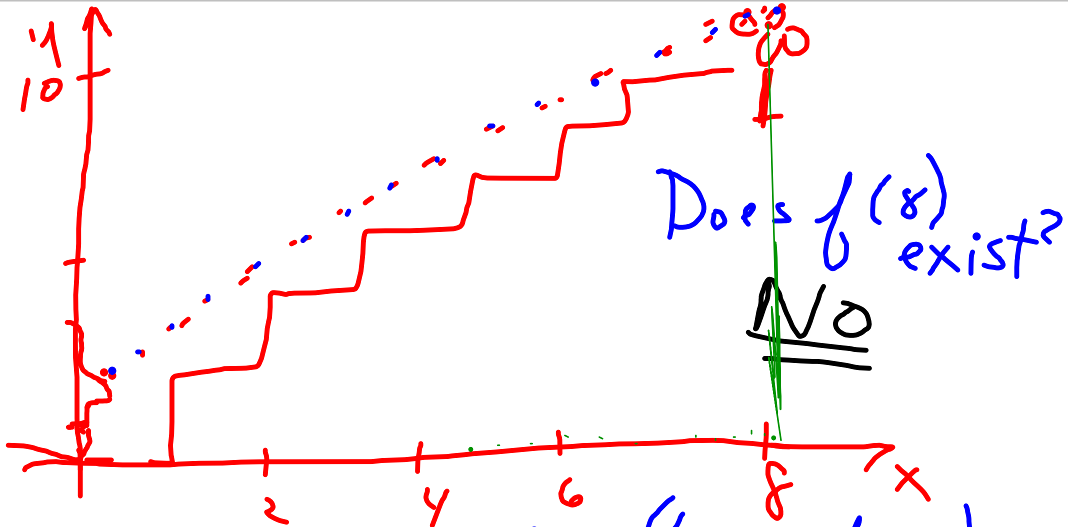
Homework for Sections:
1.2 Finding Limits Graphically and Numerically
1.3 Evaluating Limits Analytically

1.2 p. 52 Definition of Limit : aware
skip p. 53, top p. 54
p. 54-55 # 1, 7, 9-17

1.3 p. 67 # 1, 3; 5, 9, 13, ... 25, 27-37,
41-61, 67-75

Homework on the Web



$$\lim_{x \to 8} f(x) = \lim_{x \to 8} (\text{height of note}) = 10$$

Let f be a function defined on an open interval cont c (8), except possibly at c and let L (10) be a real #.

Then $\lim_{x \to c\,(8)} f(x) = L\ (10)$ means that for each $\varepsilon > 0\ \exists\ \delta > 0 \ni$ if $0 < |x - c| < \delta$, then $|f(x) - L| < \varepsilon$

x is near 8 eg: x=7.9 then δ =0.1

y is near 10 and ε is small

How Find Limit?

1. Numerically

$x \neq \pm 2$

$$\lim_{x \to 2} \frac{x-2}{x^2-4} = \frac{1}{4}$$

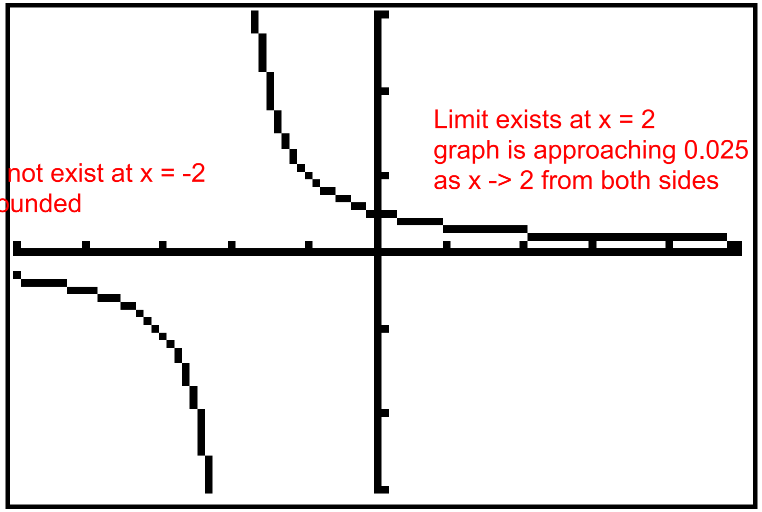
$$f(2) = \frac{2-2}{2^2-4} = \frac{0}{0} \text{ DNE}$$

x	1.9	1.99	1.999	2.001	2.01	2.1
f(x)	0.256	0.250	0.250	0.250	.249	0.244

$\to 2$

$\to 0.250$

2. Graphical Appr.: Tells us if limit exists, & suggest value



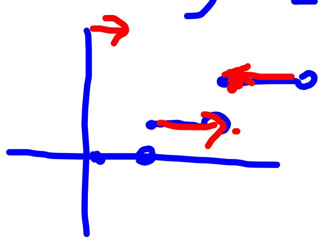
Limit does not exist at x = -2
f(x) is unbounded

