SECTION 6 Review of Trigonometric Functions

Angles and Degree Measure • Radian Measure • The Trigonometric Functions • Evaluating Trigonometric Functions • Solving Trigonometric Equations • Graphs of Trigonometric Functions

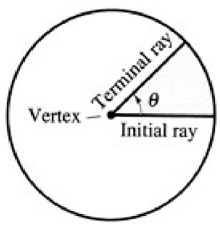


FIGURE 58
Standard position of an angle.

Angles and Degree Measure

An **angle** has three parts: an **initial ray,** a **terminal ray,** and a **vertex** (the point of intersection of the two rays), as shown in Figure 58. An angle is in **standard position** if its initial ray coincides with the positive x-axis and its vertex is at the origin. We assume that you are familiar with the degree measure of an angle.* It is common practice to use θ (the Greek lowercase letter *theta*) to represent both an angle and its measure. Angles between 0° and 90° are **acute** and angles between 90° and 180° are **obtuse.**

Positive angles are measured *counterclockwise,* and negative angles are measured *clockwise.* For instance, Figure 59 shows an angle whose measure is $-45°$. You cannot assign a measure to an angle by simply knowing where its initial and terminal rays are located. To measure an angle, you must also know how the terminal ray was revolved. For example, Figure 59 shows that the angle measuring $-45°$ has the same terminal ray as the angle measuring 315°. Such angles are **coterminal.** In general, if θ is any angle, then

$$\theta + n(360), \qquad n \text{ is a nonzero integer}$$

is coterminal with θ.

An angle that is larger than 360° is one whose terminal ray has been revolved more than one full revolution counterclockwise, as shown in Figure 47. You can form an angle whose measure is less than $-360°$ by revolving a terminal ray more than one full revolution clockwise.

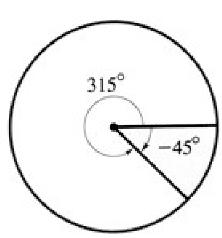


FIGURE 59
Coterminal angles

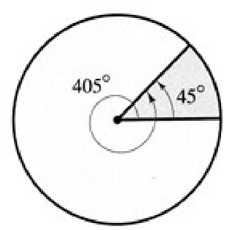


FIGURE 60
Coterminal angles

REMARK It is common to use the symbol θ to refer to both an *angle* and its *measure*. For instance, in Figure 60, you can write the measure of the smaller angle as $\theta = 45°$.

* For a more complete review of trigonometry, see *Precalculus,* 3rd edition, by Larson and Hostetler (Lexington, Mass., D. C. Heath and Company, 1993).

Radian Measure

To assign a radian measure to an angle θ, consider θ to be a central angle of a circle of radius 1, as shown in Figure 61. The **radian measure** of θ is then defined to be the length of the arc of the sector. Because the circumference of a circle is $2\pi r$, the circumference of a **unit circle** (of radius 1) is 2π. This implies that the radian measure of an angle measuring 360° is 2π. In other words, $360° = 2\pi$ radians.

Using radian measure for θ, the length s of a circular arc of radius r is $s = r\theta$, as shown in Figure 62.

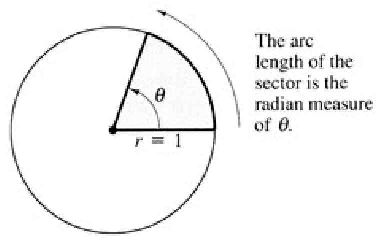


FIGURE 61
Unit circle

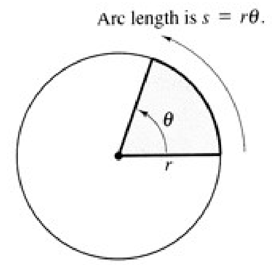


FIGURE 62
Circle of radius r

You should know the conversions of the common angles shown in Figure 63. For other angles, use the fact that 180° is equal to π radians.

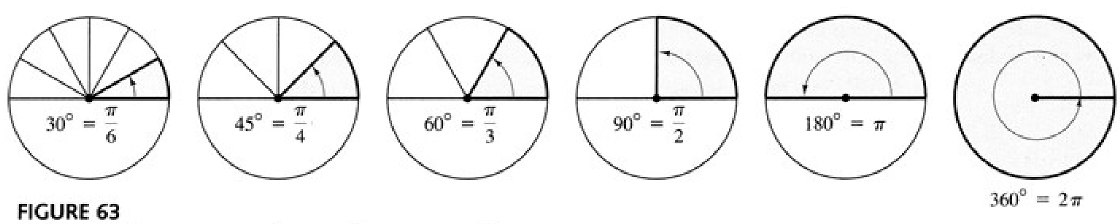


FIGURE 63
Radian and degree measure for several common angles.

EXAMPLE 1 Conversions Between Degrees and Radians

a. $40° = (40\ \text{deg})\left(\dfrac{\pi\ \text{rad}}{180\ \text{deg}}\right) = \dfrac{2\pi}{9}$ radians

b. $-270° = (-270\ \text{deg})\left(\dfrac{\pi\ \text{rad}}{180\ \text{deg}}\right) = -\dfrac{3\pi}{2}$ radians

c. $-\dfrac{\pi}{2}$ radians $= \left(-\dfrac{\pi}{2}\ \text{rad}\right)\left(\dfrac{180\ \text{deg}}{\pi\ \text{rad}}\right) = -90°$

d. $\dfrac{9\pi}{2}$ radians $= \left(\dfrac{9\pi}{2}\ \text{rad}\right)\left(\dfrac{180\ \text{deg}}{\pi\ \text{rad}}\right) = 810°$

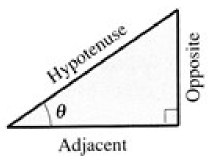


FIGURE 64
Sides of a right triangle.

