5.7 Inverse Trig Functions and Integration

HW: 5.7 p 385 # 5 - 13, 21 - 29

Class Notes: Prof. G. Battaly, Westchester Community College, NY

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Homework Part 1

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p385 # 2

$$\int \frac{3}{\sqrt{1-4x^2}}\,dx$$

Can we integrate this using $\int u^n\,du$?

$u = 1 - 4x^2$

$du = -8x\,dx$

$dx = \frac{du}{-8x}$

$$3\int \frac{du}{-8\frac{\sqrt{1-u}}{2}\cdot\sqrt{u}}$$

Does not work

$4x^2 - 1 = -u$

$x^2 = \frac{1-u}{4}$

$x = \sqrt{1-u}/2$

This is still a problem.

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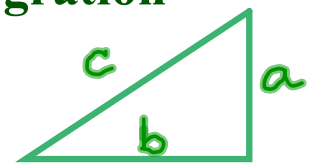
$$\int \frac{3}{\sqrt{1-4x^2}}\,dx$$



$a^2+b^2=c^2$

$b^2=c^2-a^2$

$b=\sqrt{c^2-a^2}$

$\sqrt{1^2-(2x)^2}$

What does this form suggest?

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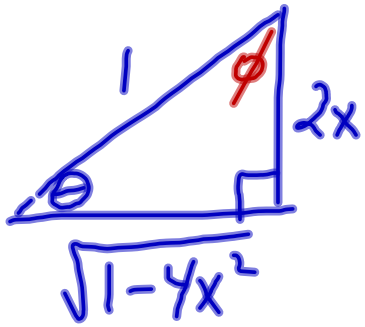
$$\int \frac{3}{\sqrt{1-4x^2}}\,dx$$



using trig:

$$\frac{3}{2}\int \frac{\cos\theta\,d\theta}{\cos\theta}$$

$$= \frac{3}{2}\int d\theta = \frac{3}{2}\theta + C$$

$$= \frac{3}{2}\arcsin 2x + C$$

$\sin\theta = 2x$

$\cos\theta\,d\theta = 2\,dx$

$\cos\theta = \sqrt{1-4x^2}$

$$\int \frac{3}{\sqrt{1-4x^2}}\,dx$$

$$\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\frac{u}{a} + C$$

using formula:

$a = 1,\ u = 2x$

$du = 2\,dx$

$$3\int \frac{dx}{\sqrt{1-4x^2}} = \frac{3}{2}\int \frac{du}{\sqrt{a^2-u^2}}$$

$$= \frac{3}{2}\arcsin\frac{u}{a} + C = \frac{3}{2}\arcsin 2x + C$$

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Is there a shorter way?

$$\int \frac{du}{\sqrt{a^2 - u^2}} = \arcsin\frac{u}{a} + c$$

$$\int \frac{du}{a^2 + u^2} = \frac{1}{a}\arctan\frac{u}{a} + c$$

$$\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a}\operatorname{arcsec}\frac{|u|}{a} + c$$

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385 #4. $\int \frac{4}{1+9x^2}\,dx = 4\int \frac{dx}{1+9x^2}$

$= \frac{4}{3}\int \frac{du}{a^2+u^2}$, $a = 1$

$\int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan\frac{|u|}{a} + C$

$u = 3x$

$du = 3\,dx$

$dx = \frac{du}{3}$

$= \frac{4}{3}\cdot\frac{1}{a}\arctan\frac{|u|}{a} + C$

$= \frac{4}{3}\cdot\arctan|3x| + C$

$$6.\ \int \frac{1}{4+(x-1)^2}\,dx \qquad \begin{aligned} a &= 2 \\ u &= x-1 \\ du &= dx \end{aligned}$$

$$= \int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan\frac{|u|}{a} + C$$

$$= \frac{1}{2}\arctan\frac{|x-1|}{2} + C$$

$$\int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan\frac{|u|}{a} + C$$

$$\int \frac{x^4-1}{x^2+1}\,dx = \int \frac{(x^2+1)(x^2-1)}{(x^2+1)}\,dx$$

$$= \int (x^2-1)\,dx = \frac{x^3}{3} - x + C$$

10, 18, 26

10. $\int \frac{t}{t^4+16}\,dt$

18. $\int \frac{4x+3}{\sqrt{1-x^2}}\,dx$

26. $\int_0^{\frac{1}{\sqrt{2}}} \frac{\arccos x}{\sqrt{1-x^2}}\,dx$

10. $\frac{1}{2}\int \frac{2t}{t^4+16}\,dt$

$\frac{1}{8}\arctan\frac{t^2}{4}+C$ ✓

$\frac{1}{t^2}\arctan\frac{|4|}{t^2}+C$

$\frac{1}{4}\arctan\frac{t^2}{4}+C$

$a=4$

$u=t^2$

$du=2t\,dt$

$= \frac{1}{2}\int \frac{du}{a^2+u^2} = \frac{1}{2}\cdot\frac{1}{a}\arctan\frac{|u|}{a}+C$

$= \frac{1}{2}\cdot\frac{1}{4}\arctan\frac{t^2}{4}+C$

18. $\int \frac{4x+3}{\sqrt{1-x^2}}\,dx$

$u = 1-x^2$

$du = -2x\,dx$

$= \int \frac{4x}{\sqrt{1-x^2}}\,dx + 3\int \frac{dx}{\sqrt{1-x^2}}$

$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$

$u = 1-x^2$

$du = -2x\,dx$

$3 \arcsin x$

$\frac{4}{-2}\int \frac{-2x\,dx}{\sqrt{1-x^2}} = -2\int \frac{du}{u^{1/2}}$

$= -2\int u^{-1/2}\,du$

$= -2\frac{u^{1/2}}{\frac{1}{2}} + C_1 = 4\left(1-x^2\right)^{\frac{1}{2}} + 3\arcsin x + C$

26. $\int_0^{\frac{1}{\sqrt{2}}} \frac{\arccos x}{\sqrt{1-x^2}}\, dx$

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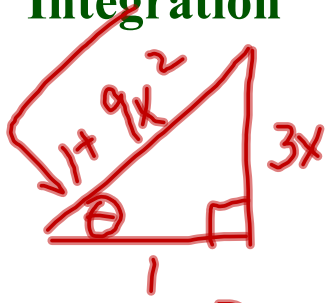
385 #4.

$$\int \frac{4}{1+9x^2}\,dx$$



$$= \frac{4}{3}\int \frac{\sec^2\theta\,d\theta}{\sec^2\theta}$$

$$= \frac{4}{3}\int d\theta = \frac{4}{3}\theta + C$$

$$\frac{4}{3}\arctan 3x + C$$

$\tan\theta = 3x$

$\sec^2\theta\,d\theta = 3\,dx$

$\frac{\sec^2\theta}{3}\,d\theta = dx$

$\cos\theta = \frac{1}{\sqrt{1+9x^2}}$

$\sec\theta = \sqrt{1+9x^2}$

$\sec^2\theta = 1+9x^2$

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formulas

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