8.2 Integration by Parts

Study 8.2 p. 531 # 5-27, 33, 35, 49

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8.2 Integration by Parts

$G: \quad y = e^x(x-1) \qquad F: dy/dx$

$$\frac{dy}{dx} = e^x(1) + (x-1)e^x$$

$$= e^x + xe^x - e^x = xe^x$$

$dy/dx = e^x x$

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8.2 Integration by Parts

What about.... $\int e^x x\,dx$ $\quad \int e^u\,du$? *No.*

Used Product Rule to get derivative.
Need a reverse process to integrate.

Product Rule:

If u and v are both differentiable functions of x $dy = \frac{dy}{dx}dx$
u = f(x) and v = g(x), then:

$$\frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

Therefore,

$$\int \frac{d(uv)}{dx}dx = \int u\frac{dv}{dx}dx + \int v\frac{du}{dx}dx$$

$$uv = \int u\,dv + \int v\,du$$

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8.2 Integration by Parts

$$uv = \int u\,dv + \int v\,du$$

$$\int x e^x\,dx$$

Rewrite as an integral:

$$\int u\,dv = uv - \int v\,du$$

Integration by Parts

So, we need to separate the integrand into parts: **u and dv**

Try again: $\int e^x x\,dx$

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8.2 Integration by Parts

$\int u\,dv = uv - \int v\,du$

$\int e^x x\,dx$

u = x dv = $\int e^x dx$

du = dx v = e^x

$= xe^x - \int e^x dx$

$= xe^x - e^x + c$

$= e^x(x-1) + c$

$$\int x e^{x}\,dx$$

$u = e^{x}$ $\quad \int dv = \int x\,dx$

$du = e^{x}\,dx$ $\quad v = \frac{x^2}{2}$

$$\int x e^{x}\,dx = \frac{e^{x}}{2} x^{2} - \frac{1}{2}\int x^{2} e^{x}\,dx$$

8.2 Integration by Parts

$\int u\,dv = uv - \int v\,du$

p. 531 #12.

$$\int \frac{2x}{e^x}\,dx = \int 2x e^{-x}\,dx$$

$$= 2\int x e^{-x}\,dx$$

$u = x \qquad \int dv = \int e^{-x}\,dx$

$du = dx \qquad v = -e^{-x}$

$$= 2\left[-xe^{-x} - \int -e^{-x}\,dx\right]$$

$$= 2\left[-xe^{-x} + \int e^{-x}\,dx\right]$$

$$= -2xe^{-x} - 2e^{-x} + C$$

$$= -2e^{-x}(x+1) + C$$

$(-)\int e^{-x}(-)\,dx$

$$-2e^{-x}(1) + (x+1)(+2e^{-x})$$

$$-2e^{-x} + 2e^{-x}x + 2e^{-x}$$

$$2e^{-x}x = \frac{2x}{e^x}$$

Title: Select u and dv (7 of 14)

8.2 Integration by Parts

$$-\int -\frac{e^{1/t}}{t^2}\,dt \qquad e^u\,du?$$

$$u = \frac{1}{t} = t^{-1}$$

$$du = -t^{-2}\,dt$$

$$= -e^{1/t} + C \qquad = -\frac{1}{t^2}\,dt$$

$$u = e^{1/t} \qquad dv = \int t^{-2}\,dt$$

$$du = e^{1/t} \cdot \frac{-1}{t^2}\,dt \qquad v = t^{-1}$$

$$\int \frac{e^{1/t}}{t^3}\,dt$$

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8.2 Integration by Parts $\int u\,dv = uv - \int v\,du$

$$\int x^4 \ln x \, dx$$

$u = \ln x \qquad \int dv = \int x^4 dx$

$du = \frac{1}{x} dx \qquad v = \frac{x^5}{5}$

$$= \frac{x^5}{5}\ln x - \int \frac{x^5}{5} \cdot \frac{1}{x} dx$$

$$= \frac{1}{5}x^5 \ln x - \frac{1}{5}\int x^4 dx$$

$$= \frac{1}{5}x^5 \ln x - \frac{1}{5}\cdot\frac{x^5}{5} + c = \frac{1}{5}x^5 \ln x - \frac{1}{25}x^5 + c$$

