7.2 Volumes of Revolution: the Disk Method

HW: 7.2 # 1-15, 19-27, 35, 63 (calc)

Step-by-Step Procedure

Practice in Problem Setup

Class Notes: Prof. G. Battaly, Westchester Community College, NY

Calculus Home Page

Homework Part 1 Homework Part 2

Homework Part 3

7.2 Volumes of Revolution: the Disk Method

From geometry, we find volumes of readily defined geometric figures. For example:

Geometric Figure	Volume
Sphere	$V = \frac{4}{3} \pi r^3$
Right Circular Cone	$V = \frac{1}{3} \pi r^2 h$
Right Circular Cylinder	$V = \pi r^2 h$

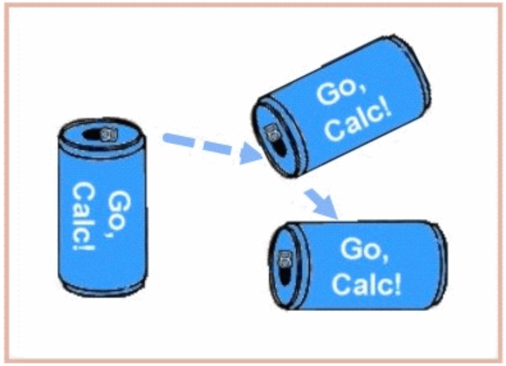
7.2 Volumes of Revolution: the Disk Method

To begin:

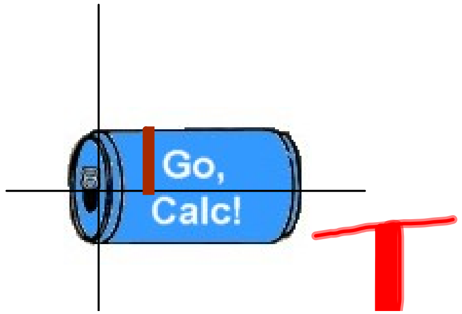
Focus on: Volume

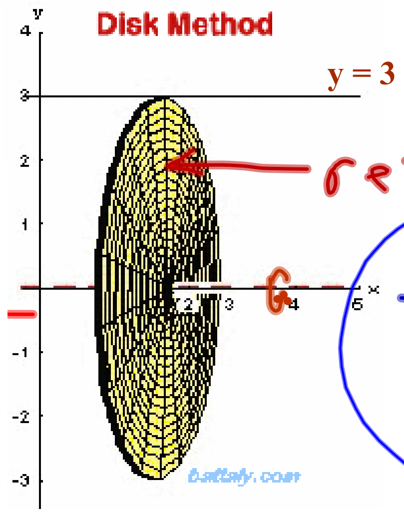
Right Circular Cylinder $V = \pi r^2 h$

Consider a soda can, and flip it onto it's side.



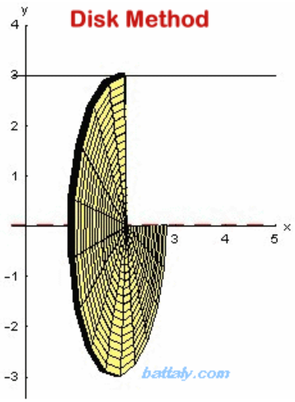
7.2 Volumes of Revolution: the Disk Method





ref. rect

⊥ axis of revolution.



