bar{x}_2$ is random and:

$$\mu_{\bar{x}_1 - \bar{x}_2} = \mu_1 - \mu_2$$

$$\sigma_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

Readings:

1. *Introductory Statistics*, N. Weiss Chapter 10, Section 1.
2. Class Notes online at:

http://www.battaly.com/stat/classnotes/Ch_10_1_Diff_2_means.pdf

Assignments:

In *Introductory Statistics*, study Chapter 10, Section 1
Study Key Fact 10.1

Chapter 10, Section 2: Pooled-t Test

Pooled-t Test, What is It?

The Pooled-t Test is performed to compare two sample means to determine if they are the same or different, or if they come from the same population or from different populations. This procedure assumes that the population standard deviations are not known, but that they are equal.

Objectives. Students will:

1. Compare two sample means when the population standard deviations, σ , are believed to be the same but are not known.
2. Use the distribution of the difference between the sample means to evaluate the samples.
3. Perform the Pooled-t Test to arrive at a conclusion: are the means from the same population or are they different?

The Pooled Standard Deviation, s_p

1. We are taking samples from two sources for which the population standard deviations are not known. There are two possibilities:
either $\sigma_1 = \sigma_2$ or $\sigma_1 \neq \sigma_2$.

The Pooled-t Test assumes that $\sigma_1 = \sigma_2$.

2. If the assumption is that $\sigma_1 = \sigma_2$, then both of the sample standard deviations, s_1 and s_2 , are estimates of the single population value, σ . This estimate is called the pooled standard deviation, s_p or s_{pooled} .
3. The formula for the Pooled Standard Deviation is:

$$s_p = s_{pooled} = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$$

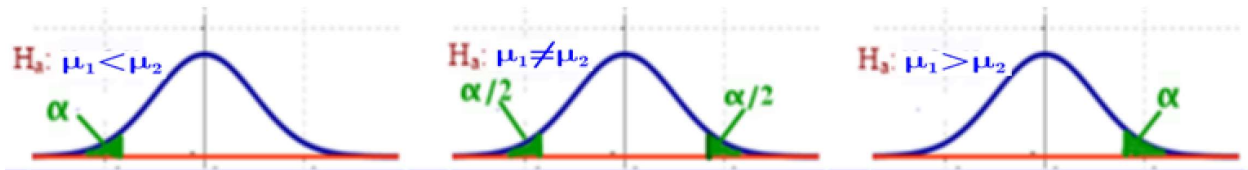
where n_1 is the size of sample 1, and n_2 is the size of sample 2

[Note: This weights each s by the size of the sample from which the s was obtained.]

To Perform a Pooled-t Test

1. Start with the Assumptions.
 - > Simple Random Sample
 - > Independent Samples

- > Normal Distribution or Large Sample Size
- > σ 's (but not known)
- 2. If the assumptions are met, state the hypotheses:
 - > $H_0: \mu_1 = \mu_2$
 - > $H_a: \mu_1 < \mu_2$ or $\mu_1 \neq \mu_2$ or $\mu_1 > \mu_2$
 This is determined by the words of the problem.
- 3. Draw a sketch that matches H_a using α from the problem.



- 4. Find the degrees of freedom, $df = n_1 + n_2 - 2$
- 5. Find the test statistic, t_t This represents data from both samples.
 - a) Write the formula

$$t_T = \frac{\bar{x}_1 - \bar{x}_2}{s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

- b) Substitute the values into the formula.
 - c) Use your calculator to find s_p and t_T (See instructions below.)
- 6. Also find s_p from your calculator, write it on your paper, and then substitute this value into the formula above.

[Note: For homework problems, write the formula for s_p .. For quizzes and tests you may leave out the formula, but be sure to write the value, clearly stated separately on the paper, as well as in the substitution for t_T .]
- 7. Find the p-value from your calculator (see below).
- 8. Decide whether or not to Reject the Null Hypothesis.
 - a) If $p \leq \alpha$ **reject** the null hypothesis, H_0 .
 - b) If $p > \alpha$ **do NOT reject** the null hypothesis, H_0 .
- 9. Interpret the answer, using the words of the problem.