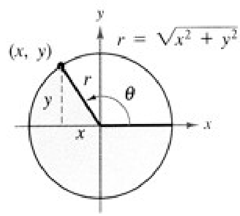


FIGURE 65
An angle in standard position.

The Trigonometric Functions

There are two common approaches to the study of trigonometry. In one, the trigonometric functions are defined as ratios of two sides of a right triangle. In the other, these functions are defined in terms of a point on the terminal side of an angle in standard position. We define the six trigonometric functions, **sine, cosine, tangent, cotangent, secant,** and **cosecant** (abbreviated as sin, cos, etc.), from both viewpoints.

Definition of the Six Trigonometric Functions

Right triangle definitions, where $0 < \theta < \frac{\pi}{2}$ (see Figure 64).

$$\sin\theta = \frac{\text{opp.}}{\text{hyp.}} \qquad \cos\theta = \frac{\text{adj.}}{\text{hyp.}} \qquad \tan\theta = \frac{\text{opp.}}{\text{adj.}}$$

$$\csc\theta = \frac{\text{hyp.}}{\text{opp.}} \qquad \sec\theta = \frac{\text{hyp.}}{\text{adj.}} \qquad \cot\theta = \frac{\text{adj.}}{\text{opp.}}$$

Circular function definitions, where θ *is any angle* (see Figure 65).

$$\sin\theta = \frac{y}{r} \qquad \cos\theta = \frac{x}{r} \qquad \tan\theta = \frac{y}{x}$$

$$\csc\theta = \frac{r}{y} \qquad \sec\theta = \frac{r}{x} \qquad \cot\theta = \frac{x}{y}$$

The following trigonometric identities are direct consequences of the definitions. (ϕ is the Greek letter *phi.*)

Trigonometric Identities [*Note that* $\sin^2\theta$ *is used to represent* $(\sin\theta)^2$.]

Pythagorean identities:

$\sin^2\theta + \cos^2\theta = 1$

$\tan^2\theta + 1 = \sec^2\theta$

$\cot^2\theta + 1 = \csc^2\theta$

Reduction formulas:

$\sin(-\theta) = -\sin\theta$

$\cos(-\theta) = \cos\theta$

$\tan(-\theta) = -\tan\theta$

$\sin\theta = -\sin(\theta - \pi)$

$\cos\theta = -\cos(\theta - \pi)$

$\tan\theta = \tan(\theta - \pi)$

Sum or difference of two angles:

$\sin(\theta \pm \phi) = \sin\theta\cos\phi \pm \cos\theta\sin\phi$

$\cos(\theta \pm \phi) = \cos\theta\cos\phi \mp \sin\theta\sin\phi$

$$\tan(\theta \pm \phi) = \frac{\tan\theta \pm \tan\phi}{1 \mp \tan\theta\tan\phi}$$

Half-angle formulas:

$\sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$

$\cos^2\theta = \frac{1}{2}(1 + \cos 2\theta)$

Double angle formulas:

$\sin 2\theta = 2\sin\theta\cos\theta$

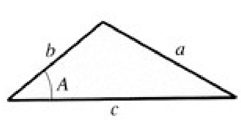
$\cos 2\theta = 2\cos^2\theta - 1$

$= 1 - 2\sin^2\theta$

$= \cos^2\theta - \sin^2\theta$

Law of Cosines:

$a^{\cdot} = b^2 + c^2 - 2bc\cos A$



Law of Cosines

Reciprocal identities:

$$\csc\theta = \frac{1}{\sin\theta}$$

$$\sec\theta = \frac{1}{\cos\theta}$$

$$\cot\theta = \frac{1}{\tan\theta}$$

Quotient identities:

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

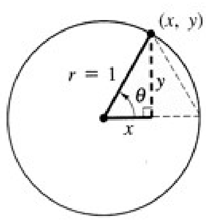
$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

Evaluating Trigonometric Functions

There are two ways to evaluate trigonometric functions: (1) decimal approximations with a calculator (or a table of trigonometric values) and (2) exact evaluations using trigonometric identities and formulas from geometry. When using a calculator to evaluate a trigonometric function, remember to set the calculator to the appropriate mode—degree mode or radian mode.

EXAMPLE 2 Exact Evaluation of Trigonometric Functions

Evaluate the sine, cosine, and tangent of $\frac{\pi}{3}$.



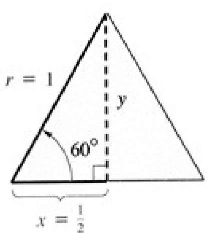


FIGURE 66
The angle $\pi/3$ in standard position.

Solution Begin by drawing the angle $\theta = \pi/3$ in the standard position, as shown in Figure 66. Then, because $60° = \pi/3$ radians, you can draw an equilateral triangle with sides of length 1 and θ as one of its angles. Because the altitude of this triangle bisects its base, you know that $x = \frac{1}{2}$. Using the Pythagorean Theorem, you obtain

$$y = \sqrt{r^2 - x^2} = \sqrt{1 - \left(\frac{1}{2}\right)^2} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}.$$

Now, knowing the values of x, y, and r, you can write the following.

