## 7.4 Arc Length and Surface of Revolution

Study 7.4 p. 483 # 1-5, 6, 15-19, 39-45

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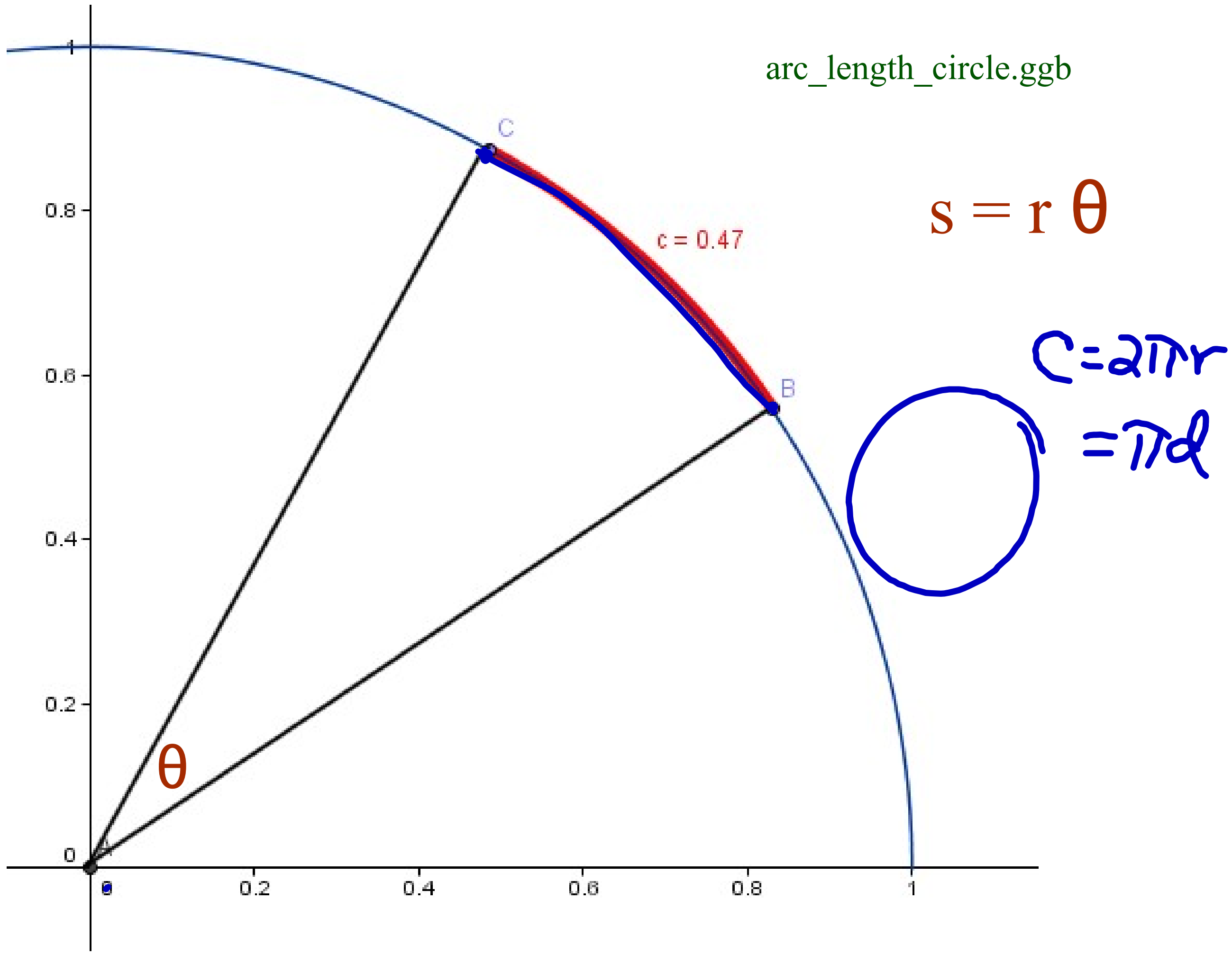
# 7.4 Arc Length and Surface of Revolution

