6.4 First Order Linear Diff Eq.

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6.4 First Order Linear Diff Eq.

Consider the following. Can we solve either?

$$\frac{1-\frac{dy}{dx}}{y} = 3x$$

$$1-\frac{dy}{dx} = 3xy$$

$$-\frac{dy}{dx} = 3xy - 1$$

$$\frac{dy}{dx} = 1 - 3xy$$

$$dy = (1-3xy)\,dx$$

Not separable. Not homogeneous.

$$2xy - \frac{dy}{dx}\ln x = y$$

$$(\ln x)\frac{dy}{dx} = y - 2xy$$

$$+\frac{dy}{dx} = \frac{-y+2xy}{\ln x} = \frac{y(2x-1)}{\ln x}$$

$$\frac{dy}{y} = \frac{2x-1}{\ln x}\,dx$$

Separable but cannot solve w present methods

6.4 First Order Linear Diff Eq.

Def: First Order Linear Diff EQ

standard form: $\frac{dy}{dx} + P(x)y = Q(x)$

where P, Q are continuous functions of x

Does this help to solve the previous diff eqs?

Need to check to see if they can be converted to this standard form.

6.4 First Order Linear Diff Eq.

Can we rearrange to standard form for 1st order linear diff eq?

#4. $$\frac{1-\frac{dy}{dx}}{y}=3x$$

$$1-\frac{dy}{dx}=3xy$$

$$-1+\frac{dy}{dx}=-3xy$$

$$\frac{dy}{dx}+3xy=1$$

$$\frac{dy}{dx}+P(x)y=Q(x)$$

$P(x)=3x$

$Q(x)=1$

Yes. is. 1st ord. line Diff Eq.

6.4 First Order Linear Diff Eq.

Can we rearrange to standard form for 1st order linear diff eq?

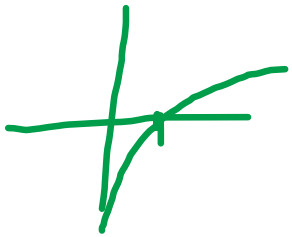
#2. $2xy - \frac{dy}{dx}\ln x = y$

$\frac{dy}{dx} + P(x)\,y = Q(x)$

$-2xy + \ln x \frac{dy}{dx} = -y$

$\frac{dy}{dx} + \frac{1-2x}{\ln x} y = 0$

$y - 2xy + \ln x \frac{dy}{dx} = 0$



$y\frac{(1-2x)}{\ln x} + \frac{dy}{dx} = 0 \;,\; x \neq 1$

6.4 First Order Linear Diff Eq.

How to solve?

http://www.battaly.com/calc/calchw.htm#Solve%20Linear%20Diff%20EQ

Solving a First Order Linear Differential Equation:

Solving a First Order Linear Differential Equation:

1. **Put the equation into standard form:** $\frac{dy}{dx} + P(x)y = Q(x)$
2. **Identify P(x) and Q(x).**
3. **Find:** $\int P(x)dx$
4. **Let** $u(x) = e^{\int P(x)dx}$
5. **Solve for y:** $y\,u(x) = \int Q(x)u(x)dx + c$

why? see page 12

5.6: $\frac{dy}{dx} + \frac{1}{x}y = 3x+4$

① $P(x) = \frac{1}{x}$ $\quad Q(x) = 3x+4$

② $\int P(x)\,dx = \int \frac{1}{x}\,dx = \ln|x| + c_1$

③ $e^{\ln|x|+c_1} = c_2 e^{\ln|x|} = c_2|x|$

$y[c|x|] = \int (3x+4)c|x|\,dx + c$

$c_2\int(3x^2+4x)\,dx = c_2\left[\frac{3x^3}{3} + \frac{4x^2}{2}\right] = c_2\left[x^3+2x^2\right]$

$c_2 y|x| = c_2\left[x^3+2x^2\right] + c$

$y|x| = x^3+2x^2+c_3$

$y = x^2+2|x|+\frac{c_3}{|x|}$

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6.4 First Order Linear Diff Eq.

$$e^{\int P(x)dx} y = \int \left(e^{\int P(x)dx} Q(x) \right) dx + c$$

$$e^{\int P(x)dx} \frac{dy}{dx} + \left[\frac{d(e^{\int P(x)dx})}{dx} \right] y = e^{\int P(x)dx} Q(x)$$

$$e^{\int P(x)dx} \frac{dy}{dx} + \left[e^{\int P(x)dx} P(x) \right] y = e^{\int P(x)dx} Q(x)$$

$$\frac{dy}{dx} + P(x)y = Q(x)$$

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