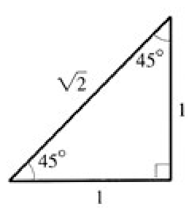
$$\sin\frac{\pi}{3} = \frac{y}{r} = \frac{\sqrt{3}/2}{1} = \frac{\sqrt{3}}{2}$$

$$\cos\frac{\pi}{3} = \frac{x}{r} = \frac{1/2}{1} = \frac{1}{2}$$

$$\tan\frac{\pi}{3} = \frac{y}{x} = \frac{\sqrt{3}/2}{1/2} = \sqrt{3}$$

REMARK All angles in the remainder of this text are measured in radians unless stated otherwise. For example, when we write $\sin 3$, we mean the sine of three radians, and when we write $\sin 3°$, we mean the sine of three degrees.

The degree and radian measures of several common angles are given in Table 4, along with the corresponding values of the sine, cosine, and tangent (see Figure 67).



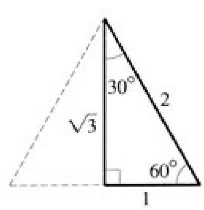


FIGURE 67
Common angles.

TABLE 4
Common First Quadrant Angles

Degrees	0	30°	45°	60°	90°
Radians	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	Undefined

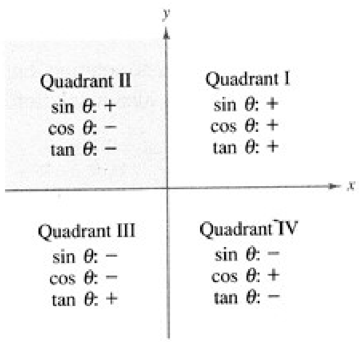


FIGURE 68
Quadrant signs for trigonometric functions.

The quadrant signs of the sine, cosine, and tangent functions are shown in Figure 68. To extend the use of Table 4 to angles in quadrants other than the first quadrant, you can use the concept of a **reference angle** (see Figure 69), with the appropriate quadrant sign. For instance, the reference angle for $3\pi/4$ is $\pi/4$, and because the sine is positive in the second quadrant, you can write

$$\sin\frac{3\pi}{4} = +\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}.$$

Similarly, because the reference angle for 330° is 30°, and the tangent is negative in the fourth quadrant, you can write

$$\tan 330^\circ = -\tan 30^\circ = -\frac{\sqrt{3}}{3}.$$

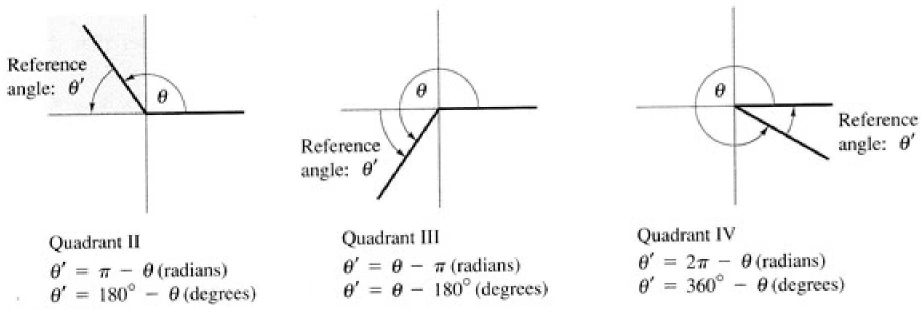


FIGURE 69

EXAMPLE 3 Trigonometric Identities and Calculators

Evaluate the trigonometric expression.

a. $\sin\left(-\frac{\pi}{3}\right)$ **b.** $\sec 60^\circ$ **c.** $\cos(1.2)$

Solution

a. Using the reduction formula $\sin(-\theta) = -\sin\theta$, you can write

$$\sin\left(-\frac{\pi}{3}\right) = -\sin\frac{\pi}{3} = -\frac{\sqrt{3}}{2}.$$

b. Using the reciprocal identity $\sec\theta = 1/\cos\theta$, you can write

$$\sec 60^\circ = \frac{1}{\cos 60^\circ} = \frac{1}{1/2} = 2.$$

c. Using a calculator, you can obtain

$$\cos(1.2) \approx 0.3624.$$

Remember that 1.2 is given in *radian* measure. Consequently, your calculator must be set in radian mode.

Solving Trigonometric Equations

How would you solve the equation $\sin\theta = 0$? You know that $\theta = 0$ is one solution, but this is not the only solution. Any one of the following values of θ is also a solution.

$$\ldots, -3\pi, -2\pi, -\pi, 0, \pi, 2\pi, 3\pi, \ldots$$

You can write this infinite solution set as $\{n\pi : n \text{ is an integer}\}$.

EXAMPLE 4 Solving a Trigonometric Equation

Solve the equation

$$\sin\theta = -\frac{\sqrt{3}}{2}.$$

Solution To solve the equation, you should consider that the sine is negative in Quadrants III and IV and that

$$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}.$$

Thus, you are seeking values of θ in the third and fourth quadrants that have a reference angle of $\pi/3$. In the interval $[0, 2\pi]$, the two angles fitting these criteria are

$$\theta = \pi + \frac{\pi}{3} = \frac{4\pi}{3} \quad \text{and} \quad \theta = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}.$$

By adding integer multiples of 2π to each of these solutions, you obtain the following general solution.

$$\theta = \frac{4\pi}{3} + 2n\pi \quad \text{or} \quad \theta = \frac{5\pi}{3} + 2n\pi, \qquad \text{where } n \text{ is an integer.}$$

(See Figure 70.)