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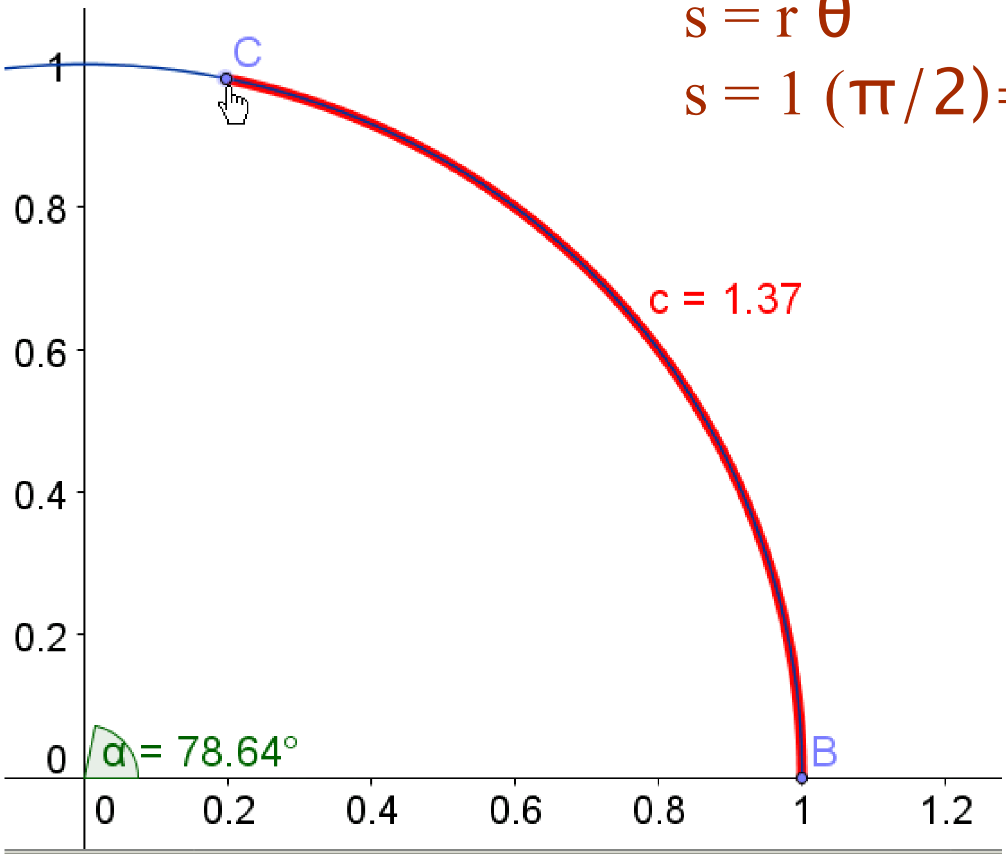
## 7.4 Arc Length and Surface of Revolution