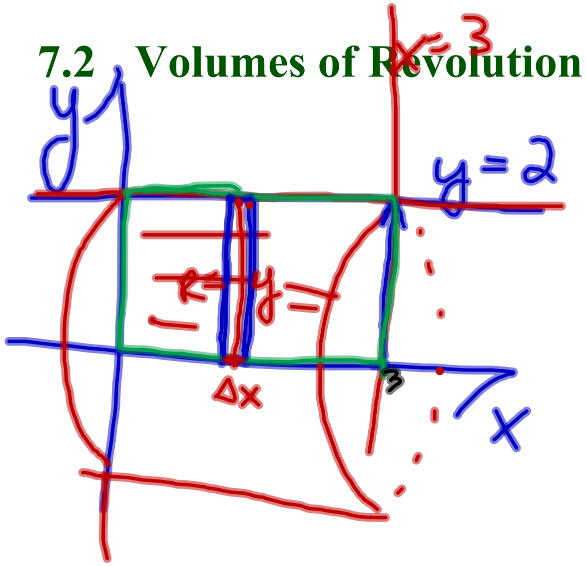
can be different curves

Class Notes: Prof. G. Battaly, Westchester Community College, NY

Calculus Home Page

Homework Part 1 Homework Part 2
Homework Part 3

7.2 Volumes of Revolution: the Disk Method



$V = \pi r^2 h$

$\pi(2)^2 3$

$= 12\pi$

$V = \pi r^2 h$

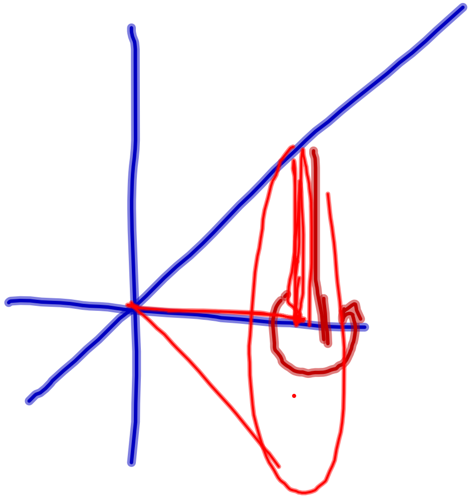
$= \pi y^2 \Delta x$

1. Volume of disk
2. Add across region
3. # disk $\to \infty$, $\Delta x \to 0$

$$V = \pi \int_a^b y^2 \, dx$$

$$= \pi \int_0^3 2^2 \, dx = 4\pi x \Big]_0^3 = 4\pi[3 - 0] = 12\pi$$

7.2 Volumes of Revolution: the Disk Method



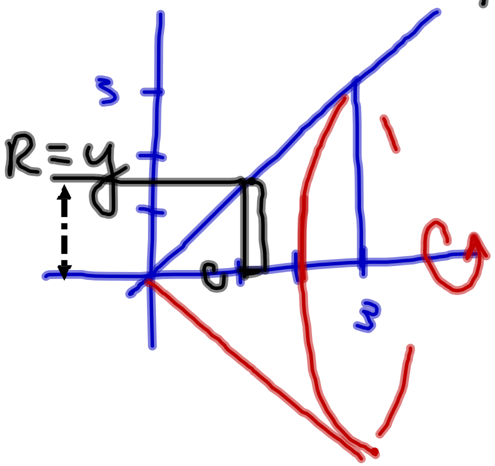
Known geometrical figures
can use a formula, or calculus.

For nonstandard geometrical
figures, there is no formula.
Instead, use calculus.

$y = x$

Vol. under curve fr. $x = 0$ to $x = 3$ ↻ x-axis.

ref. rect. ⊥ x-axis.



$$V = \pi \int_a^b R^2\,dx$$

$$= \pi \int_0^3 y^2\,dx$$

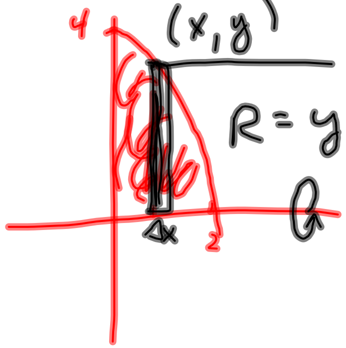
$$= \pi \int_0^3 (x)^2\,dx = \pi \frac{x^3}{3}\Bigg]_0^3$$

$$= \pi[9 - 0] = 9\pi$$

$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\, 3^2(3) = 9\pi$

compare to formula for cone

F. $\circlearrowleft$ x-axis $V = \pi \int_a^b R^2 \, \underline{dx}$



2. $G: y = 4 - x^2$

$$V = \pi \int_0^2 y^2 \, dx = \pi \int_0^2 (4-x^2)^2 \, dx$$

$$= \pi \int_0^2 (16 - 8x^2 + x^4) \, dx$$

$$= \pi \left[16x - \frac{8x^3}{3} + \frac{x^5}{5} \right]_0^2 = \pi \left[32 - \frac{64}{3} + \frac{32}{5} - (0) \right]$$

$$= \pi \left[32\left(1 + \frac{1}{5}\right) - \frac{64}{3} \right] = \pi \left[\frac{192}{5} - \frac{64}{3} \right]$$

$$\pi \left[\frac{576 - 320}{15} \right] = \pi \, \frac{256}{15}$$

7.2 Volumes of Revolution: the Disk Method

$y = \sqrt{9 - x^2}$, $y = 0$ F: volume

↻ x-axis.

7.2 Volumes of Revolution: the Disk Method

start

$y=\sqrt{x}$, $y=x^2$, $y=0$ F: volume

⟳ x-axis.

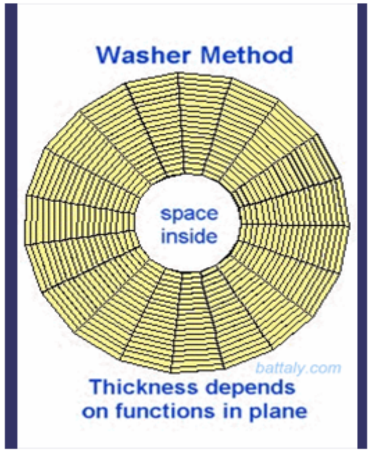
Class Notes: Prof. G. Battaly, Westchester Community College, NY

Calculus Home Page

Homework Part 1 Homework Part 2

Homework Part 3

7.2 Volumes of Revolution: the Disk/Washer Method



$$Volume =$$

$$\pi \int_a^b (R^2 - r^2)\,dx$$

$$\pi R^2 \Delta x$$

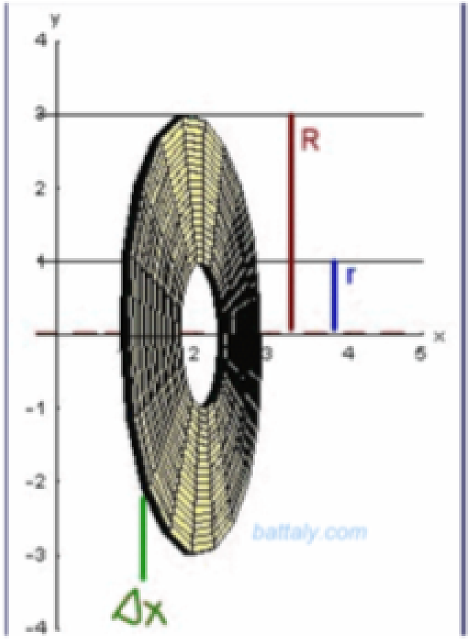
$$- \pi r^2 \Delta x$$

$$\pi (R^2 - r^2) \Delta x$$

Class Notes: Prof. G. Battaly, Westchester Community College, NY

Calculus Home Page

Homework Part 1 Homework Part 2
Homework Part 3



$$Volume =$$

$$\pi \int_a^b (R^2 - r^2)\,dx$$

7.2 Volumes of Revolution: the Disk Method

finish

$y=\sqrt{x}$, $y=x^2$, $y=0$ F: volume

⟳ x-axis.