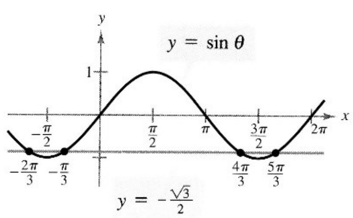


FIGURE 70

Solution points of $\sin\theta = \dfrac{-\sqrt{3}}{2}$.

EXAMPLE 5 Solving a Trigonometric Equation

Solve $\cos 2\theta = 2 - 3\sin\theta$, where $0 \le \theta \le 2\pi$.

Solution Using the double angle identity $\cos 2\theta = 1 - 2\sin^2\theta$, you can rewrite the equation as follows.

$\cos 2\theta = 2 - 3\sin\theta$	Given equation
$1 - 2\sin^2\theta = 2 - 3\sin\theta$	Trigonometric identity
$0 = 2\sin^2\theta - 3\sin\theta + 1$	Quadratic form
$0 = (2\sin\theta - 1)(\sin\theta - 1)$	Factor

If $2\sin\theta - 1 = 0$, then $\sin\theta = 1/2$ and $\theta = \pi/6$ or $\theta = 5\pi/6$. If $\sin\theta - 1 = 0$, then $\sin\theta = 1$ and $\theta = \pi/2$. Thus, for $0 \le \theta \le 2\pi$, there are three solutions.

$$\theta = \frac{\pi}{6}, \quad \frac{5\pi}{6}, \quad \text{and} \quad \frac{\pi}{2}$$

Graphs of Trigonometric Functions

A function f is **periodic** if there exists a nonzero number p such that $f(x + p) = f(x)$ for all x in the domain of f. The smallest such positive value of p (if it exists) is the **period** of f. The sine, cosine, secant, and cosecant functions each have a period of 2π, and the other two trigonometric functions have a period of π, as shown in Figure 71.

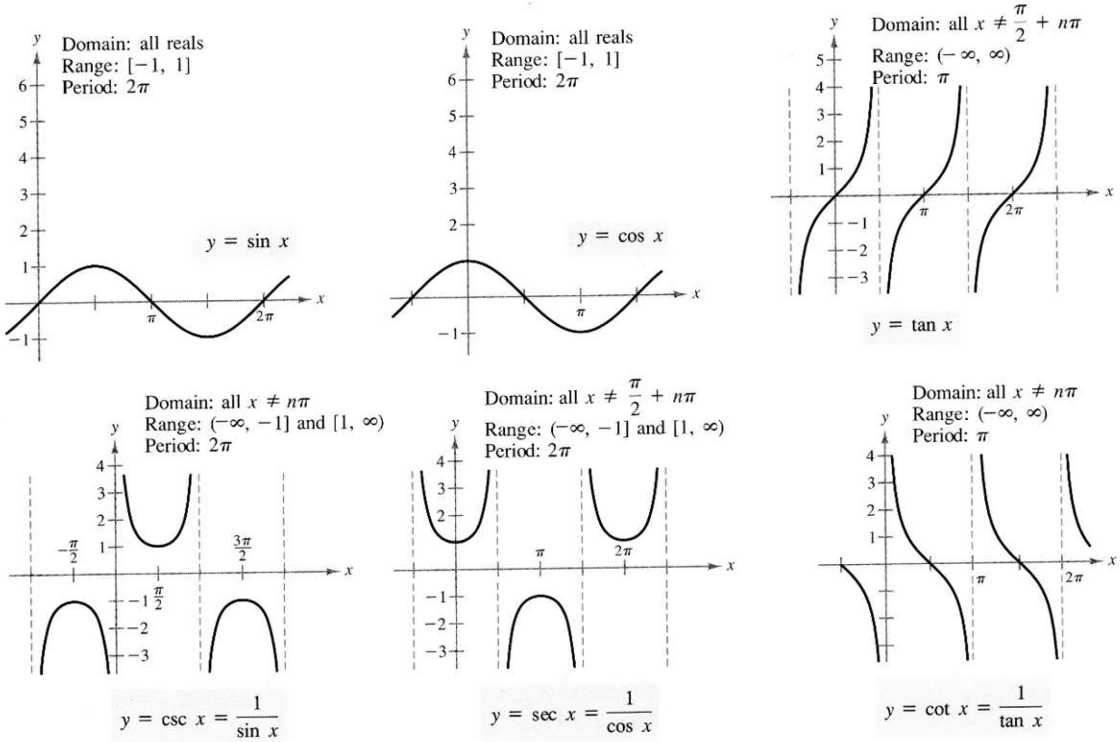


FIGURE 71
The graphs of the six trigonometric functions.

Note in Figure 71 that the maximum value of $\sin x$ and $\cos x$ is 1 and the minimum value is -1. The graphs of the functions $y = a \sin bx$ and $y = a \cos bx$ oscillate between $-a$ and a, and hence have an **amplitude** of $|a|$. Furthermore, because $bx = 0$ when $x = 0$ and $bx = 2\pi$ when $x = 2\pi/b$, it follows that the functions $y = a \sin bx$ and $y = a \cos bx$ each have a period of $2\pi/|b|$. Table 5 summarizes the amplitudes and periods for some types of trigonometric functions.

