8.4 Integration by Trig Substition

p. 549 # 5-15, 21, 27,

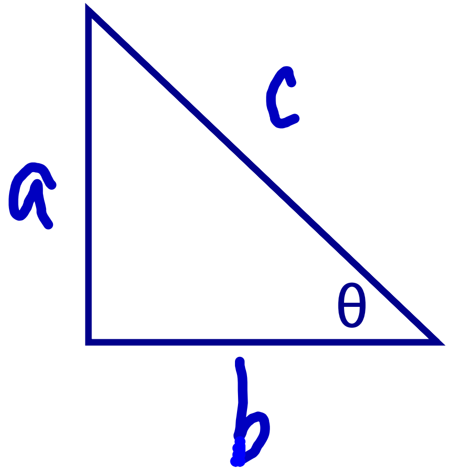
31, 35, 39, 47, 49,

Class Notes: Prof. G. Battaly, Westchester Community College, NY

8.4 Integration by Trig Substition

Pythagorian Theorum

$$a^2 + b^2 = c^2$$



Forms

$$c = \sqrt{a^2 + b^2}$$

$$b = \sqrt{c^2 - a^2}$$

Calculus Home Page

Class Notes: Prof. G. Battaly, Westchester Community College, NY

Homework Part 1 Homework Par 2

8.4 Integration by Trig Substition

p. 549 #24

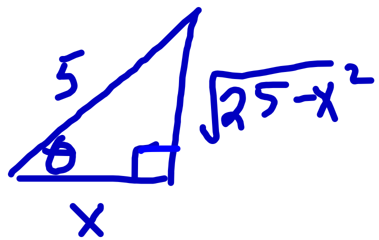
If you recognize arcsin: $u = x$ $\quad a = 5$

$du = dx$

$$\int \frac{1}{\sqrt{25 - x^2}}\,dx = \arcsin \frac{x}{5} + C$$

If you do NOT recognize arcsin, from: $a^2 + b^2 = c^2$

$$b = \sqrt{c^2 - a^2}$$

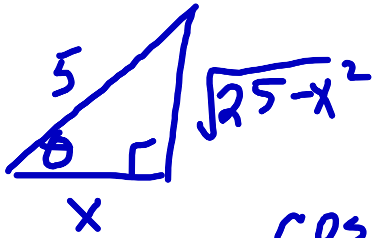


8.4 Integration by Trig Substition

$a^2+b^2=c^2$

p. 549 #24

$$\int \frac{1}{\sqrt{25-x^2}}\,dx$$



$\cos\theta = \frac{x}{5}$

$5\cos\theta = x$

$-5\sin\theta\,d\theta = dx$

$5\sin\theta = \sqrt{25-x^2}$

$$= \int \frac{1\,(-5)\sin\theta\,d\theta}{5\sin\theta}$$

$$= -\int d\theta = -\theta + C$$

$$= -\arccos\frac{x}{5} + C$$

Class Notes: Prof. G. Battaly, Westchester Community College, NY

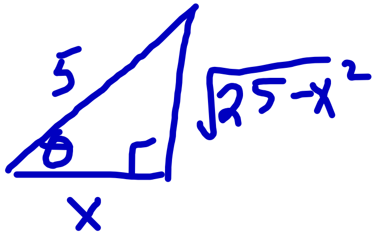
Calculus Home Page

Homework Part 1 Homework Par 2

8.4 Integration by Trig Substition

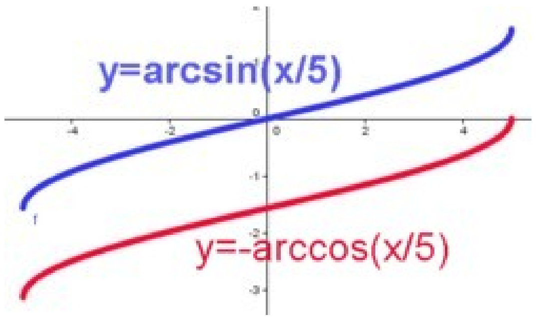
p. 549 #24

$$\int \frac{1}{\sqrt{25-x^2}}\,dx$$



But, is this the same as *arc sin (x/5)*?

$$= -\arccos\frac{x}{5} + C_1 \overset{?}{=} \arcsin\frac{x}{5} + C_2$$



Yes, but need different constants.

