8.7 Indeterminate Forms and L'Hopital's Rule

Study 8.7

p.574: 5 - 15, 19 - 25, 29, 37 - 45, 51

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8.7 Indeterminate Forms and L'Hopital's Rule

Consider:

$$\lim_{x\to 1}\frac{\overset{x(x-1)}{x^2-x}}{(x^2+1)(x-1)} \longrightarrow ?\ \frac{0}{0}$$

$$= \lim_{x\to 1}\frac{x}{x^2+1} = \frac{1}{2}$$

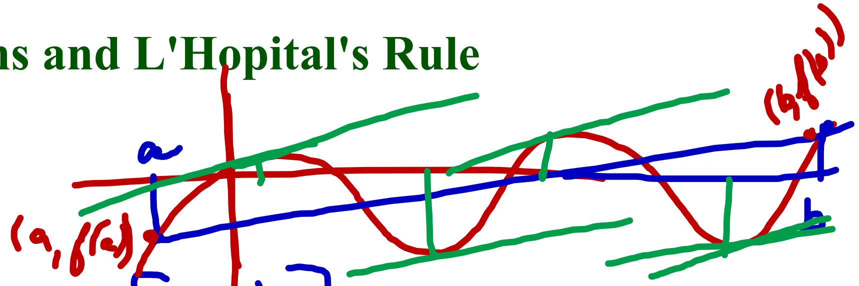
$$\frac{1}{0}\ \infty$$

8.7 Indeterminate Forms and L'Hopital's Rule

What about:
p. 574 #2

$$\lim_{x \to 0} \frac{1 - e^{x}}{x} \to \frac{0}{0}$$

8.7 Indeterminate Forms and L'Hopital's Rule
from Calc 1, Ch 3



MVT

① f(x) is cont [a,b]

② f(x) is diff. (a,b)

then $\exists$ at least one c $\ni$ $f'(c) = \dfrac{f(b)-f(a)}{b-a}$

Extended MVT: Given that both f(x), g(x)

1 continuous on [a,b],

2. differentiable on (a,b)

and 3. g ' (x)≠ 0 for any x on (a,b)

Then there exists c on (a,b) such that

$$\frac{f'(c)}{g'(c)} = \frac{f(b) - f(a)}{g(b) - g(a)}$$

Used to prove L'Hopital'sRule

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L'Hopital's Rule Let f(x), g(x) be

1. **differentiable on (a,b)** containing c, except possibly at c itself.
2. **g ' (x) ≠ 0** for any x on (a,b), except possibly at c itself.

If :

$$\lim_{x \to c} \frac{f(x)}{g(x)} \Rightarrow \frac{0}{0} \text{ or } \frac{\infty}{\infty}$$

Th

$$\lim_{x \to c} \frac{f(x)}{g(x)} = \lim_{x \to c} \frac{f'(x)}{g'(x)}$$

provided the limit exists or is infinite.

8.7 Indeterminate Forms and L'Hopital's Rule

Indeterminate Forms:

$$\frac{0}{0} \qquad \frac{\infty}{\infty} \qquad \infty - \infty$$

$$0\infty \qquad 0^0 \qquad 1^\infty \qquad \infty^0$$

Determinate Forms:

$$\infty + \infty \Rightarrow \infty$$

$$-\infty - \infty \Rightarrow -\infty$$

$$0^\infty \Rightarrow 0$$

$$0^{-\infty} \Rightarrow \infty$$

$$\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x = e$$

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$$\lim_{x\to 0}\frac{1-e^{x}}{x} \to \frac{0}{0}$$

① Cont

② $g'(x) \neq 0$

$$\overset{LH}{=} \lim_{x\to 0}\frac{-e^{x}}{1} = -1$$

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8. $\lim_{x\to 0} \frac{\sin 4x}{2x} \to \frac{0}{0}$

$\overset{LH}{=} \lim_{x\to 0} \frac{4\cos 4x}{2} = 2$

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14. $\lim_{x \to 2^-} \dfrac{\sqrt{4-x^2}}{x-2} \to \dfrac{0}{0}$ $-2 \le x \le 2$

$\overset{LH}{=} \lim_{x \to 2^-} \dfrac{\frac{1}{\cancel{2}} \frac{-\cancel{2}x}{\sqrt{4-x^2}}}{1} \to \dfrac{-2}{0} = -\infty$

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$$\lim_{x\to 1} \frac{\ln x^2}{x^2-1} \to \frac{0}{0}$$

$$= \lim_{x\to 1} \frac{2\ln x}{x^2-1} \overset{LH}{=} \lim_{x\to 1} \frac{2\cdot\frac{1}{x}}{2x}$$

$$= \lim_{x\to 1} \frac{1}{x^2} = 1$$

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$$\lim_{x \to 0^+} \frac{e^x - (1+x)}{x^3} \quad \overset{-1-x}{} \quad \to \frac{1-1}{0} = \frac{0}{0}$$

$$\overset{LH}{=} \lim_{x \to 0^+} \frac{e^x - 1}{3x^2} \to \frac{1-1}{0} = \frac{0}{0}$$

$$\overset{LH}{=} \lim_{x \to 0^+} \frac{e^x}{6x} \to \frac{1}{+0} = \infty$$

8.7 Indeterminate Forms and L'Hopital's Rule

24. $\lim_{x\to\infty} \dfrac{x-1}{x^2+2x+3} \longrightarrow \dfrac{\infty}{\infty}$

$\overset{LH}{=} \lim_{x\to\infty} \dfrac{1}{2x+2} \longrightarrow \dfrac{1}{\infty} = 0$

8.7 Indeterminate Forms and L'Hopital's Rule

26. $\lim_{x \to \infty} \dfrac{x^3}{x+2} \to \dfrac{\infty}{\infty}$

$\overset{LH}{=} \lim_{x \to \infty} \dfrac{3x^2}{1} = +\infty$

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28. $\lim_{x\to\infty} \frac{x^2}{e^x} \to \frac{\infty}{\infty}$

$\overset{LH}{=} \lim_{x\to\infty} \frac{2x}{e^x} \to \frac{\infty}{\infty}$

$\overset{LH}{=} \lim_{x\to\infty} \frac{2}{e^x} \to \frac{2}{\infty} = 0$

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$$\lim_{x\to\infty}(1+x)^{1/x} \to \infty^0$$

$$y=\lim_{x\to\infty}(1+x)^{1/x}$$

$$\ln y=\lim_{x\to\infty}\ln(1+x)^{1/x}$$

$$=\lim_{x\to\infty}\frac{1}{x}\ln(1+x) \to \frac{\infty}{\infty}$$

$$\overset{LH}{=}\lim_{x\to\infty}\frac{1}{1+x}=\frac{1}{\infty}=0$$

$\ln y=0$ $\qquad$ $y=e^0=1$

$$y=2^x$$

$$\ln y = x\ln 2$$

$$\frac{\ln y}{\ln 2}=x$$

52. $\lim_{x \to 2^+} \left(\frac{1}{x^2-4} - \frac{\sqrt{x-1}}{x^2-4} \right) \to \infty - \infty$

$= \lim_{x \to 2^+} \frac{1 - \sqrt{x-1}}{x^2-4} \to \frac{0}{0}$

$x - 1 \geq 0$

$x \geq 1$

$\overset{LH}{=} \lim_{x \to 2^+} \frac{-\frac{1}{2}\frac{1}{\sqrt{x-1}}}{2x} = \frac{-\frac{1}{2} \cdot \frac{1}{1}}{4} = -\frac{1}{8}$