22 $\int \frac{x^3 e^{x^2}}{(x^2+1)^2}\,dx$

$u = x^2 e^{x^2}$

$\int dv = \frac{1}{2}\int \frac{2x}{(x^2+1)^2}\,dx$

$du = [x^2 e^{x^2}(2x) + e^{x^2}(2x)]\,dx$

$= 2x e^{x^2}(x^2+1)\,dx$

$= \frac{1}{2}(x^2+1)^{-1}$

$= \frac{x^2 e^{x^2}}{2(x^2+1)} - \frac{1}{2}\int \frac{2x e^{x^2}(x^2+1)}{(x^2+1)}\,dx$

$= \frac{x^2 e^{x^2}}{2(x^2+1)} - \frac{1}{2}\int 2x e^{x^2}\,dx = \frac{x^2 e^{x^2}}{2(x^2+1)} - \frac{e^{x^2}}{2} + C$

$u = x^2$

$du = 2x\,dx$

8.2 Integration by Parts

$\int u\,dv = uv - \int v\,du$

33. $\int \arctan x\,dx$

$u = \arctan x \qquad \int dv = \int dx$

$du = \frac{1}{1+x^2}dx \qquad v = x$

$= x\arctan x - \frac{1}{2}\int \frac{2x}{1+x^2}dx$

$= x\arctan x - \frac{1}{2}\ln|1+x^2| + C$

21. $\int \frac{xe^{2x}}{(2x+1)^2}\,dx$

$\int u\,dv = uv - \int v\,du$

$u = xe^{2x}$

$\int dv = \frac{1}{2}\int \frac{2}{(2x+1)^2}\,dx$

$du = (2xe^{2x} + e^{2x})\,dx$

$= e^{2x}(2x+1)\,dx$

$v = -\frac{1}{2}(2x+1)^{-1}$

$= -\frac{1}{2}\frac{xe^{2x}}{(2x+1)} + \frac{1}{2}\cdot\frac{1}{2}\int \frac{(2x+1)}{2x+1}e^{2x}(2)\,dx = -\frac{xe^{2x}}{2(2x+1)} + \frac{1}{4}e^{2x} + C$

8.2 Integration by Parts

$$\int u\,dv = uv - \int v\,du$$

35. $\int e^{2x} \sin x\,dx$

$u = \sin x$, $\int dv = \frac{1}{2}\int e^{2x}\, 2dx$

$du = \cos x\,dx$, $v = \frac{1}{2}e^{2x}$

$$= \frac{1}{2}e^{2x}\sin x - \frac{1}{2}\int e^{2x}\cos x\,dx$$

$u = \cos x$, $\int dv = \int e^{2x}\,dx$

$du = -\sin x\,dx$, $v = \frac{1}{2}e^{2x}$

$$= \frac{1}{2}e^{2x}\sin x - \frac{1}{2}\left[\frac{1}{2}e^{2x}\cos x + \frac{1}{2}\int e^{2x}\sin x\,dx\right]$$

$$\int e^{2x}\sin x\,dx = \frac{1}{2}e^{2x}\sin x - \frac{1}{4}e^{2x}\cos x - \frac{1}{4}\int e^{2x}\sin x\,dx$$

$$+\frac{1}{4}\int \qquad = \qquad +\frac{1}{4}\int$$

$$\frac{5}{4}\int e^{2x}\sin x\,dx = \frac{1}{2}e^{2x}\sin x - \frac{1}{4}e^{2x}\cos x$$

$$\int e^{2x}\sin x\,dx = \frac{4}{5}\left[\frac{1}{2}e^{2x}\sin x - \frac{1}{4}e^{2x}\cos x\right]$$

$$\int_0^1 \ln(1+x^2)\,dx$$