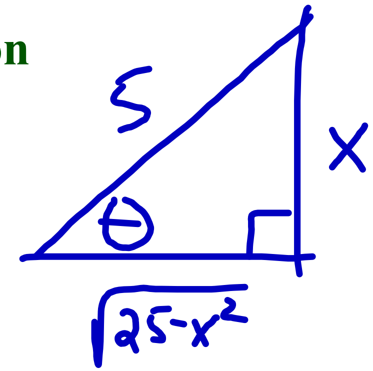
Class Notes: Prof. G. Battaly, Westchester Community College, NY

Calculus Home Page

Homework Part 1 Homework Par 2

8.4 Integration by Trig Substition

$$\int \frac{1}{\sqrt{25-x^2}}\,dx$$



What if we label the triangle differently, reversing *a* and *b*?

$$5\sin\theta = x$$

$$5\cos\theta\,d\theta = dx$$

$$5\cos\theta = \sqrt{25-x^2}$$

$$= \int \frac{5\cos\theta\,d\theta}{5\cos\theta}$$

$$= \int d\theta = \theta + C = \arcsin\frac{x}{5} + C$$

8.4 Integration by Trig Substition

22. $\int \frac{x}{\sqrt{9-x^2}}\,dx = -\frac{1}{2}\int (9-x^2)^{-1/2}(-2)x\,dx$

$u = 9-x^2$

$du = -2x\,dx$

$= -\frac{1}{2}\frac{(9-x^2)^{1/2}}{1/2} + C = -(9-x^2)^{1/2} + C$

No need for trig substitution. Use u^n

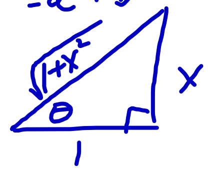
Class Notes: Prof. G. Battaly, Westchester Community College, NY

Calculus Home Page

Homework Part 1 Homework Par 2

8.4 Integration by Trig Substition

$c^2 = a^2 + b^2$



16. $$\int \frac{x^2}{(1+x^2)^2}\,dx$$

$\tan\theta = x$

$\sec^2\theta\, d\theta = dx$

$\cos\theta = \frac{1}{\sqrt{1+x^2}}$

$\sec\theta = \sqrt{1+x^2}$

$\sec^2\theta = 1+x^2$

$$= \int \frac{\tan^2\theta\ \sec^2\theta\, d\theta}{(\sec^2\theta)(\sec^2\theta)}$$

$$= \int \frac{\tan^2\theta}{\sec^2\theta}\, d\theta = \int \frac{\sin^2\theta}{\cos^2\theta} \cdot \cos^2\theta\, d\theta = \int \sin^2\theta\, d\theta$$

$$= \frac{1}{2}\int (1-\cos 2\theta)\, d\theta = \frac{1}{2}\int d\theta - \frac{1}{2}\cdot\frac{1}{2}\int \cos 2\theta\, 2\,d\theta$$

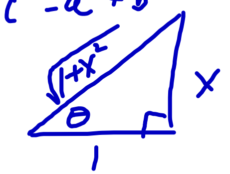
Class Notes: Prof. G. Battaly, Westchester Community College, NY

Calculus Home Page

Homework Part 1 Homework Par 2

$$= \frac{1}{2}\theta - \frac{1}{4}\sin 2\theta + c$$

$c^2 = a^2 + b^2$



!