Class Notes: Prof. G. Battaly, Westchester Community College, NY

Calculus Home Page

Homework Part 1 Homework Part 2

Homework Part 3

7.2 Volumes of Revolution: the Disk Method

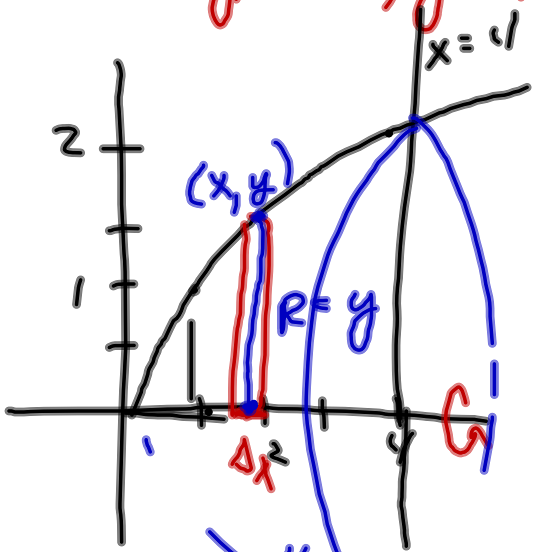
#11 G: $y = \sqrt{x}$, $y = 0$, $x = 4$ F: Vol of

a) x-axis

b) y-axis

c) $x = 4$

d) $x = 6$



$$V = \pi \int_a^b R^2 \, dx$$

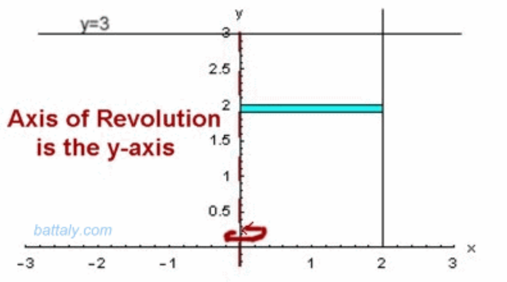
$$V = \pi \int_0^4 y^2 \, dx = \pi \int_0^4 (\sqrt{x})^2 \, dx$$

$$V = \pi \int_0^4 x \, dx = \pi \frac{x^2}{2}\Big]_0^4 = 8\pi - 0 = 8\pi$$

7.2 Volumes of Revolution: the Disk Method

Rotate around the y-axis
(or a vertical line)

$$V = \pi \int_c^d R^2 \, dy$$



Class Notes: Prof. G. Battaly, Westchester Community College, NY

Calculus Home Page

Homework Part 1 Homework Part 2
Homework Part 3

7.2 Volumes of Revolution: the Disk Method

#11 G: $y = \sqrt{x}$, $y = 0$, $x = 4$ F: Vol of

a) x-axis

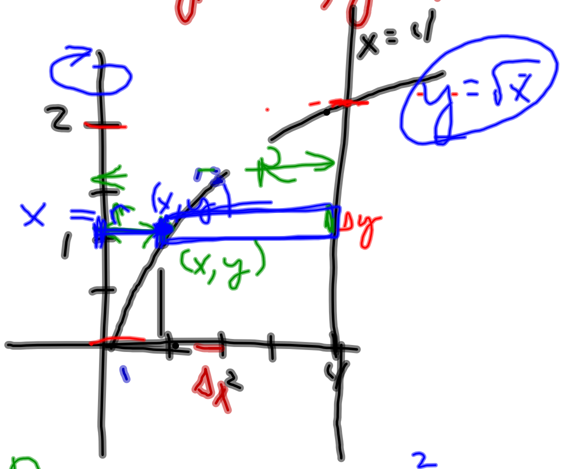
b) y-axis

c) $x = 4$

d) $x = 6$

$y = \sqrt{x}$

$x = y^2$



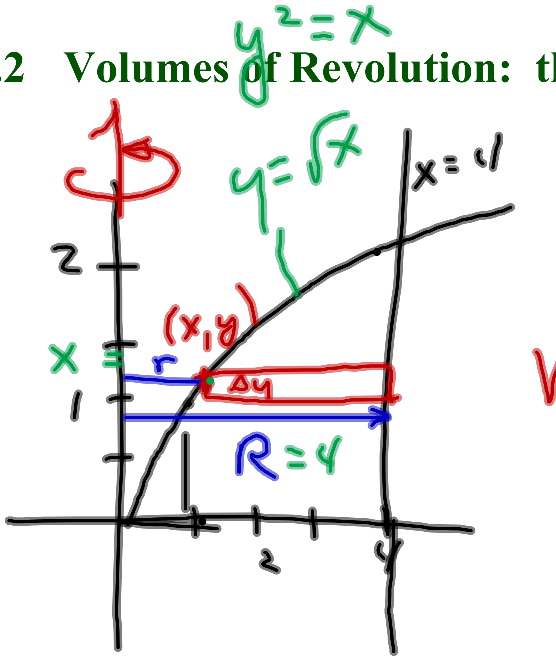
$$V = \pi \int_c^d \left(R^2 - r^2\right) dy$$

$R = 4$ $r = x = y^2$

$$= \pi \int_0^2 \left[16 - (y^2)^2\right] dy$$

$$= \pi \int_0^2 \left(16 - y^4\right) dy = \pi \left[16y - \frac{y^5}{5}\right]_0^2 =$$

$$\pi\left(32 - \frac{32}{5} - 0\right) = 32\pi\left(1 - \frac{1}{5}\right) = \frac{128\pi}{5}$$



b) ↻ y-axis

$$V = \pi \int_c^d (R^2 - r^2)\,dy$$

$$= \pi \int_0^2 (4^2 - x^2)\,dy$$

$$= \pi \int_0^2 [16 - (y^2)^2]\,dy$$

$$\pi R^2 dy - \pi r^2 dy$$

$$V = \pi \int_0^2 (16 - y^4)\,dy = \pi \left[16y - \frac{y^5}{5}\right]_0^2 =$$

$$= \pi \left[32 - \frac{32}{5} - 0\right] = 32\pi \left\{1 - \frac{1}{5}\right\} = \frac{4}{5} \cdot 32\pi =$$

$$\frac{128}{5}\pi$$

7.2 Volumes of Revolution: the Disk Method

Volumes of Revolution - Disk Method

1. Sketch the curves and identify the region, using the points of intersection.

2. Locate the axis of revolution on the sketch.

3. Decide whether to use a horizontal or vertical rectangle. The rectangle should be perpendicular to the axis of revolution.

