7.3 Volumes of Revolution: the Shell Method

Study 7.3 p. 472 # 1-9, 13, 15, 21, 27
7.3 Volumes of Revolution: the Shell Method

Consider: \( y = x^3 - 3x + 3, \ x = 0, \ y = 0, \ x = 2 \)

Easy to revolve about x-axis: Use disk method

BUT, what about revolving about y axis?
7.3 Volumes of Revolution: the Shell Method

Consider:  \( y = x^3 - 3x + 3, \ x = 0, \ y = 0, \ x = 2 \)

What about revolving about y axis?

Reference rectangle for disk method is not consistent and does not have an easy algebraic representation.
7.3 Volumes of Revolution: the Shell Method

Consider: \( y = x^3 - 3x + 3, x = 0, y = 0, x = 2 \)

Could divide into 3 regions.  
Then add the volumes.
7.3 Volumes of Revolution: the Shell Method

Consider: \( y = x^3 - 3x + 3, \ x = 0, \ y = 0, \ x = 2 \)

For Region 1:

\[
V = 2\pi \int_0^1 R^2 \, dy
\]

\[ R = x \]

We have \( x \), but we need an integrand to match the \( dy \).

Could divide into 3 regions.
7.3 Volumes of Revolution: the Shell Method

Consider: \( y = x^3 - 3x + 3, \ x = 0, \ y = 0, \ x = 2 \)

For \( dy \), we need to solve the cubic function for \( x \) in terms of \( y \) to express the integrand algebraically.
7.3 Volumes of Revolution: the Shell Method

Use alternate method: the Shell Method.

Start with reference rectangle, but this time the Reference Rectangle is parallel to the axis of revolution.
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Start with reference rectangle, but this time the Reference Rectangle is parallel to the axis of revolution.
7.3 Volumes of Revolution: the Shell Method

\[ V = 2\pi \int_{a}^{b} p(x)h(x)\,dx \]

or

\[ V = 2\pi \int_{c}^{d} p(y)h(y)\,dy \]

\[ V = 2\pi \int_{a}^{b} p(x)h(x)\,dy \]

\( p = \) average radius of shell
\( h = \) height
\( dx \) or \( dy = \) thickness
7.3 Volumes of Revolution: the Shell Method

\[ V = 2\pi \int_a^b p(x)h(x)\,dx \quad \text{or} \quad V = 2\pi \int_c^d p(y)h(y)\,dy \]

Volume of the shell = volume of the outer cylinder - volume of the inner cylinder.

\[ V = \pi \left( p + \frac{w}{2} \right)^2 h - \pi (p - \frac{w}{2})^2 h \]

\[ = \pi h \left[ \left( p + \frac{w}{2} \right)^2 - (p - \frac{w}{2})^2 \right] \]

\[ = \pi h \left[ (p^2 + wp + \frac{w^2}{4}) - (p^2 - wp + \frac{w^2}{4}) \right] \]

\[ = \pi h (2wp) = 2\pi p h w \]

\[ V = 2\pi \int_a^b p(x)h(x)\,dx \]

\( w \) (delta x) is the width of the reference shell. Add the volumes of adjacent shells, and let delta x \( \rightarrow 0 \). Results in representation of the thickness of the shell as dx or dy.
7.3 When to Use the Shell Method

Volumes of Revolution - Which Method?

1. Sketch the curves and identify the region, using the points of intersection.

2. Locate the axis of revolution on the sketch.

3. Decide whether to use a horizontal or vertical rectangle. Select the orientation that requires the least number of separate sections.

4. Decide whether to use the Disc Method or the Shell Method:
   a) If the rectangle is perpendicular to the axis of revolution, use the Disc Method.
   b) If the rectangle is parallel to the axis of revolution, use the Shell Method.
Volumes of Revolution - Shell Method

1. Complete Steps 1 to 4 in Volumes of Revolution, which Method? noted above.

2. Be sure that your rectangle is parallel to the axis of revolution.

3. Determine the variable of integration:
   a) If the rectangle is horizontal, then integrate with respect to y (use \(dy\)). The integrand must be in terms of y.
   b) If the rectangle is vertical, then integrate with respect to x (use \(dx\)). The integrand must be in terms of x.

4. Determine the integrand: \(p(x)h(x)\) or \(p(y)h(y)\)?
   a) If the rectangle is horizontal, identify \(p(y)\), the distance of the rectangle from the axis of revolution, and \(h(y)\), the length of the rectangle. Use: \(V = 2\pi \int p(y)h(y)dy\)
   b) If the rectangle is vertical, identify \(p(x)\), the distance of the rectangle from the axis of revolution, and \(h(x)\), the length of the rectangle. Use: \(V = 2\pi \int p(x)h(x)dx\)
7.3 **Volumes of Revolution: the Shell Method**

Consider: \( y = x^3 - 3x + 3, \ x = 0, \ y = 0, \ x = 2 \)

Which formula?

\[
V = 2\pi \int_a^b p(x)h(x)\,dx
\]

\[
V = \int_a^b p(y)h(y)\,dy
\]

* Use the formula with \( dx \)!

Which method?

Shell Method.
Can't find \( x = f(y) \).
7.3 Volumes of Revolution: the Shell Method

\[ V = 2\pi \int_a^b \rho(x) h(x) \, dx \]

\[ V = 2\pi \int_0^2 x (x^3 - 3x + 3) \, dx \]

\[ = 2\pi \int_0^2 (x^4 - 3x^2 + 3x) \, dx \]

\[ = 2\pi \left[ \frac{x^5}{5} - \frac{3x^3}{3} + \frac{3x^2}{2} \right]_0^2 \]

\[ = 2\pi \left[ \frac{32}{5} - \frac{8}{1} + \frac{6}{2} \right] \]

\[ = 2\pi \left[ \frac{32}{5} - 4 + 3 \right] = 2\pi \left[ \frac{23}{5} \right] = \frac{46\pi}{5} \]

\[ h = \text{height of rev.} \]

\[ p = \text{distance fr. rev.} \]

\[ \text{axis of revolution.} \]

Title: example (14 of 16)
#20. \( G: y = x^2, y = 4x - x^2 \) \( F: \text{Vol. } \forall x = 2 \)

\[
V = 2\pi \int_a^b \rho(x) h(x) \, dx
\]

\[
= 2\pi \int_0^2 (2-x)(4x-2x^2) \, dx
\]

\[
\rho = 2 - x
\]

\[
h = y_u - y_l = 4x - x^2 - x^2 = 4x - 2x^2
\]
7.3 Volumes of Revolution: the Shell Method