$$= \frac{1}{2}\arctan x - \frac{2}{4}\sin\theta\cos\theta + c$$

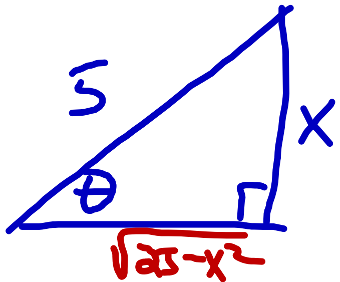
$\sin 2u = 2\sin u \cos u$

$$= \frac{1}{2}\arctan x - \frac{1}{2}\cdot\frac{x}{\sqrt{1+x^2}}\cdot\frac{1}{\sqrt{1+x^2}} + c$$

$$= \frac{1}{2}\arctan x - \frac{x}{2(1+x^2)} + c$$

8.4 Integration by Trig Substition

$$\int \frac{10}{x^2\sqrt{25-x^2}}\,dx$$



$$5\sin\theta = x$$

$$5\cos\theta\, d\theta = dx$$

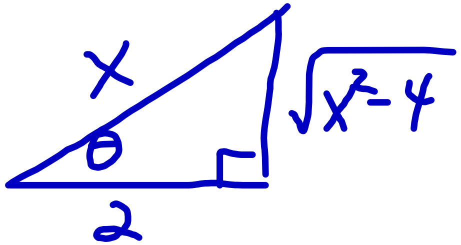
$$5\cos\theta = \sqrt{25-x^2}$$

$$10\int \frac{\cancel{5\cos\theta}\, d\theta}{25\sin^2\theta\cdot \cancel{5\cos\theta}}$$

$$= \frac{10}{25}\int \csc^2\theta\, d\theta = -\frac{2}{5}\cot\theta + C = -\frac{2}{5}\frac{\sqrt{25-x^2}}{x} + C$$

8.4 Integration by Trig Substition

12. $\displaystyle\int \frac{x^3}{\sqrt{x^2-4}}\,dx$



$\cos\theta = \frac{2}{x}$

$\sec\theta = \frac{x}{2}$

$x = 2\sec\theta$

$\tan\theta = \frac{\sqrt{\ \ }}{2}$

$$= \int \frac{2^3\sec^3\theta \cdot \cancel{2}\sec\theta\,\cancel{\tan\theta}\,d\theta}{\cancel{2\tan\theta}}$$

$$= 8\int \sec^4\theta\,d\theta$$

8.4 Integration by Trig Substition

Class Notes: Prof. G. Battaly, Westchester Community College, NY

Calculus Home Page

Homework Part 1   Homework Par 2

8.4 Integration by Trig Substition

Class Notes: Prof. G. Battaly, Westchester Community College, NY

Calculus Home Page

Homework Part 1   Homework Par 2

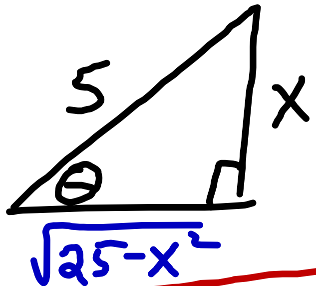
8.4 Integration by Trig Substition

8.4 Integration by Trig Substition

8.4 Integration by Trig Substition

5. $\displaystyle\int \frac{1}{(25-x^2)^{3/2}}\,dx$

$(\sqrt{25-x^2})^3$



$5\cos\theta = \sqrt{25-x^2}$

$5\sin\theta = x$

$5\cos\theta\,d\theta = dx$

$$= \int \frac{5\cos\theta\,d\theta}{(5\cos\theta)^3}$$

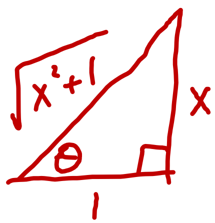
$$= \frac{5}{5^3}\int \frac{\cos\theta}{\cos^3\theta}\,d\theta = \frac{1}{25}\int \frac{1}{\cos^2\theta}\,d\theta$$

$$= \frac{1}{25}\int \sec^2\theta\,d\theta = \frac{1}{25}\tan\theta + C$$

$$= \frac{1}{25}\cdot\frac{x}{\sqrt{25-x^2}} + C$$

Title: Apr 21-6:21 PM (1 of 4)