TABLE 5

Function	*Period*	*Amplitude*
$y = a \sin bx$ or $y = a \cos bx$	$\dfrac{2\pi}{\lvert b \rvert}$	$\lvert a \rvert$
$y = a \tan bx$ or $y = a \cot bx$	$\dfrac{\pi}{\lvert b \rvert}$	Not applicable
$y = a \sec bx$ or $y = a \csc bx$	$\dfrac{2\pi}{\lvert b \rvert}$	Not applicable

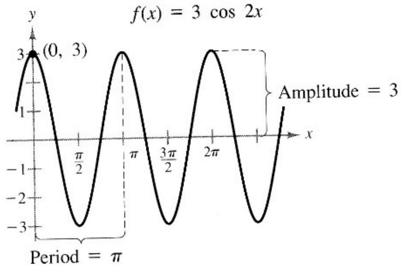


FIGURE 72

EXAMPLE 6 Sketching the Graph of a Trigonometric Function

Sketch the graph of $f(x) = 3\cos 2x$.

Solution The graph of $f(x) = 3\cos 2x$ has an amplitude of 3 and a period of $2\pi/2 = \pi$. Using the basic shape of the graph of the cosine function, sketch one period of the function on the interval $[0, \pi]$, using the following pattern.

Maximum: $(0, 3)$ Minimum: $\left(\frac{\pi}{2}, -3\right)$ Maximum: $[\pi, 3]$

By continuing this pattern, you can sketch several cycles of the graph, as shown in Figure 72.

The discussion of horizontal shifts, vertical shifts, and reflections given in Section 5 can be applied to the graphs of trigonometric functions, as illustrated in Example 7.

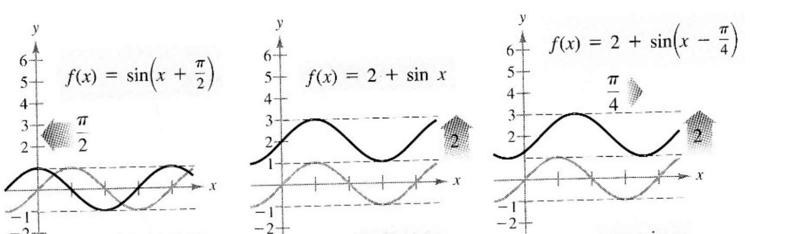
EXAMPLE 7 Shifts of Graphs of Trigonometric Functions

Sketch the graph of the following functions.

a. $f(x) = \sin\left(x + \frac{\pi}{2}\right)$ **b.** $f(x) = 2 + \sin x$ **c.** $f(x) = 2 + \sin\left(x - \frac{\pi}{4}\right)$

Solution

a. To sketch the graph of $f(x) = \sin(x + \pi/2)$, shift the graph of $y = \sin x$ to the left $\pi/2$ units, as shown in Figure 73(a).
b. To sketch the graph of $f(x) = 2 + \sin x$, shift the graph of $y = \sin x$ up 2 units, as shown in Figure 73(b).
c. To sketch the graph of $f(x) = 2 + \sin(x - \pi/4)$, shift the graph of $y = \sin x$ up 2 units and to the right $\pi/4$ units, as shown in Figure 73(c).



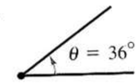
(a) Horizontal shift to the left (b) Vertical shift upward (c) Horizontal and vertical shift

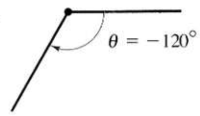
FIGURE 73
Transformations of the graph of $y = \sin x$.

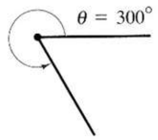
EXERCISES for Section 6

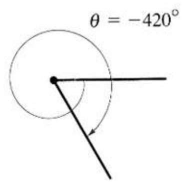
TECHNOLOGY Laboratory Guide Lab 0.6

In Exercises 1 and 2, determine two coterminal angles (one positive and one negative) for each given angle. Express your answers in degrees.

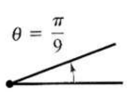
1. a.


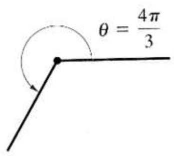
b.


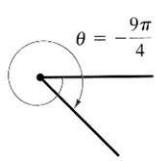
2. a.


b.


In Exercises 3 and 4, determine two coterminal angles (one positive and one negative) for each given angle. Express your answers in radians.

3. a.


b.


4. a.


b.


In Exercises 5 and 6, express the angles in radian measure as multiples of π and as decimals accurate to three decimal places.

5. a. 30° **b.** 150° **c.** 315° **d.** 120°

6. a. -20° **b.** -240° **c.** -270° **d.** 144°

In Exercises 7 and 8, express the angles in degree measure.

7. a. $\frac{3\pi}{2}$ **b.** $\frac{7\pi}{6}$ **c.** $-\frac{7\pi}{12}$ **d.** -2.367

8. a. $\frac{7\pi}{3}$ **b.** $-\frac{11\pi}{30}$ **c.** $\frac{11\pi}{6}$ **d.** 0.438

9. Let r represent the radius of a circle, θ the central angle (measured in radians), and s the length of the arc subtended by the angle. Use the relationship $s = r\theta$ to complete the table.

r	8 ft	15 in.	85 cm		
s	12 ft			96 in.	8642 mi
θ		1.6	$\frac{3\pi}{4}$	4	$\frac{2\pi}{3}$

10. *Instrumentation* The pointer on a voltmeter is 2 inches in length (see figure). Find the angle through which the pointer rotates when it moves $\frac{1}{2}$ inch on the scale.

