## 7.1 Area of Region Bounded by 2 Curves

HW: p. 452 # 1-9, 19, 21-27,

33, 35, 39

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## 7.1 Area of Region Bounded by 2 Curves

(background, see also Class Notes, Ch 42.)

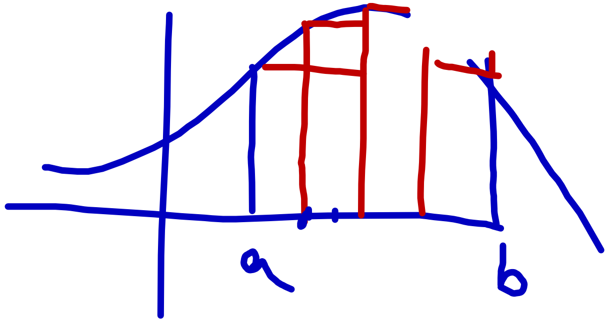
Area under a curve.

f continuous and non-negative

$$A = \lim_{n \to \infty} \sum_{i=1}^{n} f(c_i)\,\Delta x_i$$

$$\Delta x_i = \frac{b-a}{n}$$

$$x_i < c_i \le x_{i+1}$$

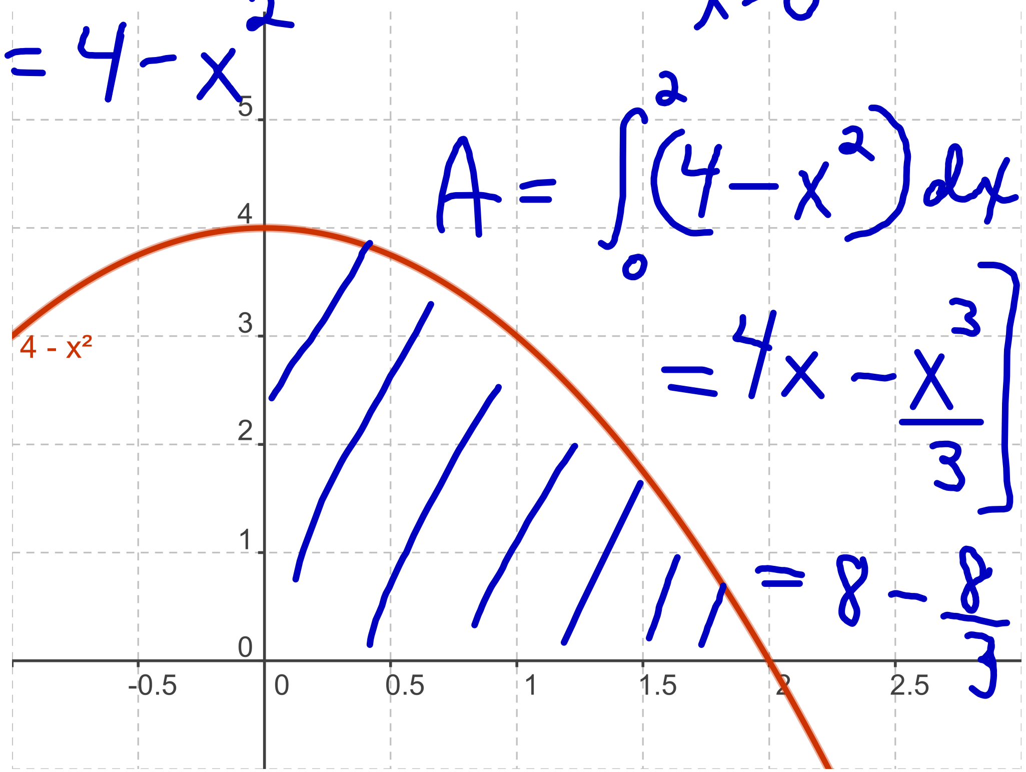


# 7.1 Area of Region Bounded by 2 Curves

(background, see also Class Notes, Ch 4.2)

