8.3 Integrals with Trig

Study 8.3 p. 540# 1-9, 13, 17
25-29, 43, 69, 71, 91

8.3 Integrals with Trig

$$-\int \cos^5 x(-)\sin x\,dx$$

$u = \cos x$

$du = -\sin x\,dx$

$$-\int u^5\,du = -\frac{u^6}{6} + C$$

$$= -\frac{\cos^6 x}{6} + C$$

Consider the Basic Rules

$$u^n\,du$$

$$e^u\,du$$

$$\frac{du}{u}$$

8.3 Integrals with Trig

$\sin^2 u + \cos^2 u = 1$

$$\int \cos^5 x \sin^2 x \, dx$$

$$= \int \sin^2 x \cos^4 x \cos x \, dx$$

$$= \int \sin^2 x (\cos^2 x)^2 \cos x \, dx$$

$$= \int \sin^2 x (1 - \sin^2 x)^2 \cos x \, dx$$

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Calculus Home Page

Homework Part 1

$$= \int \sin^2 x (1 - 2\sin^2 x + \sin^4 x) \cos x \, dx$$

$$= \int (\sin^2 x - 2\sin^4 x + \sin^6 x) \cos x \, dx$$

$$= \int \sin^2 x \cos x \, dx - 2\int \sin^4 x \cos x \, dx + \int \sin^6 x \cos x \, dx$$

$$= \frac{\sin^3 x}{3} - \frac{2\sin^5 x}{5} + \frac{\sin^7 x}{7} + C$$

8.3 Integrals with Trig

#8. $\int \sin^3 x\,dx = \int \sin^2 x \sin x\,dx$

$= \int (1-\cos^2 x)\sin x\,dx = \int \sin x\,dx + \int \cos^2 x \sin x\,dx$

$u = \cos x$

$du = -\sin x\,dx$

$= -\cos x + \frac{\cos^3 x}{3} + C$

$u = 1 - \cos^2 x$

$du = -2\cos x \sin x\,dx$

Does not work because do not have the resulting du.

8.3 Integrals with Trig

10. $\int \cos^3 \frac{x}{3}\,dx = \int \cos^2 \frac{x}{3} \cdot \cos \frac{x}{3}\,dx$

$= \int \left(1 - \sin^2 \frac{x}{3}\right) \cos \frac{x}{3}\,dx = 3\int \cos \frac{x}{3} \cdot \frac{1}{3}\,dx - 3\int \sin^2 \frac{x}{3} \cdot \frac{1}{3} \cos \frac{x}{3}\,dx$

$= 3\sin \frac{x}{3} - 3\,\frac{\sin^3 \frac{x}{3}}{3} + C$

$= 3\sin \frac{x}{3} - \sin^3 \frac{x}{3} + C$

$u = \sin \frac{x}{3}$

$du = \frac{1}{3} \cos \frac{x}{3}\,dx$

$$26\ \frac{1}{2}\int \sec^2(2x-1)^2\,dx = \frac{1}{2}\tan(2x-1)+c$$

$$\int \sin^2 x\,dx = \int \sin x \sin x\,dx$$

$$\frac{1}{2}\int (1-\cos 2x)\,dx = \frac{1}{2}x - \frac{1}{4}\sin 2x + C$$

$u = \sin x \qquad \int dv = \int \sin x\,dx$

$du = \cos x\,dx \qquad v = -\cos x$

$$= -\sin x \cos x + \int \cos^2 x\,dx$$

$$-\sin x \cos x + \int (1-\sin^2 x)\,dx = -\sin x \cos x + x - \int \sin^2 x\,dx$$

$$\int \sin^2 x\,dx = \frac{1}{2}(-\sin x \cos x + x)$$

$$\cos^2\theta = \frac{1+\cos 2\theta}{2}$$

$$\sin^2\theta = \frac{1-\cos 2\theta}{2}$$

✓ Power Reducing Formulas.

$$\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1$$

$$= 1 - 2\sin^2\theta$$

p. 540 #17

$$\int x \sin^2 x \, dx$$

$$= \int x \left(\frac{1-\cos 2x}{2}\right) dx$$

$$= \frac{1}{2}\int (x - x\cos 2x)\, dx$$

$$= \frac{1}{2}\int x\, dx - \frac{1}{2}\int x\cos 2x\, dx$$

$$= \frac{1}{2}\cdot\frac{x^2}{2} - \frac{1}{2}\left[\frac{1}{2}x\sin 2x - \frac{1}{2}\int \sin 2x\, dx\right]$$

$$= \frac{1}{4}x^2 - \frac{1}{4}x\sin 2x + \frac{1}{4}\cdot\frac{1}{2}(-\cos 2x) + C$$

$$= \frac{1}{4}x^2 - \frac{1}{4}x\sin 2x - \frac{1}{8}\cos 2x + C$$

$u = x \quad dv = \sin^2 x\, dx$

$u = \sin^2 x \quad dv = x\, dx$

$u = x\sin x \quad dv = \sin x\, dx$

$(x\cos x + \sin x)\, dx \quad v = \cos x$

$\cos^2 x = \frac{1+\cos 2x}{2}$

$\sin^2 x = \frac{1-\cos 2x}{2}$

$u = x \quad du = dx$

$dv = \frac{1}{2}\int \cos 2x\, dx \quad (2)$

$v = \frac{1}{2}\sin 2x$

$$\int_{-\pi/2}^{\pi/2} \cos^3 x\,dx = \int_{-\pi/2}^{\pi/2} \cos^2 x \cos x\,dx$$

$$= \int_{-\pi/2}^{\pi/2} (1-\sin^2 x)\cos x\,dx = \int_{-\pi/2}^{\pi/2} \cos x\,dx - \int_{-\pi/2}^{\pi/2} \sin^2 x \cos x\,dx$$

$$= \sin x \Big]_{-\pi/2}^{\pi/2} - \frac{\sin^3 x}{3} \Big]_{-\pi/2}^{\pi/2} = \sin\frac{\pi}{2} - \sin\left(-\frac{\pi}{2}\right) - \left[\frac{1}{3}\left(\sin\frac{\pi}{2}\right)^3 - \frac{1}{3}\left(\sin\frac{-\pi}{2}\right)^3\right]$$

$$= 1-(-1) - \frac{1}{3}\left[1-(-1)^3\right]$$

$$= 1+1-\frac{1}{3}\cdot 2 = 2 - \frac{2}{3} = \frac{4}{3}$$