15. $\displaystyle\int \frac{1}{(1+x^2)^2}\,dx$



$\tan\theta = x$

$dx = \sec^2\theta\, d\theta$

$\cos\theta = \frac{1}{\sqrt{x^2+1}}$

$\sec\theta = \sqrt{x^2+1}$

$\sec^4\theta = \left(\sqrt{x^2+1}\right)^4 = (x^2+1)^2$

$$= \int \frac{\sec^2\theta\, d\theta}{\sec^4\theta}$$

$$= \int \frac{1}{\sec^2\theta}\,d\theta$$

$$= \int \cos^2\theta\, d\theta$$

$$= \frac{1}{2}\int (1+\cos 2\theta)\,d\theta$$

$$= \frac{1}{2}\int d\theta + \frac{1}{2}\cdot\frac{1}{2}\int \cos 2\theta (2)\, d\theta$$

$$= \frac{1}{2}\theta + \frac{1}{2}\cdot\frac{1}{2}\sin 2\theta + C$$

$$= \frac{1}{2}\theta + \frac{1}{4}\sin 2\theta + C = \frac{1}{2}\arctan x + \frac{1}{2}\cdot\frac{x}{\sqrt{x^2+1}}\cdot\frac{1}{\sqrt{x^2+1}} + C$$

$$\frac{1}{4}\cdot 2\sin\theta\cos\theta$$

$$= \frac{1}{2}\arctan x + \frac{1}{2}\,\frac{x}{(x^2+1)} + C$$

Title: Apr 21-6:33 PM (2 of 4)

35. $\frac{1}{2}\int \underline{\underline{2e^{2x}}}\sqrt{\underline{1+e^{2x}}}\,\underline{\underline{dx}}$

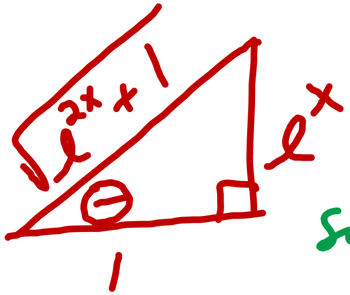
$u = 1+e^{2x}$

$du = 2e^{2x}dx$

$= \frac{1}{2}\int u^{1/2}\,du = \frac{1}{2}\frac{u^{3/2}}{3/2} + c$

$= \frac{1}{3}(1+e^{2x})^{3/2} + c$

$$\int e^{2x}\sqrt{1+e^{2x}}\,dx$$



$$\sec\theta = \sqrt{e^{2x}+1}$$

$$\tan\theta = e^x$$

$$\sec^2\theta\,d\theta = e^x\,dx$$

$$dx = \frac{\sec^2\theta\,d\theta}{e^x} = \frac{\sec^2\theta\,d\theta}{\tan\theta}$$

$$= \frac{1}{\cos^2\theta}\cdot\frac{\cos\theta}{\sin\theta}\,d\theta$$

$$= \frac{1}{\cos\theta\sin\theta}\,d\theta$$

$$= \int \tan^2\theta\,\sec\theta\,\frac{1}{\cos\theta\sin\theta}\,d\theta$$

$$= \int \frac{\sin^{\cancel{2}}\theta}{\cos^2\theta}\cdot\frac{1}{\cos\theta}\cdot\frac{1}{\cos\theta\cancel{\sin\theta}}\,d\theta$$

$$= \int \frac{\sin\theta}{\cos^4\theta}\,d\theta = \frac{\cos^{-3}\theta}{-3} + c$$

$$= -\frac{1}{3}\sec^3\theta + C$$

$$= -\frac{1}{3}\left(e^{2x}+1\right)^{3/2} + C$$