F: Area fr:
$x = 0$ to $x = 2$

$y = 4 - x^2$




$$A = \int_0^2 (4 - x^2)\,dx$$

$$= 4x - \frac{x^3}{3}\Big]_0^2$$

$$= 8 - \frac{8}{3} - 0$$

$$= \frac{16}{3}$$

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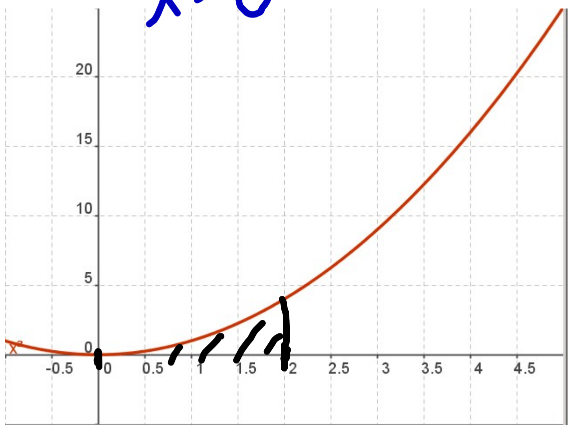
# 7.1 Area of Region Bounded by 2 Curves

(background, see also Class Notes, Ch 4.2)

F: Area fr.: $x=0$ to $x=2$




$y = x^2$

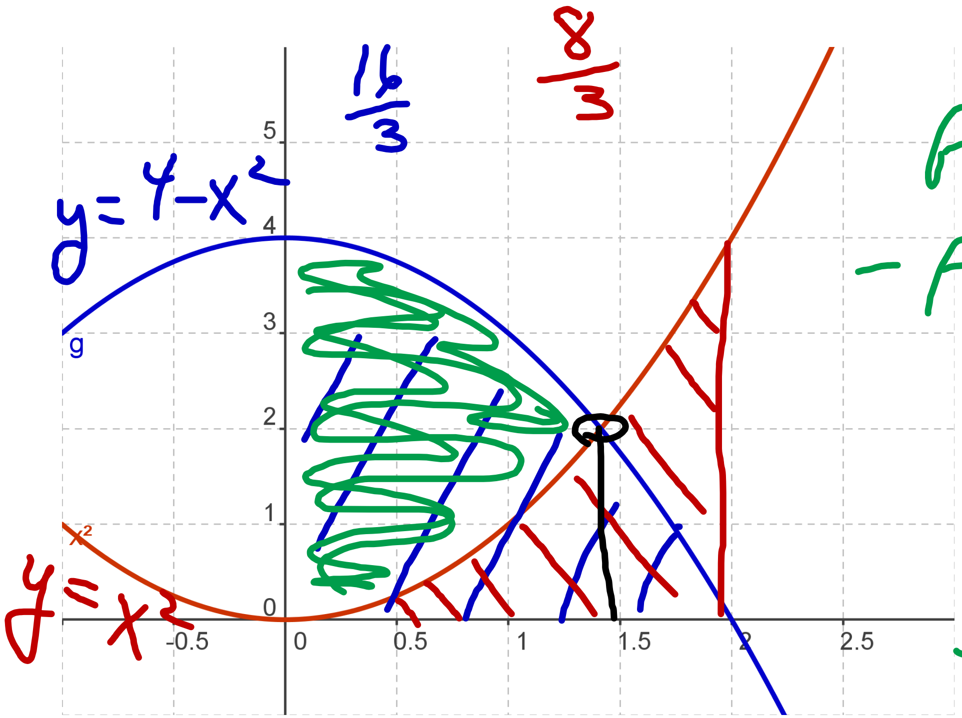
F: A under $x=0$ to $x=2$

$$A = \int_0^2 x^2 \, dx$$

$$A = \frac{x^3}{3}\Big]_0^2 = \frac{8}{3}$$

## 7.1 Area of Region Bounded by 2 Curves

F: Area of bounded region in Quadrant I




A of $4-x^2$

− A of $x^2$

only if confined to bounded region.