$s = r\,\theta$

$s = 1\,(\pi/2) = 1.57$

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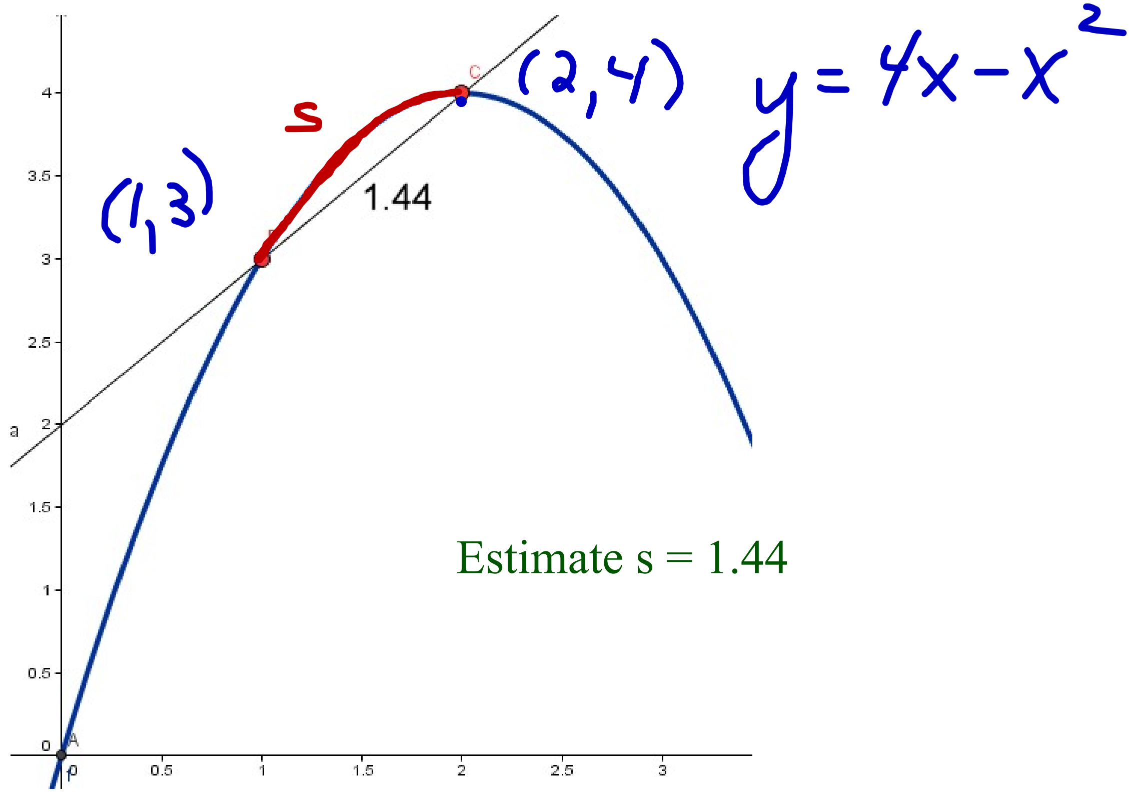
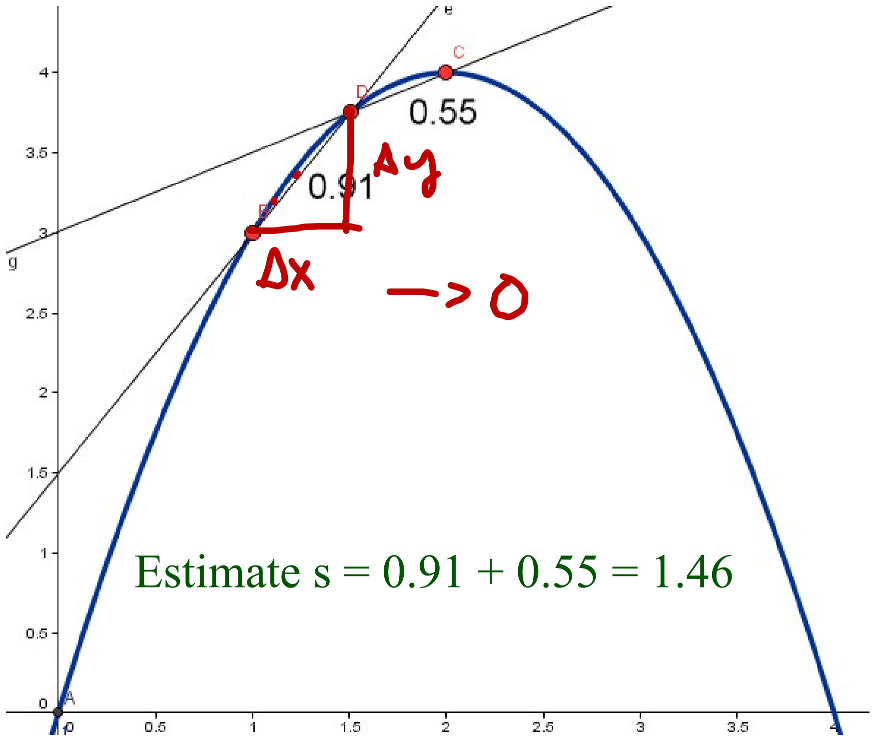



Estimate s = 1.44

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$d = \sqrt{(2-1)^2 + (4-3)^2} = \sqrt{2} = 1.41$$




Sum of two line segments gets a better estimate.
Can increase the number of line segments and let delta x approach 0.