4. Sketch the rectangle and determine the variable of integration.

• If the rectangle is horizontal, then integrate with respect to y (use *dy*). The integrand must be in terms of y.

• If the rectangle is vertical, then integrate with respect to x (use *dx*). The integrand must be in terms of x.

5. **Determine the integrand: R^2, or $R^2 - r^2$?**

a) **If the rectangle touches the axis of revolution**, identify R as the length of the rectangle. Find **R** in terms of the appropriate variable (see above), and **use R^2** as the integrand.

b) **If the rectangle does not touch the axis of revolution**, identify **R** as the distance of the furthest end of the rectangle from the axis of revolution and **r** as the distance of the closest end of the rectangle from the axis of revolution. **Use $R^2 - r^2$** as the integrand.

7.2 Volumes of Revolution: the Disk Method

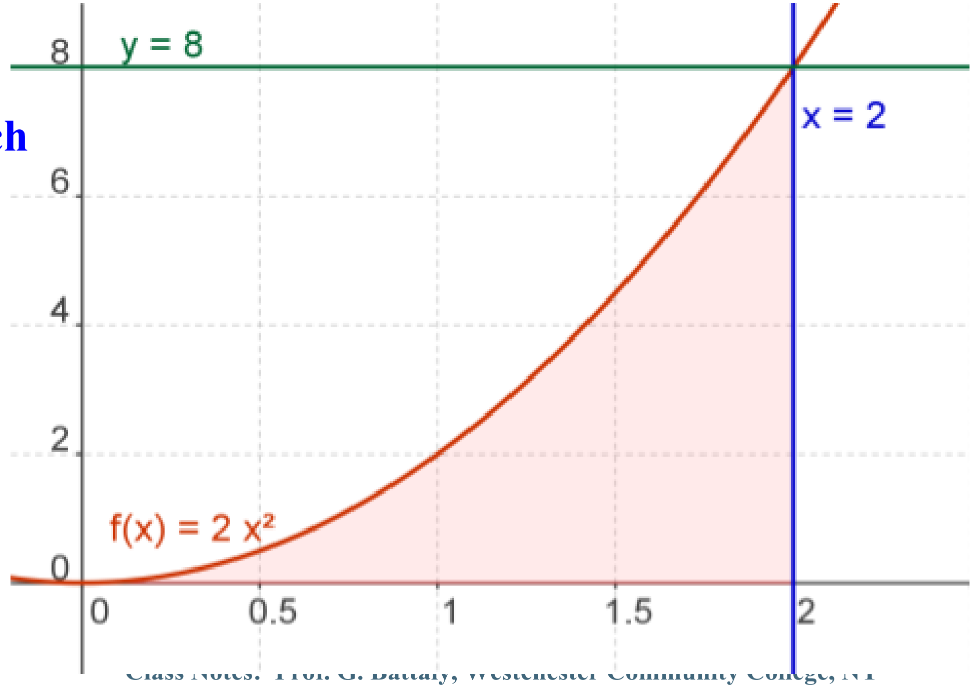
Find the volume of the solids generated by revolving the region bounded by:

$y = 2x^2 \qquad y = 0 \qquad x = 2$

about the given axes.

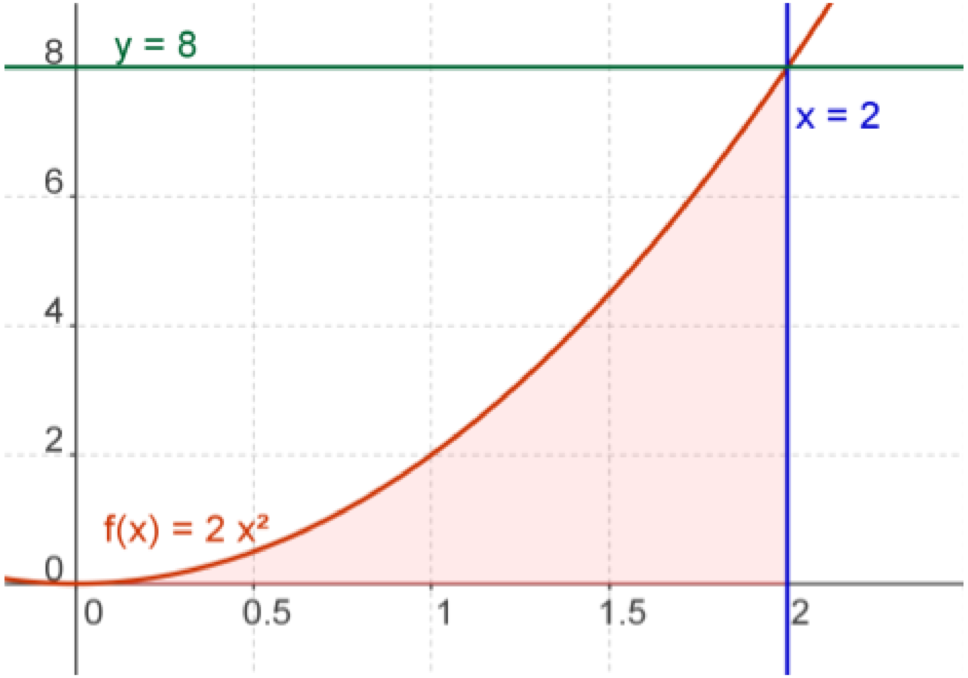
a) y-axis b) x-axis c) $y = 8$ d) $x = 2$