11. *Electric Hoist* An electric hoist is being used to lift a piece of equipment (see figure). The diameter of the drum on the hoist is 8 inches and the equipment must be raised 1 foot. Find the number of degrees through which the drum must rotate.

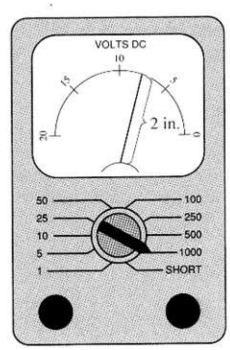


FIGURE FOR 10

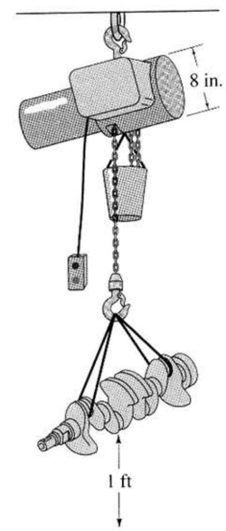


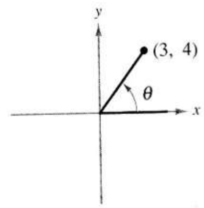
FIGURE FOR 11

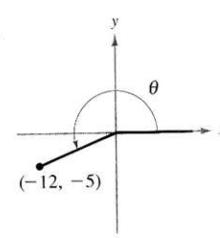
12. *Angular Speed* A car is moving at the rate of 50 miles per hour, and the diameter of its wheels is 2.5 feet.

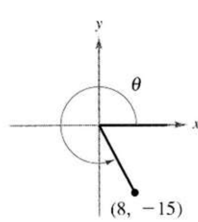
a. Find the number of revolutions per minute that the wheels are rotating.

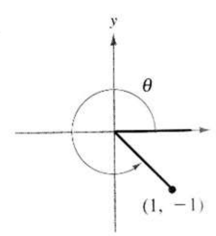
b. Find the angular speed of the wheels in radians per minute.

In Exercises 13 and 14, determine all six trigonometric functions for the angle θ.

13. a.


b.


14. a.


b.


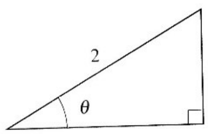
In Exercises 15 and 16, determine the quadrant in which θ lies.

15. a. $\sin\theta < 0$ and $\cos\theta < 0$
b. $\sec\theta > 0$ and $\cot\theta < 0$

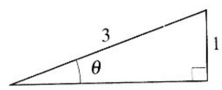
16. a. $\sin\theta > 0$ and $\cos\theta < 0$
b. $\csc\theta < 0$ and $\tan\theta > 0$

In Exercises 17–22, evaluate the trigonometric function.

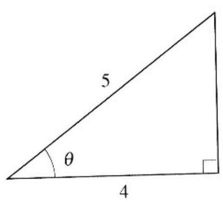
17. $\sin\theta = \frac{1}{2}$
$\cos\theta = ?$



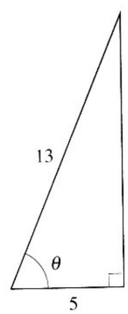
18. $\sin\theta = \frac{1}{3}$
$\tan\theta = ?$



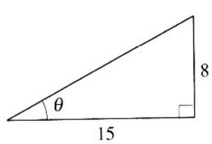
19. $\cos\theta = \frac{4}{5}$
$\cot\theta = ?$



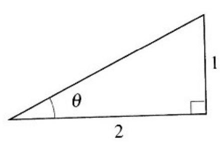
20. $\sec\theta = \frac{13}{5}$
$\csc\theta = ?$



21. $\cot\theta = \frac{15}{8}$
$\sec\theta = ?$



22. $\tan\theta = \frac{1}{2}$
$\sin\theta = ?$



In Exercises 23–26, evaluate the sine, cosine, and tangent of each angle *without* using a calculator.

23. a. $60°$
b. $120°$
c. $\frac{\pi}{4}$
d. $\frac{5\pi}{4}$

24. a. $-30°$
b. $150°$
c. $-\frac{\pi}{6}$
d. $\frac{\pi}{2}$

25. a. $225°$
b. $-225°$
c. $\frac{5\pi}{3}$
d. $\frac{11\pi}{6}$

26. a. $750°$
b. $510°$
c. $\frac{10\pi}{3}$
d. $\frac{17\pi}{3}$

In Exercises 27–30, use a calculator to evaluate the trigonometric functions to four significant digits.

27. a. $\sin 10°$
b. $\csc 10°$

28. a. $\sec 225°$
b. $\sec 135°$

29. a. $\tan\frac{\pi}{9}$
b. $\tan\frac{10\pi}{9}$

30. a. $\cot(1.35)$
b. $\tan(1.35)$

In Exercises 31–34, find two solutions of each equation. Express the results in radians ($0 \le \theta < 2\pi$). Do not use a calculator.