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Calculus Home Page

Homework Part 1 Homework Part 2

## 7.1 Area of Region Bounded by 2 Curves

Find upper limit by finding x coordinate of point of intersection.

$$4 - x^2 = x^2$$

$$4 = 2x^2$$

$$x^2 = 2$$

$$x = +\sqrt{2}$$

$$\int_0^{\sqrt{2}} (4 - x^2)\,dx$$ upper

–

$$-\int_0^{\sqrt{2}} x^2\,dx$$ lower

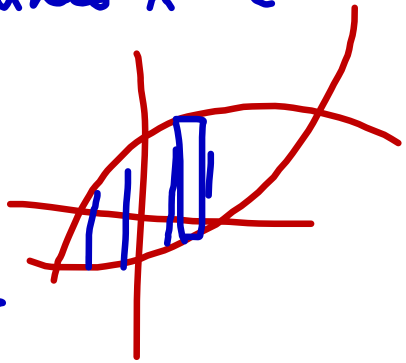
$$= \int_0^{\sqrt{2}} (\underset{\text{upper}}{4 - x^2} - \underset{\text{lower}}{x^2})\,dx$$

$$= \int_0^{\sqrt{2}} (4 - 2x^2)\,dx = 4x - \frac{2x^3}{3}\Big]_0^{\sqrt{2}} = 4\sqrt{2} - \frac{2}{3}2^{3/2} - 0$$

## 7.1 Area of Region Bounded by 2 Curves

If f and g cont [a, b]
and $g(x) \le f(x)$ for all x on [a, b]
then the area of the region bounded by
f and g and the vertical lines x = a
and x = b is

$$A = \int_a^b [f(x) - g(x)]\,dx$$



upper - lower

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# 7.1 Area of Region Bounded by 2 Curves

on the web:

*Finding the Area of the Region Bounded by 2 or More Curves:*

1. Sketch the curves:
   - Find the points of intersection of the curves.
   - If the curves are close and orientation is difficult to determine, substitute values between those of the points of intersection to determine which is above (or to the right of) the other.
2. Use the sketch to determine which integral to use:
   - If each curve passes the vertical line test in the bounded region, use vertical rectangles, the x variable, and the integral:

     $$A = \int_a^b (upper - lower)\,dx$$

   - If a curve fails the vertical line test but passes the horizontal line test in the bounded region, use horizontal rectangles, the variable y, and the integral:

     $$A = \int_c^d (right - left)\,dy$$

3. If the bounded area contains more than one distinct region, write the area as the sum of the areas of each distinct region.
4. Limits of integration:
   - Use the coordinates of the points of intersection.
   - If $x = k_1$ or $y = k_2$ is given this may be one of the limits.

## 7.1 Area of Region Bounded by 2 Curves

G: $f(x) = x^2 + 2x + 1 = (x+1)^2$

$g(x) = 2x + 5$

F: area of bounded region

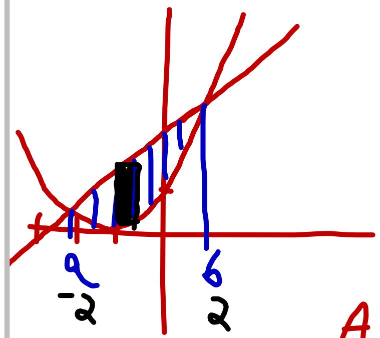
$x^2 + 2x + 1 = 2x + 5$

$x^2 - 4 = 0$

$(x+2)(x-2) = 0$

$x = -2, 2$

$x = 0: \; f(0) = 1, \; g(0) = 5$




g(x) - f(x)

$$A = \int_{-2}^{2} \left[ (2x+5) - (x^2 + 2x + 1) \right] dx$$

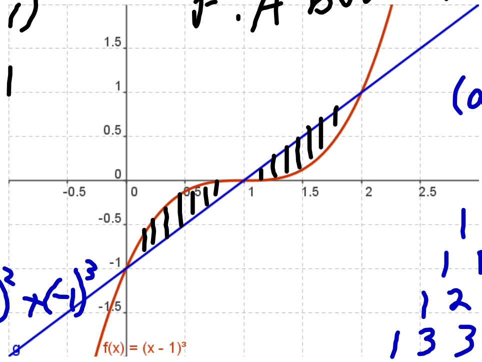
$$= \int_{-2}^{2} (4 - x^2)\, dx = 4x - \frac{x^3}{3} \Big]_{-2}^{2}$$

$$4(2) - \frac{2^3}{3} - \left( 4(-2) - \frac{(-2)^3}{3} \right)$$

$$8 - \frac{8}{3} - \left( -8 + \frac{8}{3} \right) = 16 - \frac{16}{3} = \frac{48 - 16}{3} = \frac{32}{3}$$