1st) sketch



7.2 Volumes of Revolution: the Disk Method

2nd) axis of rev	link a) y-axis vertical	link b) x-axis horizontal	link c) y = 8 horizontal	link d) x = 2 vertical
3rd) ref. rectangle: _|_ to axis rev				



Class Notes: Prof. G. Battaly, Westchester Community College, NY

Calculus Home Page

Homework Part 1 Homework Part 2
Homework Part 3

7.2 Volumes of Revolution: the Disk Method

2nd)
axis of rev

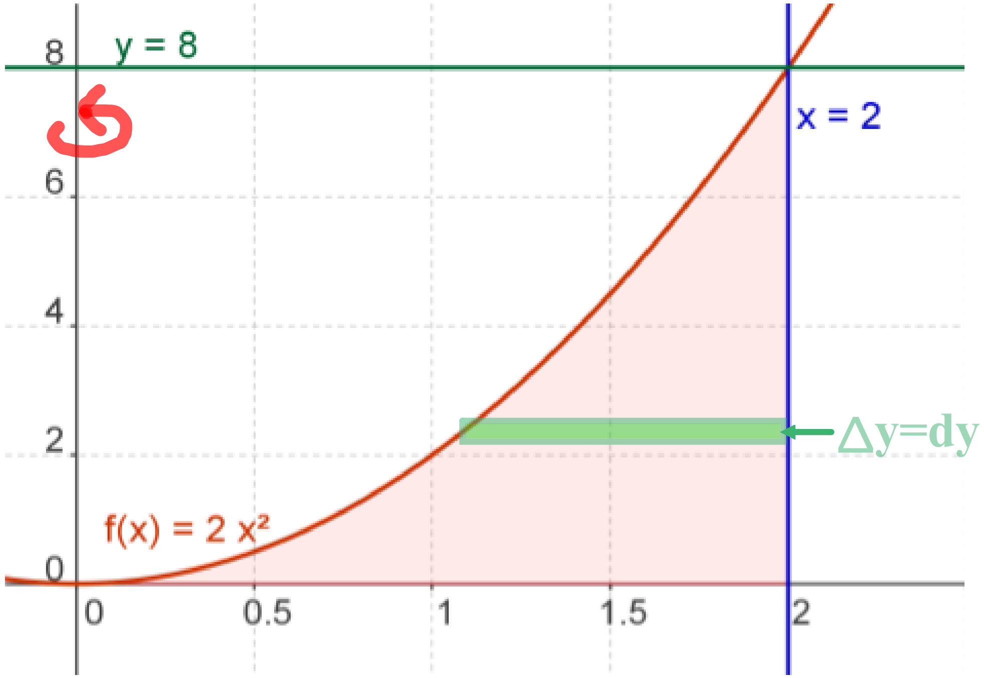
a) y-axis
vertical

3rd)
ref. rectangle: _|_ to axis rev

4th)
decide: dy or dx?

$\Delta y = dy$

$$\int_c^d (\qquad)dy$$



5th)
decide: R^2 or $R^2 - r^2$?

ref. rectangle does NOT touch the axis of revol. Use $\mathbf{R^2 - r^2}$

return to problem

Class Notes: Prof. G. Battaly, Westchester Community College, NY

Calculus Home Page

Homework Part 1 Homework Part 2
Homework Part 3

7.2 Volumes of Revolution: the Disk Method

2nd) axis of rev

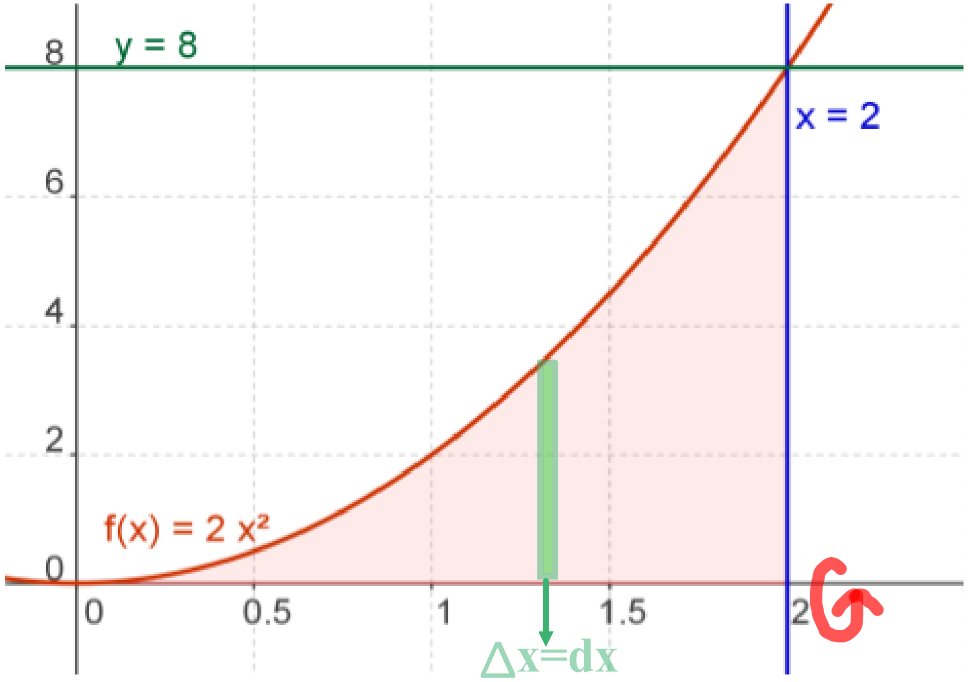
b) x-axis
horizontal

3rd) ref. rectangle: _|_ to axis rev

4th) decide: dy or dx?