31. a. $\cos\theta = \frac{\sqrt{2}}{2}$
b. $\cos\theta = -\frac{\sqrt{2}}{2}$

32. a. $\sec\theta = 2$
b. $\sec\theta = -2$

33. a. $\tan\theta = 1$
b. $\cot\theta = -\sqrt{3}$

34. a. $\sin\theta = \frac{\sqrt{3}}{2}$
b. $\sin\theta = -\frac{\sqrt{3}}{2}$

In Exercises 35–42, solve the equation for θ ($0 \le \theta < 2\pi$).

35. $2\sin^2\theta = 1$

36. $\tan^2\theta = 3$

37. $\tan^2\theta - \tan\theta = 0$

38. $2\cos^2\theta - \cos\theta = 1$

39. $\sec\theta\csc\theta = 2\csc\theta$

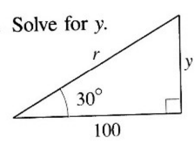
40. $\sin\theta = \cos\theta$

41. $\cos^2\theta + \sin\theta = 1$

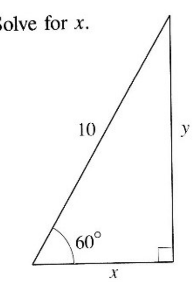
42. $\cos\frac{\theta}{2} - \cos\theta = 1$

In Exercises 43–46, solve for *x*, *y*, or *r* as indicated.

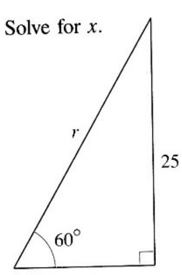
43. Solve for y.



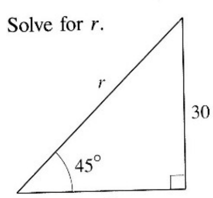
44. Solve for x.



45. Solve for x.



46. Solve for r.



47. *Airplane Ascent* An airplane leaves the runway climbing at 18° with a speed of 275 feet per second (see figure). Find the altitude of the plane after 1 minute.

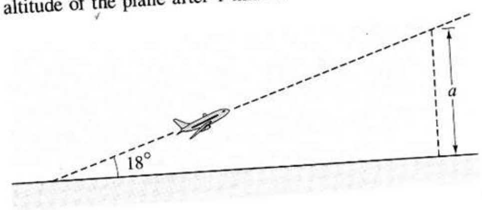


FIGURE FOR 47

48. *River Width* A biologist wants to know the width w of a river in order to set instruments properly to study the pollutants in the water. From point A, the biologist walks downstream 100 feet and sights point C to determine that $\theta = 50°$ (see figure). How wide is the river?

49. *Machine Shop Calculations* A tapered shaft has a diameter of 2 inches at the small end and is 6 inches long (see figure). The taper is 3°. Find the diameter d of the large end of the shaft.

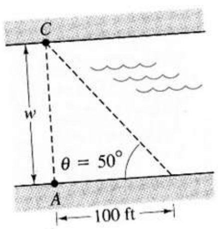


FIGURE FOR 48

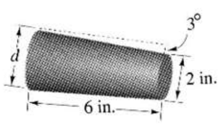


FIGURE FOR 49

50. *Height of a Mountain* In traveling across flat land, you notice a mountain directly in front of you. Its angle of elevation (to the peak) is 3.5°. After you drive 13 miles closer to the mountain, the angle of elevation is 9°. Approximate the height of the mountain.

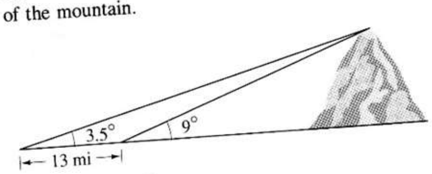
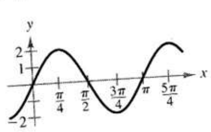


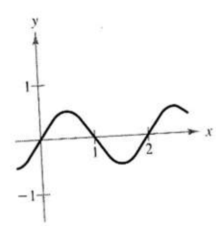
FIGURE FOR 50

In Exercises 51–54, determine the period and amplitude of each function.

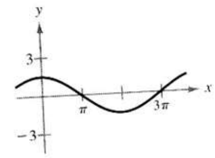
51. a. $y = 2\sin 2x$



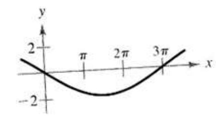
b. $y = \frac{1}{2}\sin \pi x$



52. a. $y = \frac{3}{2}\cos\frac{x}{2}$



b. $y = -2\sin\frac{x}{3}$



53. $y = 3\sin 4\pi x$

54. $y = \frac{2}{3}\cos\frac{\pi x}{10}$

In Exercises 55–58, find the period of the function.

55. $y = 5\tan 2x$

56. $y = 7\tan 2\pi x$

57. $y = \sec 5x$

58. $y = \csc 4x$

C *Essay* **In Exercises 59 and 60, use a graphing utility to graph each function f on the same set of coordinate axes when $c = -2$, $c = -1$, $c = 1$, and $c = 2$. Give a written description of the change in the graph caused by changing c.**

59. a. $f(x) = c\sin x$
b. $f(x) = \cos(cx)$
c. $f(x) = \cos(\pi x - c)$

60. a. $f(x) = \sin x + c$
b. $f(x) = -\sin(2\pi x - c)$
c. $f(x) = c\cos x$

In Exercises 61–72, sketch the graph of the function.