Using the Calculator to Perform a Pooled-t Test

On your calculator, find and select the following:

- 1. STAT / TESTS
- 2. 4:2-SampTTest..
- 3. Inpt: Data Stats

Select Stats if you already have the sample mean

Select Data if you have the sample data

4. At this point the window will be different depending on whether you selected Stats (here) or Data (below)

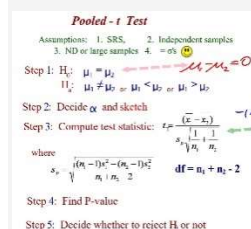
either	Stats
sample 1 mean	\bar{x}_1
sample 1 standard deviation	s_1
sample 1 size	n_1
sample 2 mean	\bar{x}_2
sample 2 standard deviation	s_2
sample 2 size	n_2
OR	Data
List1:	L1
List2:	L2

5. $\mu_1 \neq \mu_2$ $\mu_1 < \mu_2$ $\mu_1 > \mu_2$ Select the H_a for the problem
 6. Pooled: Yes
 6. Calculate

Calculator T-Test Output This varies slightly by Data or Stats

The H_a you selected	H_a
test statistic from the sample	t_t
p-value	p
degrees of freedom	df
sample 1 mean	\bar{x}_1
sample 2 mean	\bar{x}_2
sample 1 standard deviation	s_1
sample 2 standard deviation	s_2
pooled standard deviation	s_p
sample 1 size	n
sample 2 size	n

VIDEO: Pooled-t Test (start at 16:30 in 1 hr video) Apr, 2020
http://www.battaly.com/stat/classnotes/video/10.2_pooled-t_zoom_04132020.mp4



Readings:

1. *Introductory Statistics*, N. Weiss Chapter 10, Section 2.
2. Class Notes online at:

http://www.battaly.com/stat/classnotes/Ch_10_2_pooled_t_test.pdf

Assignments:

In *Introductory Statistics*, study Chapter 10, Section 2
Complete problems #33-43, 48, 49

Assignment: http://www.battaly.com/stat/homework/10_2_pooled-t.htm

Chapter 10, Section 3: Non-Pooled t Test

Non-Pooled t Test, What is It?

Like the Pooled-t Test, the Non-Pooled t Test is performed to compare two sample means to determine if they are the same or different, or if they come from the same population or from different populations. However, the Non-Pooled procedure assumes that the population standard deviations are not known, and that they are also NOT equal.

Objectives. Students will:

1. Compare two sample means when the population standard deviations, σ , are not known and are not expected to be equal.
2. Use the distribution of the difference between the sample means to evaluate the samples.
3. Perform the Non-Pooled t Test to arrive at a conclusion: are the means from the same population or are they different?

Degrees of Freedom, Δ

1. We are taking samples from two sources for which the population standard deviations are not known. There are two possibilities:
either $\sigma_1 = \sigma_2$ or $\sigma_1 \neq \sigma_2$.

The Non-Pooled t Test assumes that $\sigma_1 \neq \sigma_2$.

2. If the assumption is that $\sigma_1 \neq \sigma_2$, then the sample standard deviations, s_1 and s_2 , are estimates of the population values, σ_1 and σ_2 . Thus, this procedure requires estimating four parameters - both of the population standard deviations, as well as the population means, μ_1 and μ_2 . For this we have a more complex formula for the Degrees of Freedom, indicated by the Greek delta, Δ .

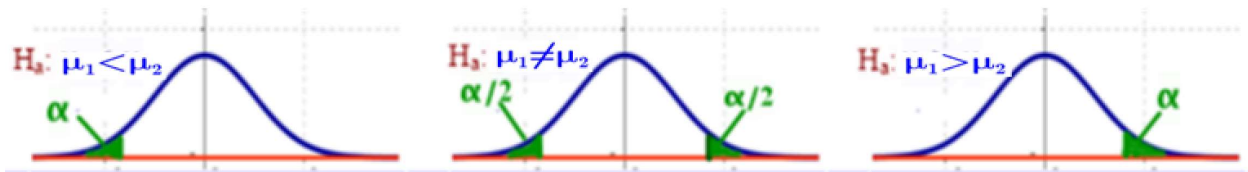
3. The formula for Degrees of Freedom, Δ is:

$$\Delta = \frac{\left[\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2} \right]^2}{\frac{\left(\frac{s_1^2}{n_1} \right)^2}{n_1 - 1} + \frac{\left(\frac{s_2^2}{n_2} \right)^2}{n_2 - 1}}$$

where n_1 and s_1 are for sample 1, and n_2 and s_2 are for sample 2

To Perform a Non-Pooled t Test

1. Start with the Assumptions.
 - > Simple Random Sample
 - > Independent Samples
 - > Normal Distribution or Large Sample Size
 - > $\neq \sigma$'s (and not known)
2. If the assumptions are met, state the hypotheses:
 - > $H_0: \mu_1 = \mu_2$
 - > $H_a: \mu_1 < \mu_2$ or $\mu_1 \neq \mu_2$ or $\mu_1 > \mu_2$ This is determined by the words of the problem.
3. Draw a sketch that matches H_a using α from the problem.