6? $f(x)=(x-1)^3$ F: A boundary.

$g(x)=x-1$




$(a+b)^n$

| n | |
|---|---|
| 0 | 1 |
| 1 | 1 1 |
| 2 | 1 2 1 |
| 3 | 1 3 3 1 |

$(x-1)^3 = x-1$

$x^3+3x^2(-1)+3x(-1)^2+(-1)^3$

$x^3-3x^2+3x-1 = x-1$

$x^3-3x^2+2x = x(x^2-3x+2) = x(x-1)(x-2) = 0 \therefore x = 0, 1, 2$

$A = \int_0^1 [(x-1)^3-(x-1)]dx + \int_1^2 [x-1-(x-1)^3]dx$

$= \int_0^1 (x^3-3x^2+2x)dx + \int_1^2 (-x^3+3x^2-2x)dx$

$= \frac{x^4}{4} - \frac{3x^3}{3} + \frac{2x^2}{2}\Big]_0^1 + -\frac{x^4}{4} + \frac{3x^3}{3} - \frac{2x^2}{2}\Big]_1^2$

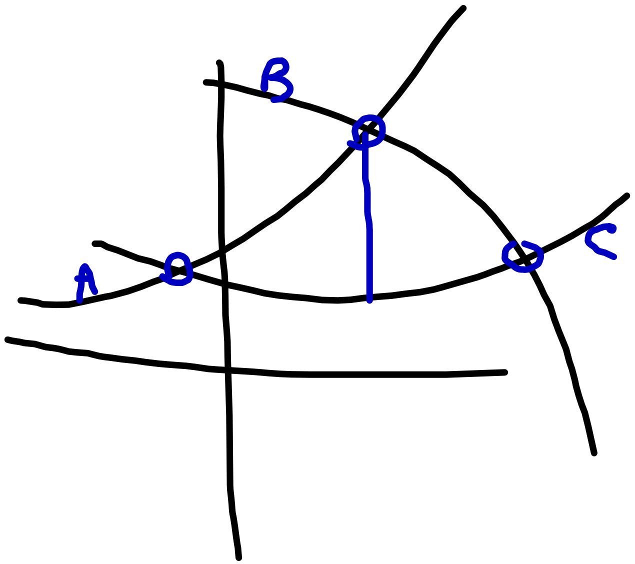
$= \frac{1}{4} - 1 + 1 - 0 + (-4+8-4) - (-\frac{1}{4}+1-1)$

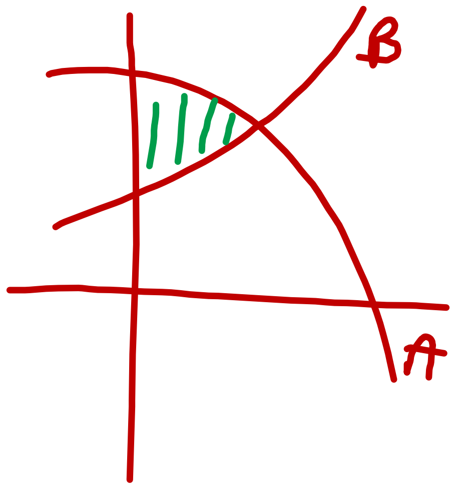
$= \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$



Title: 3 points of intersection (10 of 12)

# 7.1 Area of Region Bounded by 3 Curves




Need to arrange the bounded region by dividing into appropriate sections.

## 7.1 Area of Region Bounded by 2 Curves