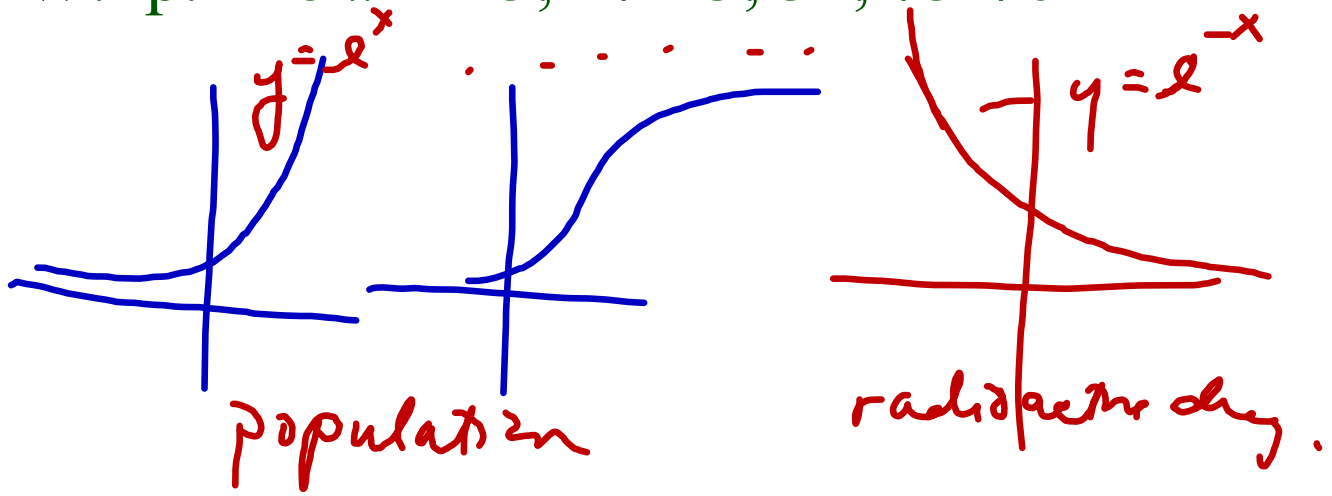
6.2 Differential Equations: Growth & Decay

Exponential

HW: p. 418 # 1-13, 17-23, 31, 73-76



6.2 Differential Equations: Growth & Decay

Consider the problem:

Let $P(t)$ represent the number of wolves in a population at time t years, when $t \geq 0$. The population $P(t)$ is increasing at a rate directly proportional to $800 - P(t)$, where the constant of proportionality is k.

(a) If $P(0) = 500$, find $P(t)$ in terms of t and k.

$$\frac{dP}{dt} = k(800 - P)$$

b) If P(2) = 700, find k.

c) Find $\lim_{t \to \infty} P(t)$

p. 418 # 12.6: The rate of change of P with respect to t is proportional to $10 - t$.

F: Solve

I.F: $P(t) = \ldots$

$$\frac{dP}{dt} = k(10 - t)$$

$$dP = \frac{dP}{dt}\,dt$$

$$\int dP = \int k(10 - t)\,dt$$

$$P = k\left[10t - \frac{t^2}{2}\right] + C$$

inversely : $\frac{dP}{dt} = \frac{k}{10-t}$

14. The rate of change of y with respect to x varies jointly as x and $L-y$.

$$\frac{dy}{dx} = kx(L-y)$$

2. $\frac{dy}{dx} = 4 - x$ Soln:

$dy = (4 - x)\,dx$

Easy (this is what we have done already): rate of change varies with independent variable

4. $\frac{dy}{dx} = 4 - y \Rightarrow dy = \frac{dy}{dx}\,dx$

Different: rate of change varies directly with dependent variable.

$\int dy = \int (4 - y)\,dx$

$-\int \frac{-dy}{4-y} = \int dx \Rightarrow u = 4 - y,\ du = -dy$

$-\ln|4-y| = x + C_1$

$a^m a^n = a^{m+n}$

$e^{-x-C_1} = e^{-x}e^{-C_1} = C_2 e^{-x}$

Class Notes: Prof. G. Battaly, Westchester Community College, NY
Calculus Home Page
Homework Part II

$\ln|4-y| = -x - C_1$

$e^{-x-C_1} = 4 - y$

$4 - y = C_2 e^{-x}$

$-y = C_2 e^{-x} - 4$

$y = 4 - C_2 e^{-x}$

$y = 4 + C e^{-x}$

6.2 Differential Equations: Growth & Decay

$y = Ce^{kt}$ $k > 0$ growth

$k < 0$ decay

6. $y' = \frac{\sqrt{x}}{3y}$

$dy = y' dx$

$dy = \frac{\sqrt{x}}{3y} dx$

$\int 3y\, dy = \int \sqrt{x}\, dx$

8. $y' = x(1+y)$

$dy = y' dx$

$dy = x(1+y) dx$

$\int \frac{dy}{1+y} = \int x\, dx$

Exponential Growth or Decay:

Rate of change varies directly with the dependent variable.

Examples:

$dy/dx = ky$ $\qquad$ $dP/dt = kP$

$dy/dx = k(y+1)$ $\qquad$ $dP/dt = k(200-P)$

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