$\Delta x = dx$

$$\int_a^b (\qquad) dx$$



5th) decide: R^2 or $R^2 - r^2$?

ref. rectangle touches the axis of revol. Use R^2

return to problem

7.2 Volumes of Revolution: the Disk Method

2nd) **axis of rev**

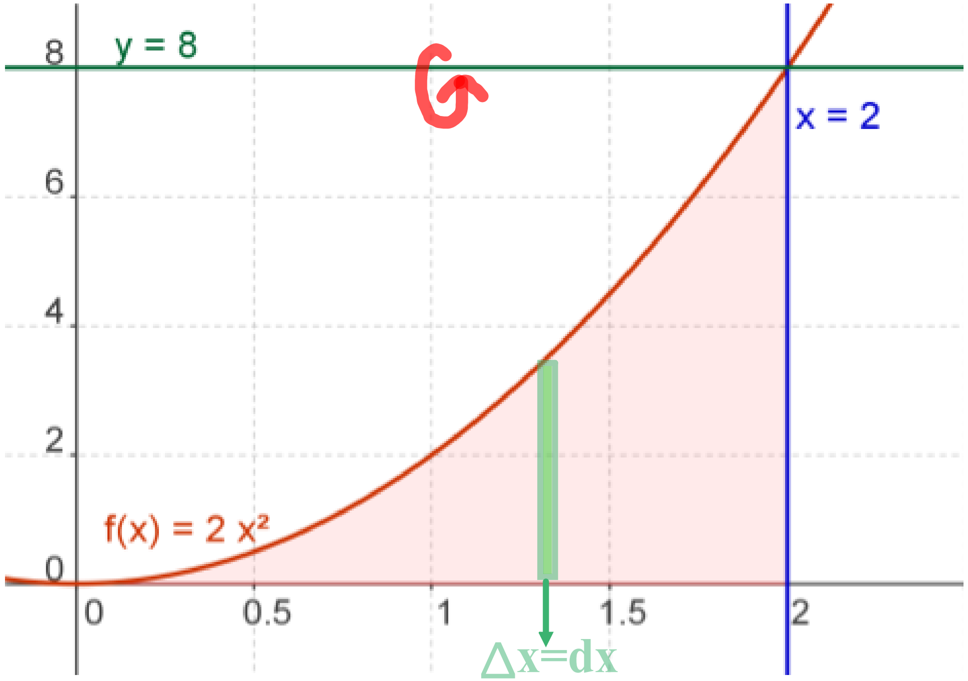
c) $y = 8$
horizontal

3rd) **ref. rectangle: _|_ to axis rev**

4th) **decide: dy or dx?**

$\Delta x = dx$

$$\int_a^b (\qquad) dx$$



5th) **decide: R^2 or $R^2 - r^2$?**

ref. rectangle does NOT touch the axis of revol.
Use $R^2 - r^2$

return to problem

Class Notes: Prof. G. Battaly, Westchester Community College, NY

Calculus Home Page

Homework Part 1 Homework Part 2
Homework Part 3

7.2 Volumes of Revolution: the Disk Method

2nd)
axis of rev

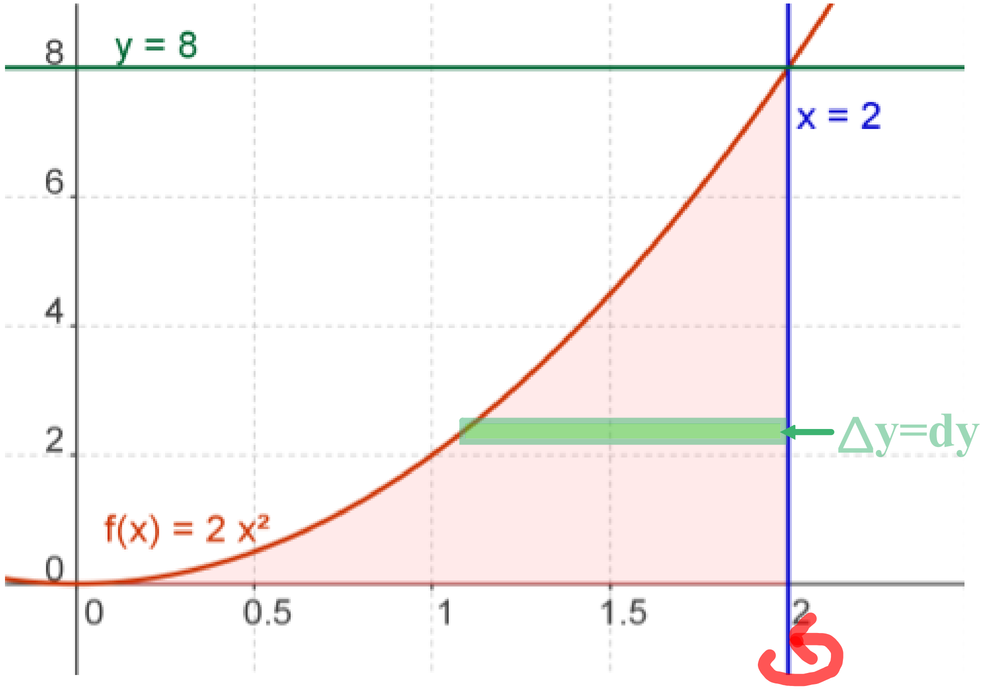
d) $x = 2$
vertical

3rd)
ref. rectangle:
_|_ to axis rev

4th)
decide:
dy or dx?