Limit exists at x = 2
graph is approaching 0.025
as x -> 2 from both sides

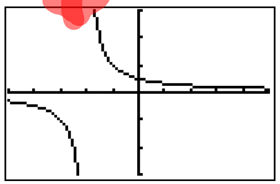
When Does Limit NOT exist?

1. $\lim\limits_{x\to c^-} f(x) \neq \lim\limits_{x\to c^+} f(x)$

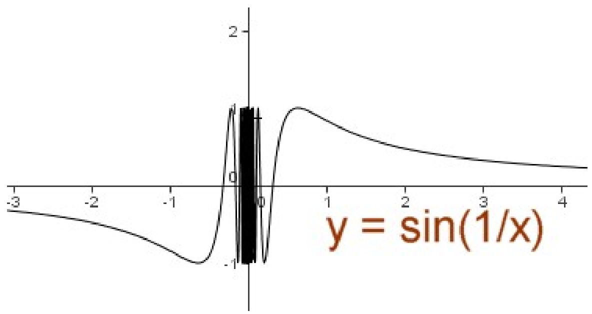
$y = [x]$



2. lim unbounded

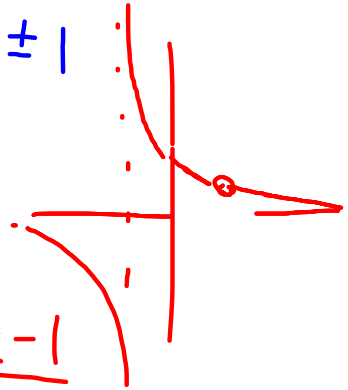


3. f oscillating



Find limit analytically.

$$f(x) = \frac{x-1}{x^2-1}, \quad x \neq \pm 1$$



Subst: $f(1) = \frac{1-1}{1-1} = \frac{0}{0}$

$$f(x) = \frac{\cancel{(x-1)}}{(x+1)\cancel{(x-1)}} = \frac{1}{x+1} \cdot \frac{x-1}{x-1}$$

$$\frac{x-1}{x^2-1} \stackrel{?}{=} \frac{1}{x+1} \leftarrow \left(1, \tfrac{1}{2}\right)$$

The function on the left, f(x), is not defined at x=1 or x=-1.
The function on the right, 1/(x+1), is not defined at x= -1, but it is defined at x=1, at the point (1, 0.5).
So, these functions are equal, except for the point (1,0.5).
That means that as x -> 1, the y values of both are equal, and they are approaching the same limit, 0.5.

$$\lim_{x \to 1} \frac{x-1}{x^2-1} = \lim_{x \to 1} \frac{1}{x+1} = \frac{1}{2}$$

$$\lim_{x \to -1} \frac{x-1}{x^2-1} = \lim_{x \to -1} \frac{1}{x+1} \to \frac{1}{0}$$

DNE

unbdy.

Unbounded

54. $\lim_{x \to 0} \frac{\sqrt{2+x} - \sqrt{2}}{x} = \lim_{x \to 0} \frac{1}{\sqrt{2+x} + \sqrt{2}} = \frac{1}{2\sqrt{2}}$

$$\frac{\sqrt{2+x} - \sqrt{2}}{x} \cdot \frac{\sqrt{2+x} + \sqrt{2}}{\sqrt{2+x} + \sqrt{2}}$$

$$\frac{(\cancel{2}+x) - \cancel{2}}{x(\sqrt{2+x} + \sqrt{2})} = \frac{\cancel{x}^{1}}{\cancel{x}_{1}(\sqrt{2+x} + \sqrt{2})} = \frac{1}{\sqrt{2+x} + \sqrt{2}}$$

Title: example w radicals (6 of 6)

Summary: **To Find a Limit**

1. **Substitute x = c**

    a) If finite number, L, then the limit is L.

    b) If results in form, k / 0 , then f is unbounded and the limit DNE

2. **If indeterminate, use algebra to find a function that is equivalent at all but the undefined point, and substitute again.**

3. **If still indeterminate, consider special limits:**

$$\lim_{x \to 0} \frac{\sin x}{x} = 1 \qquad \lim_{x \to 0} \frac{1 - \cos x}{x} = 0$$