4. The degrees of freedom is found using the formula above. You should find Δ using your calculator.
5. Find the test statistic, t_t This represents data from both samples.
 - a) Write the formula

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

- b) Substitute the values into the formula.
 - c) Use your calculator to find t_t (See instructions below.)
6. Find the p-value from your calculator (see below).
 7. Decide whether or not to Reject the Null Hypothesis.
 - a) If $p \leq \alpha$ **reject** the null hypothesis, H_0 .
 - b) If $p > \alpha$ **do NOT reject** the null hypothesis, H_0 .
 8. Interpret the answer, using the words of the problem.

Using the Calculator to Perform a Non-Pooled t Test

On your calculator, find and select the following:

1. STAT / TESTS
2. 4:2-SampTTest..
3. Inpt: Data Stats

Select Stats if you already have the sample means

Select Data if you have the sample data

4. At this point the window will be different depending on whether you selected Stats (here) or Data (below)

either	Stats
sample 1 mean	\bar{x}_1
sample 1 standard deviation	s1
sample 1 size	n1
sample 2 mean	\bar{x}_2
sample 2 standard deviation	s2
sample 2 size	n2
OR	Data
List1:	L1
List2:	L2

5. $\mu_1 \neq \mu_2$ $< \mu_2$ $> \mu_2$ Select the H_a for the problem

6. Pooled: **NO**

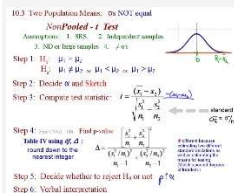
6. Calculate

Calculator T-Test Output This varies slightly by Data or Stats

The H_a you selected	H_a
test statistic from the sample	t_t
p-value	p
degrees of freedom	df
sample 1 mean	\bar{x}_1
sample 2 mean	\bar{x}_2
sample 1 standard deviation	s1
sample 2 standard deviation	s2
sample 1 size	n1
sample 2 size	n2

VIDEO: Non-Pooled t Test (37 min video) Apr, 2020

http://www.battaly.com/stat/classnotes/video/10.3_NONpooled-t_04152020_zoom.mp4



Readings:

1. *Introductory Statistics*, N. Weiss Chapter 10, Section 3.
2. Class Notes online at:

http://www.battaly.com/stat/classnotes/Ch_10_3_NONpooled_t_test.pdf

Assignments:

In *Introductory Statistics*, study Chapter 10, Section 3
Complete problems #67-70all, 73-77 (noCI), 81, 83

Assignment: http://www.battaly.com/stat/homework/10_3_non-pooled-t.htm

Chapter 10, Section 4: Mann-Whitney Test

Mann-Whitney Test, What is It?

The Mann-Whitney Test is performed to compare two sets of independent data when the data is neither normally distributed nor large enough samples to assume that sample means are approximately normally distributed.

The Mann-Whitney Test is performed on the ranks of the data rather than the data itself.

Objectives. Students will:

1. Use the Mann-Whitney test to analyze data that is not normally distributed and from small samples.
2. Use overall ranking to weight the data in the two data sets while keeping the sets separate.
3. Compute the test statistic as the sum of the ranks of the first data set.
4. Use critical values as the criteria for hypothesis testing.
5. Find the right critical value using Table VI.
6. Find a left critical value using a formula and the right critical value.
7. Complete the testing procedure by using the critical value approach.

How to Rank the Data

1. Overview

- a) The total number of ranks is $n_1 + n_2$, and all the ranks must be used.
- b) Keep the two data sets separate, but assign ranks as if they were a single group
- c) Identical values of data are assigned the same rank, an average of the ranks that would be assigned to the data if they were not equivalent.

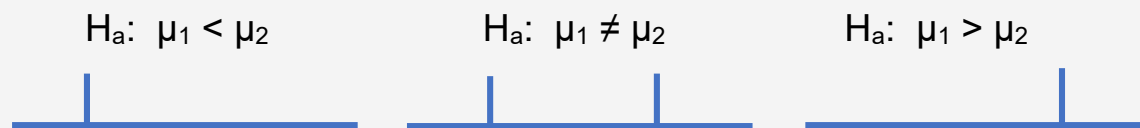
2. What to do

- a) Sequence each data set independently.
- b) Assign Rank 1 to the lowest value in either group. Assign Rank 2 to the next largest item of data in either group.