$\Delta y = dy$

$$\int_c^d (\qquad)dy$$

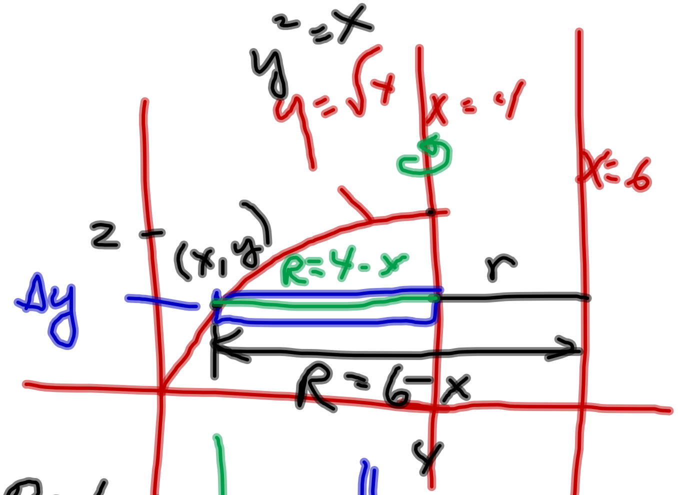


5th)
decide:
R^2 or $R^2 - r^2$?

ref. rectangle does
touches the axis of revol.
Use $\mathbf{R^2}$

return to problem

7.2 Volumes of Revolution: the Disk Method



$$V = \pi \int_c^d (R^2 - r^2)\,dy$$

$$= \pi \int_0^2 \left[(6-y^2)^2 - 2^2 \right] dy$$

$R = 6 - x$
$= 6 - y^2$

$r = 2$

$(6-y^2)^2 - 2^2$

$36 - 12y^2 + y^4 - 4$

$$= \pi \int_0^2 (32 - 12y^2 + y^4)\,dy$$

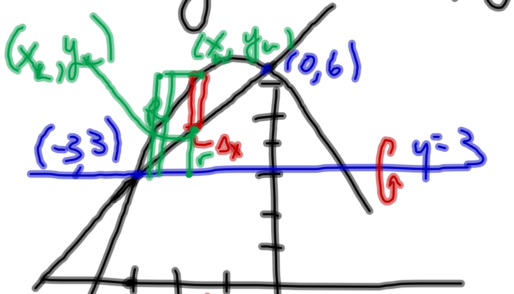
$$= \pi \left[32y - \frac{\overset{4}{\cancel{12}}\, y^3}{\cancel{3}} + \frac{y^5}{5} \right]_0^2$$

$$= \pi \left[64 - 4(8) + \frac{32}{5} - 0 \right]$$

$$= \pi \left(32 + \frac{32}{5} \right) = 32\pi \left(1 + \frac{1}{5} \right) = \frac{192\pi}{5}$$

7.2 Volumes of Revolution: the Disk Method

Ex. G: $y=6-2x-x^2$, $y=x+6$ Find vol. @ $y=3$



$$6-2x-x^2=x+6$$
$$-2x-x^2=x$$
$$+3x+x^2=0$$
$$x(3+x)=0$$
$$\boxed{x=0,\ x=-3}$$ calc

$$V=\pi\int_a^b (R^2-r^2)\,dx$$

$$=\pi\int_{-3}^{0}\left[(3-2x-x^2)^2-(x+3)^2\right]dx$$

$R:\ y_u=6-2x-x^2$

$r:\ y_L=x+6$

$R=y_u-3;\ r=y_L-3$

Class Notes: Prof. G. Battaly, Westchester Community College, NY

Calculus Home Page

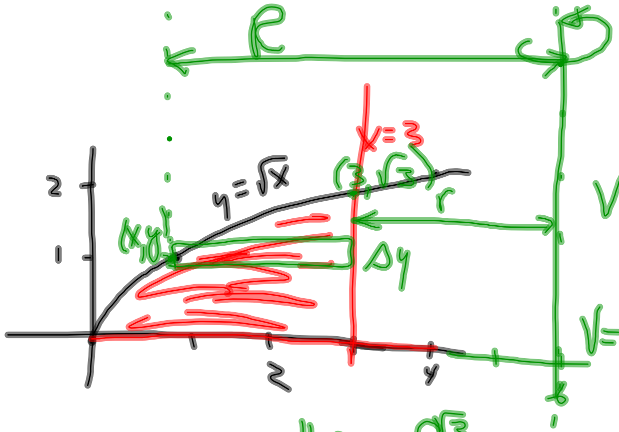
Homework Part 1 Homework Part 2

Homework Part 3

$$R=6-2x-x^2-3=3-2x-x^2$$

$$r=x+6-3=x+3$$

$$\begin{array}{r} 3-2x-x^2 \\ 3-2x-x^2 \\ \hline 9-6x-3x^2 \\ -6x+4x^2+2x^3 \end{array}$$



$$V=\pi\int_c^d (R^2-r^2)\,dy$$

$$V=\pi\int_0^{\sqrt{3}}\left[(6-y^2)^2-3^2\right]dy$$

$R=6-x=6-y^2$

$r=6-3=3$

$$V=\pi\int_0^{\sqrt{3}}\left[36-12y^2+y^4-9\right]dy$$

$$=\pi\int_0^{\sqrt{3}}\left[27-12y^2+y^4\right]dy$$

$$=\pi\left[27y-\frac{\overset{4}{\cancel{12}}y^3}{\cancel{3}}+\frac{y^5}{5}\right]_0^{\sqrt{3}}$$