# 7.4 Arc Length and Surface of Revolution

$$d = \sqrt{(x_i - x_{i-1})^2 + (y_i - y_{i-1})^2}$$

$$d = \sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{(\Delta x)^2 + (\Delta y)^2 \frac{(\Delta x)^2}{(\Delta x)^2}}$$

$$= \sqrt{(\Delta x)^2 + \frac{(\Delta y)^2}{(\Delta x)^2}(\Delta x)^2} = \sqrt{(\Delta x)^2 + \left[\frac{\Delta y}{\Delta x}\right]^2 (\Delta x)^2}$$

$$\frac{\Delta y}{\Delta x}$$

$$d = \sqrt{\left[1 + \left(\frac{\Delta y}{\Delta x}\right)^2\right](\Delta x)^2} = \sqrt{\left[1 + \left(\frac{\Delta y}{\Delta x}\right)^2\right]}(\Delta x)$$

$$S = \lim_{\|\Delta\| \to 0} \sum_{i=1}^{n} \sqrt{1 + \left[\frac{\Delta y_i}{\Delta x_i}\right]^2}\ \Delta x_i$$

$$s = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

## 7.4 Arc Length and Surface of Revolution

Let y = f(x) be a smooth curve on [a,b].
Then the arc length of f between a and b is

$$s = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

Let x = g(y) be a smooth curve on [c,d].
Then the arc length of g between c and d is

$$s = \int_c^d \sqrt{1 + (g'(y))^2}\, dy$$

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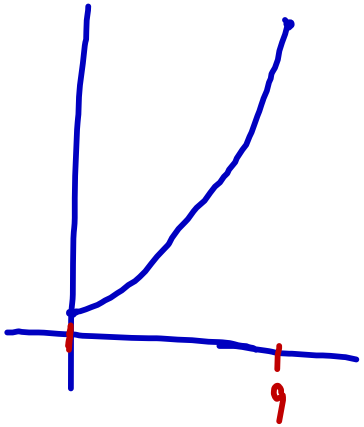
G: $y = 2x^{3/2} + 3$  F: s $[0, 9]$




$$s = \int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

$$\frac{dy}{dx} = 2\cdot\frac{3}{2}x^{1/2} = 3x^{1/2}$$

$$= \int_0^9 \sqrt{1+(3x^{1/2})^2}\,dx$$

$$= \frac{1}{9}\int_0^9 \sqrt{1+9x}\,(9)\,dx$$

$u = 1+9x$

$du = 9\,dx$

54.93

$x=9$, $x=0$

$$= \frac{1}{9}\int_1^{82} u^{1/2}\,du = \frac{1}{9}\,\frac{u^{3/2}}{3/2}\Big]_1^{82} = \frac{2}{27}u^{3/2}\Big]_1^{82}$$

$$= \frac{2}{27}\left(82^{3/2}-1\right)$$

$$= \frac{1}{9}\cdot\frac{2}{3}(1+9x)^{3/2}\Big]_0^9$$

## 7.4 Arc Length and Surface of Revolution

#8 $y = \ln(\cos x)$ F: S $[0, \pi/3]$

$$S = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$\frac{dy}{dx} = \frac{1}{\cos x} \cdot (-\sin x)$$

$$\frac{dy}{dx} = -\frac{\sin x}{\cos x} = -\tan x$$

$$= \int_0^{\pi/3} \sqrt{1 + (-\tan x)^2}\, dx$$

$$= \int_0^{\pi/3} \sqrt{1 + \tan^2 x}\, dx$$

$$\frac{\sin^2}{\cos^2} + \frac{\cos^2}{\cos^2} = \frac{1}{\cos^2}$$

$$\tan^2 + 1 = \sec^2$$

$$= \int_0^{\pi/3} \sqrt{\sec^2 x}\, dx$$

$$= \int_0^{\pi/3} \sec x\, dx = \ln|\sec x + \tan x| \Big]_0^{\pi/3}$$

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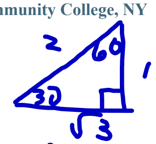
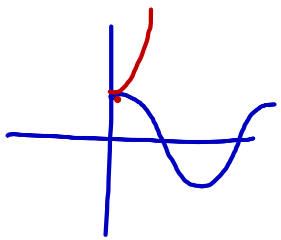
$$= \ln\left|\tfrac{1}{2} + \sqrt{3}\right| - \ln|1 + 0|$$

$$= \ln\left|\tfrac{1}{2} + \sqrt{3}\right|$$




$$\sec x \cdot \frac{\sec x + \tan x}{\sec x + \tan x} = \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x}$$