61. $y = \sin\frac{x}{2}$

62. $y = 2\cos 2x$

63. $y = -\sin\frac{2\pi x}{3}$

64. $y = 2\tan x$

65. $y = \csc\frac{x}{2}$

66. $y = \tan 2x$

67. $y = 2\sec 2x$

68. $y = \csc 2\pi x$

69. $y = \sin(x + \pi)$

70. $y = \cos\left(x - \frac{\pi}{3}\right)$

71. $y = 1 + \cos\left(x - \frac{\pi}{2}\right)$

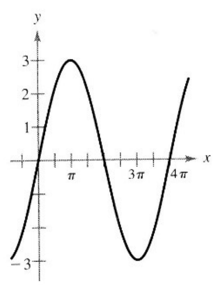
72. $y = 1 + \sin\left(x + \frac{\pi}{2}\right)$

C 73. Use a graphing utility to graph the functions $f(x) = 2\cos 2x + 3\sin 3x$ and $g(x) = 2\cos 2x + 3\sin 4x$.
a. Use the graphs to find the period of each function.
b. If α and β are positive integers, is the function $h(x) = A\cos\alpha x + B\sin\beta x$ periodic? Explain your answer.

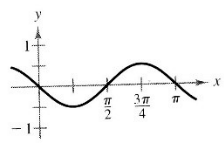
74. Two trigonometric functions f and g have periods of 2, and their graphs intersect when $x = 5.35$.
a. Give one smaller and one larger positive value of x where the functions have the same value.
b. Determine one negative value of x where the graphs intersect.
c. Is it true that $f(13.35) = g(-4.65)$? Give a reason for your answer.

In Exercises 75 and 76, find $a, b,$ and c so that the graph of the function matches the graph in the figure.

75. $y = a\cos(bx - c)$



76. $y = a\sin(bx - c)$



77. *Temperature* The temperature T (°F) in Orlando, Florida for a 24-hour period is listed in the table (time t in hours, with $t = 0$ corresponding to midnight).

t	0	1	2	3	4	5
T	57	56	55	55	57	59

t	6	7	8	9	10	11
T	63	66	70	74	77	81

t	12	13	14	15	16	17
T	83	84	85	84	83	81

t	18	19	20	21	22	23
T	77	74	70	66	63	59

a. Select an appropriate scale and plot the data on a rectangular coordinate system.

b. Estimate the constants a, b, ω, and δ so that the function

$$T(t) = a + b\sin(\omega t - \delta)$$

is a reasonable model for the data.

c. Use a graphing utility to plot the data and the model found in part b.

78. *Ferris Wheel* The model for the height h of a Ferris wheel car is

$$h = 50 + 50\sin 8\pi t$$

where t is measured in minutes. (The Ferris wheel has a radius of 50 feet.) This model yields a height of 50 feet when $t = 0$. Alter the model so that the height of the car is 0 feet when $t = 0$.

79. *Piano Tuning* When tuning a piano, a technician strikes a tuning fork for the A above middle C. This creates a sound (a type of wave motion) that can be approximated by

$$y = 0.001\sin 880\pi t$$

where y is measured in inches and t is the time in seconds.

a. What is the period p of this function?

b. What is the frequency f of this note $(f = 1/p)$?

c. Sketch the graph of this function.

80. *Blood Pressure* The function

$$P = 100 - 20\cos\frac{5\pi t}{3}$$

approximates the blood pressure P in millimeters of mercury at time t in seconds for a person at rest.

a. Find the period of the function.

b. Find the number of heartbeats per minute.

c. Sketch the graph of the pressure function.

In Exercises 81 and 82, use a graphing utility to compare the graph of f with the given graph. In each case, the graph of f is a Fourier approximation of the given graph. Try to improve the approximation by adding a term to $f(x)$. Use a graphing utility to verify that your new approximation is better than the original. Can you find other terms to add to make the approximation even better? What is the pattern? (In Exercise 81, sine terms can be used to improve the approximation and in Exercise 82, cosine terms can be used.)

81. $f(x) = \dfrac{4}{\pi}\left(\sin \pi x + \dfrac{1}{3}\sin 3\pi x\right)$

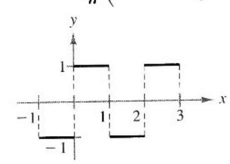


FIGURE FOR 81

82. $f(x) = \dfrac{1}{2} - \dfrac{4}{\pi^2}\left(\cos \pi x + \dfrac{1}{9}\cos 3\pi x\right)$

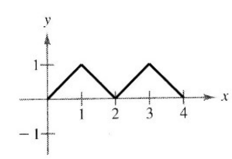


FIGURE FOR 82

83. *Normal Temperatures* The normal monthly high temperature for Erie, Pennsylvania is approximated by

$$H(t) = 54.33 - 20.38\cos\frac{\pi t}{6} - 15.69\sin\frac{\pi t}{6}$$

and the normal monthly low temperature is approximated by

$$L(t) = 39.36 - 15.70\cos\frac{\pi t}{6} - 14.16\sin\frac{\pi t}{6}$$

where t is the time in months with $t = 1$ corresponding to January. (*Source:* NOAA) Use a graphing utility to graph the functions over a period of one year, and use the graphs to answer the following questions.

a. During what part of the year is the difference between the normal high and low temperatures greatest? When is it smallest?

b. The sun is the farthest north in the sky around June 21, but the graph shows the warmest temperatures at a later date. Approximate the lag time of the temperatures relative to the position of the sun.