$$=\pi\left[27\sqrt{3}-4\cdot 3\sqrt{3}+\frac{9\sqrt{3}}{5}-0\right]$$

$$=3\sqrt{3}\,\pi\left[\overset{5}{\cancel{9-4}}+\frac{3}{5}\right]$$

$$=\frac{3(28)\sqrt{3}\,\pi}{5}=\frac{84\sqrt{3}\,\pi}{5}$$

..... E-15

4 3 $\quad 10^{-15}$

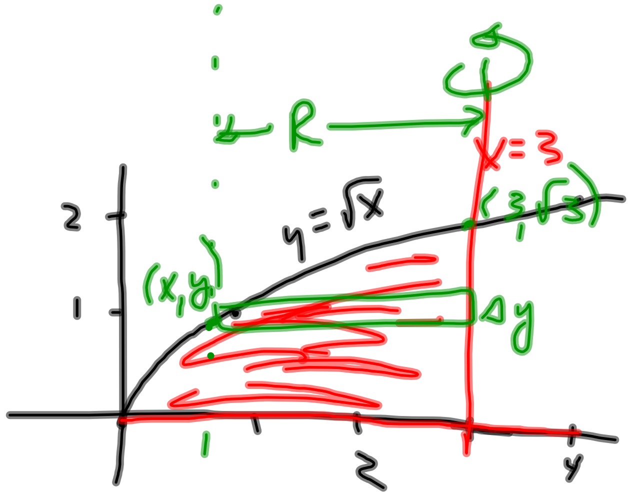
$6-2x-x^2=x+6$

$-3x-x^2=0$

$x^2+3x=0$

$x(x+3)=0$

$x=0,-3$



$$V = \pi \int_c^d R^2 \, dy$$

$$= \pi \int_0^{\sqrt{3}} (3 - y^2)^2 \, dy$$

$R = 3 - x = 3 - y^2$

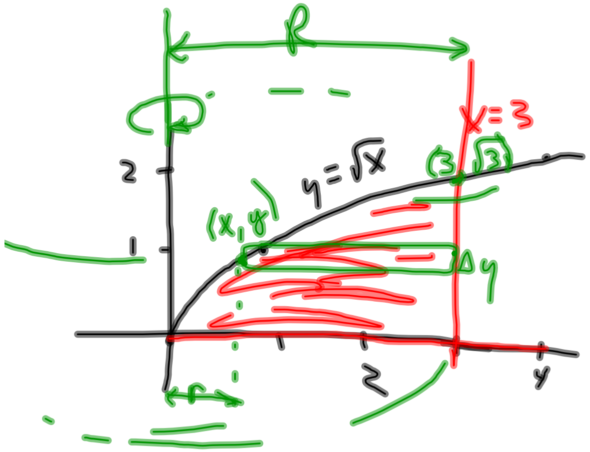
$x = y^2$

$$= \pi \int_0^{\sqrt{3}} (9 - 6y^2 + y^4) \, dy$$

$$= \pi \left[9y - \frac{\cancel{6}y^3}{\cancel{3}} + \frac{y^5}{5} \right]_0^{\sqrt{3}}$$

$$= \pi \left[9\sqrt{3} - \overset{-6\sqrt{3}}{2(3\sqrt{3})} + \frac{9\sqrt{3}}{5} - 0 \right]$$

$$= \pi \left[3\sqrt{3} + \frac{9\sqrt{3}}{5} \right] = 3\pi\sqrt{3}\left(1 + \frac{3}{5}\right) = \frac{\overset{24}{\cancel{8}}\pi\sqrt{3}}{5}$$



$$V = \pi \int^{d} (R^2 - r^2)\, dy$$

$$= \pi \int_0^{\sqrt{3}} \left(3^2 - (y^2)^2\right) dy$$

$$= \pi \int_0^{\sqrt{3}} (9 - y^4)\, dy$$

$R = 3$

$r = x = y^2$

$y = \sqrt{x}$

$y^2 = x$

$$= \pi \left[9y - \frac{y^5}{5} \right]_0^{\sqrt{3}} = \pi \left[9\sqrt{3} - \frac{(\sqrt{3})^4 \sqrt{3}}{5} - 0 \right]$$

$$= \pi \left[9\sqrt{3} - \frac{\overset{3}{\cancel{9}}\sqrt{3}}{5} \right]$$

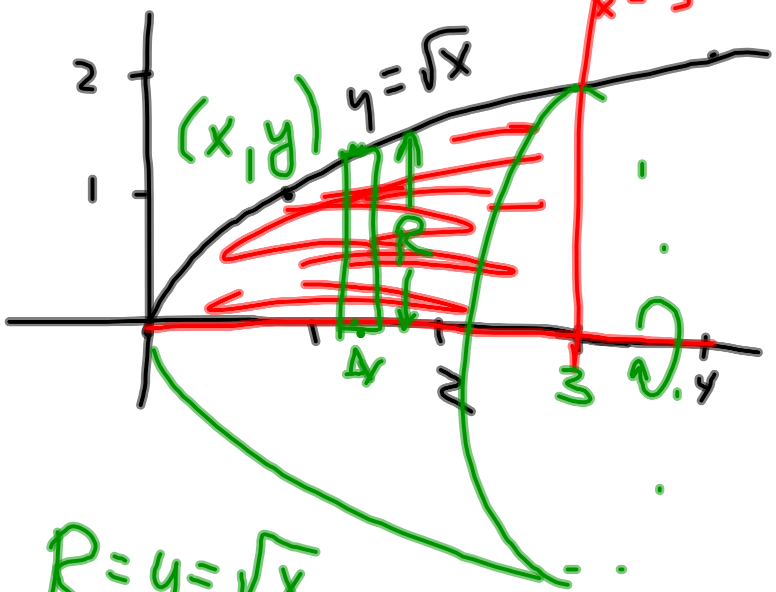
39.178

$$= 9\sqrt{3}\pi\left(1 - \frac{1}{5}\right) = 9\sqrt{3}\pi \frac{4}{5}$$

$$\frac{36\sqrt{3}\pi}{5}$$

$$= \frac{14\sqrt{3}}{5}\pi$$

11. G: $y=\sqrt{x}$, $y=0$, $x=3$ F: v.r.
a) ⟲ x-axis ✓
b) ⟲ y-axis
c) ⟲ $x=3$
d) ⟲ $x=6$



$R=y=\sqrt{x}$

R^2

a) $V=\pi\int_a^b R^2\,dx$

$=\pi\int_0^3 (\sqrt{x})^2\,dx$

$=\pi\int_0^3 x\,dx=\pi\frac{x^2}{2}\Big]_0^3=\frac{9}{2}\pi$