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$



Title: example (9 of 14)

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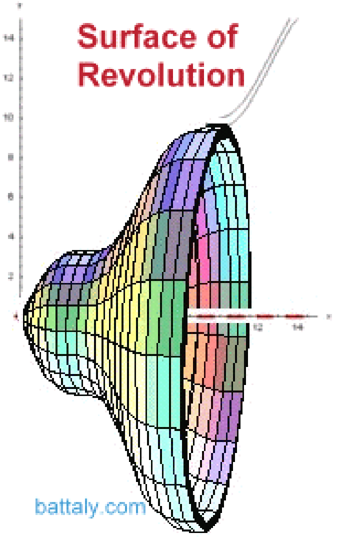
# 7.4 Arc Length and Surface of Revolution




surface_revolution.swf

If we start with an arc length, and rotate it around an axis of revolution, we have a surface of revolution.

Surface measurement:
Surface Area

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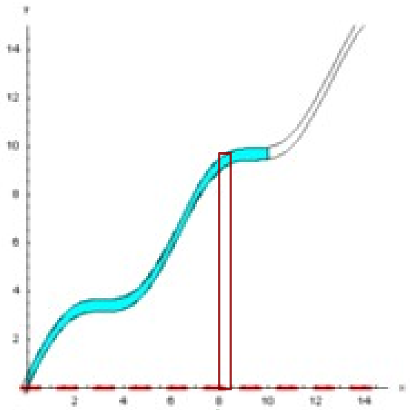



Let $\Delta x$ be small enough so that the surface being rotated is like the surface of a cylinder.

Then the surface area is

$$S = 2\pi \, r \, L$$

where L is the arc length.

$$S = 2\pi\int_a^b r(x)\sqrt{1+[f\,'(x)]^2}\,dx$$

## 7.4 Arc Length and Surface of Revolution

Let y = *f(x)* has continuous derivative on [a,b]. The Area S of the surface of revolution formed by revolving the graph of *f* about a horizontal axis is:

$$S = 2\pi \int_a^b r(x)\sqrt{1+[f\,'(x)]^2}\,dx$$

where r(x) = distance between *f* and the axis of revolution.

If *x=g(y)* on [c,d]. The Area S of the surface of revolution formed by revolving the graph of *g* about a verticle axis is:

$$S = 2\pi \int_c^d r(y)\sqrt{1+[g\,'(y)]^2}\,dy$$

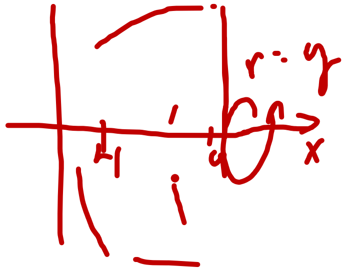
where r(y) = distance between g and the axis of revolution.

## 8.4 Arc Length and Surface of Revolution

40. G: $y = 2\sqrt{x}$  F: SA ⟲ x-axis $[4, 9]$  $\frac{dy}{dx} = x^{-1/2}$

$$SA = 2\pi \int_a^b r(x) \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$= 2\pi \int_4^9 y \sqrt{1 + \left(\frac{1}{x^{1/2}}\right)^2}\, dx$$




$$= 2\pi \int_4^9 2x^{1/2} \sqrt{1 + \frac{1}{x}}\, dx$$

$$= 2\pi \int_4^9 2x^{1/2} \sqrt{\frac{x+1}{x}}\, dx = 2\pi \int_4^9 2x^{1/2} \frac{\sqrt{x+1}}{x^{1/2}}\, dx$$

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$$= 4\pi \int_4^9 (x+1)^{1/2}\, dx$$

$u = x + 1$
$du = dx$

$$= 4\pi \cdot \frac{2}{3} (x+1)^{3/2} \Big]_4^9 = \frac{8}{3}\pi \left[10^{3/2} - 5^{3/2}\right]$$