- c) Continue to assign ranks sequentially until every item is ranked.
- d) If there are 2 items of data that have the same value, both get assigned the average of the two ranks that would be used for those values if they were not equal. For example,
 - > If the 2 items would be ranked 4 and 5, then each gets assigned the rank of 4.5 and the next rank to use is 6.
 - [Note: $(4+5)/2 = 4.5$]
- e) If there are 3 items of data that have the same value, all 3 get assigned the average of the 3 ranks that would be used for those values if they were not equal. For example,
 - > If the 3 items would be ranked 4, 5, and 6, then each gets assigned the rank of 5 and the next rank to use is 7.
 - [Note: $(4+5+6)/3 = 5$]
- f) If more than 3 items of data have the same value, follow the same patterns as above, using the average of the ranks that would be assigned if they did not have the same value.

To Perform a Mann-Whitney Test:

1. Start with the Assumptions.
 - > Simple Random Paired Sample
 - > Independent Samples
 - > Populations have the same shape.
2. If the assumptions are met, state the hypotheses:
 - > $H_0: \mu_1 = \mu_2$ (or $\eta_1 = \eta_2$)
 - > $H_a: \mu_1 < \mu_2$ or $\mu_1 \neq \mu_2$ or $\mu_1 > \mu_2$
 - This is determined by the words of the problem.
3. Draw a sketch that matches H_a using α from the problem.



4. Construct a Table of sequenced data and overall ranks

Sample 1	Rank	Sample 2	Rank

5. Find the test statistic, M_T . This is the sum of the ranks for Sample 1.
6. Find the critical value, M_R , using Table VI. Use the top columns for Sample 1 and the left rows for Sample 2, along with the significance level required for the problem.
7. If the alternative hypothesis requires a left tail, then use the formula below to find the critical value for the left tail:

$$M_L = n_1(n_1 + n_2 + 1) - M_R$$

8. Decide whether or not to Reject the Null Hypothesis.
 - a) If $M_T \geq M_R$ **reject** the null hypothesis, H_0 .
 - b) If $M_T \leq M_L$ **reject** the null hypothesis, H_0 .
 - c) If $M_L \leq M_T \leq M_R$ **do NOT** reject the null hypothesis, H_0 .

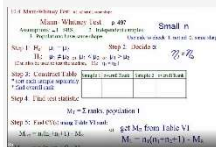
Generally, if the test statistic is in the tail, reject.

If the test statistic is not in the tails, do not reject.

9. Interpret the answer, using the words of the problem.

VIDEO: [Mann-Whitney Test](http://www.battaly.com/stat/classnotes/video/10.4_M-W-part1_11182020.mp4) (36 min video) Nov, 2020

http://www.battaly.com/stat/classnotes/video/10.4_M-W-part1_11182020.mp4



Readings:

1. *Introductory Statistics*, N. Weiss Chapter 10, Section 4.
2. Class Notes online at:

http://www.battaly.com/stat/classnotes/Ch_10_4_MannWhitney.pdf

Assignments:

In *Introductory Statistics*, study Chapter 10, Section 4

Complete problems #105, 106, 107, 111, 112; 113-121, 2 of 123-127

Assignment http://www.battaly.com/stat/homework/10_4_MannWhitney.htm

Chapter 10, Section 5: Paired-t Test

Paired-t Test, What is It?

The Paired-t Test is performed to compare dependent data.

Examples of paired data include: 1) before and after a treatment, 2) data obtained from the same individuals or objects but obtained in different ways or at different times.

The Paired-t Test is equivalent to a one mean t-test (Ch 9 Section 5) for the difference of the paired data, d .

Objectives. Students will:

1. Recognize problems with paired data (dependent data).
2. Consider the distribution of the difference of paired data, d .
3. Analyze normally distributed paired data using a single sample t-test and $H_0: \mu_1 = \mu_2$ or $d = 0$.

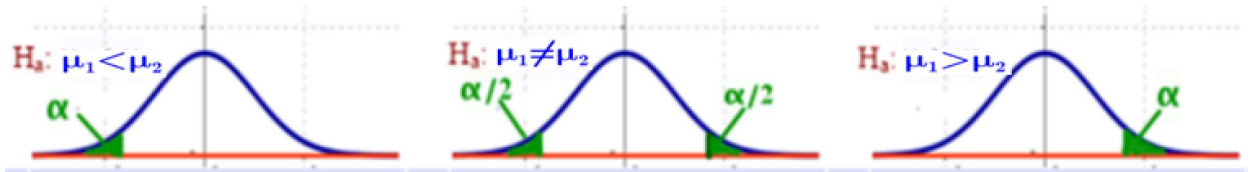
Recognizing Paired Data

1. Paired Data – What it is.
 - a) Paired Data are from dependent samples.
 - b) **Dependent samples are paired measurements for one set of items.**
 - c) examples:
 - >before and after a treatment
 - >different measurements of same items
 - >blood levels of vitamin D before and after a person drinks fortified OJ for a month
2. **Not** paired data – what it is not.
 - a) Independent samples are measurements made on two different sets of items.
 - b) example: blood levels of vitamin D are compared for a randomly selected group that does not drink fortified OJ and a different group that does drink fortified OJ

To Perform a Paired-t Test: This test is equivalent to a one mean t-test (Ch 9 Section 5) for the difference, d .

1. Start with the Assumptions.
 - > Simple Random Paired Sample

- > Normal Distribution or Large Sample Size
- 2. If the assumptions are met, state the hypotheses:
 - > $H_0: \mu_1 = \mu_2$ (or $d = 0$)
 - > $H_a: \mu_1 < \mu_2$ or $\mu_1 \neq \mu_2$ or $\mu_1 > \mu_2$ (or $d < 0$ or $d \neq 0$ or $d > 0$)
 This is determined by the words of the problem.
- 3. Draw a sketch that matches H_a using α from the problem.



$df = n - 1$, where n = number of pairs

- 4. Construct a Table and calculate the paired differences

Sample 1	Sample 2	$d = \text{Sample 1} - \text{Sample 2}$

- 5. Find the test statistic, t_t This represents the difference, d .

- a) Write the formula

$$t_T = \frac{\bar{d}}{s_d / \sqrt{n}}$$

- b) Substitute the values into the formula.

- c) Use your calculator to find t_t (See instructions below.)

- 6. Also find s from your calculator, and substitute this value into the formula above.

- 7. Find the p-value from your calculator (see below).

- 8. Decide whether or not to Reject the Null Hypothesis.

- a) If $p \leq \alpha$ **reject** the null hypothesis, H_0 .

- b) If $p > \alpha$ **do NOT reject** the null hypothesis, H_0 .

- 9. Interpret the answer, using the words of the problem.

Using the Calculator to Perform a Pooled-t Test

On your calculator, find and select the following:

- 1. STAT / TESTS

2. 2: T-Test

3. Inpt: Data Stats

Select Stats if you already have the sample mean

Select Data if you have the sample data

4. At this point the window will be different depending on whether you selected Stats (here) or Data (below)

either	Stats
sample 1 mean	$\mu_d = 0$
sample 1 standard deviation	s_d
sample 1 size	n
OR	Data
List1:	L1

5. $\mu_1 \neq \mu_2$ $< \mu_2$ $> \mu_2$ Select the H_a for the problem

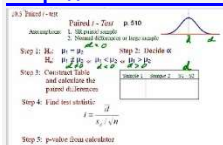
6. Calculate

Calculator T-Test Output This varies slightly by Data or Stats

The H_a you selected	H_a
test statistic from the sample	t_t
p-value	p
sample	\bar{d}
sample standard deviation	s_d
sample size	n

VIDEO: Paired-t Test (25 min video) Apr, 2020

http://www.battaly.com/stat/classnotes/video/10.5_paired-t_zoom_04202020.mp4



VIDEO 2: Paired-t Test (continued) (12 min) Apr, 2020

http://www.battaly.com/stat/classnotes/video/10.5_paired-t_continued.mp4

8 students were given a diagnostic test before studying the Pythagorean Theorem and again after completing the topic. Their scores on the tests are listed below.

At the 5% α , do the data provide sufficient evidence to conclude that, on average, studying improved their understanding of the Pythagorean Theorem?

	Pre-Test	Post-Test	$d = \text{post} - \text{pre}$
1	18	22	4
2	16	17	1
3	22	24	2
4	17	20	3
5	14	15	1
6	18	18	0
7	12	18	6
8	17	16	-1

Readings:

1. *Introductory Statistics*, N. Weiss Chapter 10, Section 5.
2. Class Notes online at:

http://www.battaly.com/stat/classnotes/Ch_10_5_Paired_t_test.pdf

Assignments:

In *Introductory Statistics*, study Chapter 10, Section 5
Complete problems #139, 141, 146, 149, 153, 155*, 157*

*Word problems

Assignment: http://www.battaly.com/stat/homework/10_5_Paired-t.htm