

## 5.4 Exponential Functions: Differentiation and Integration

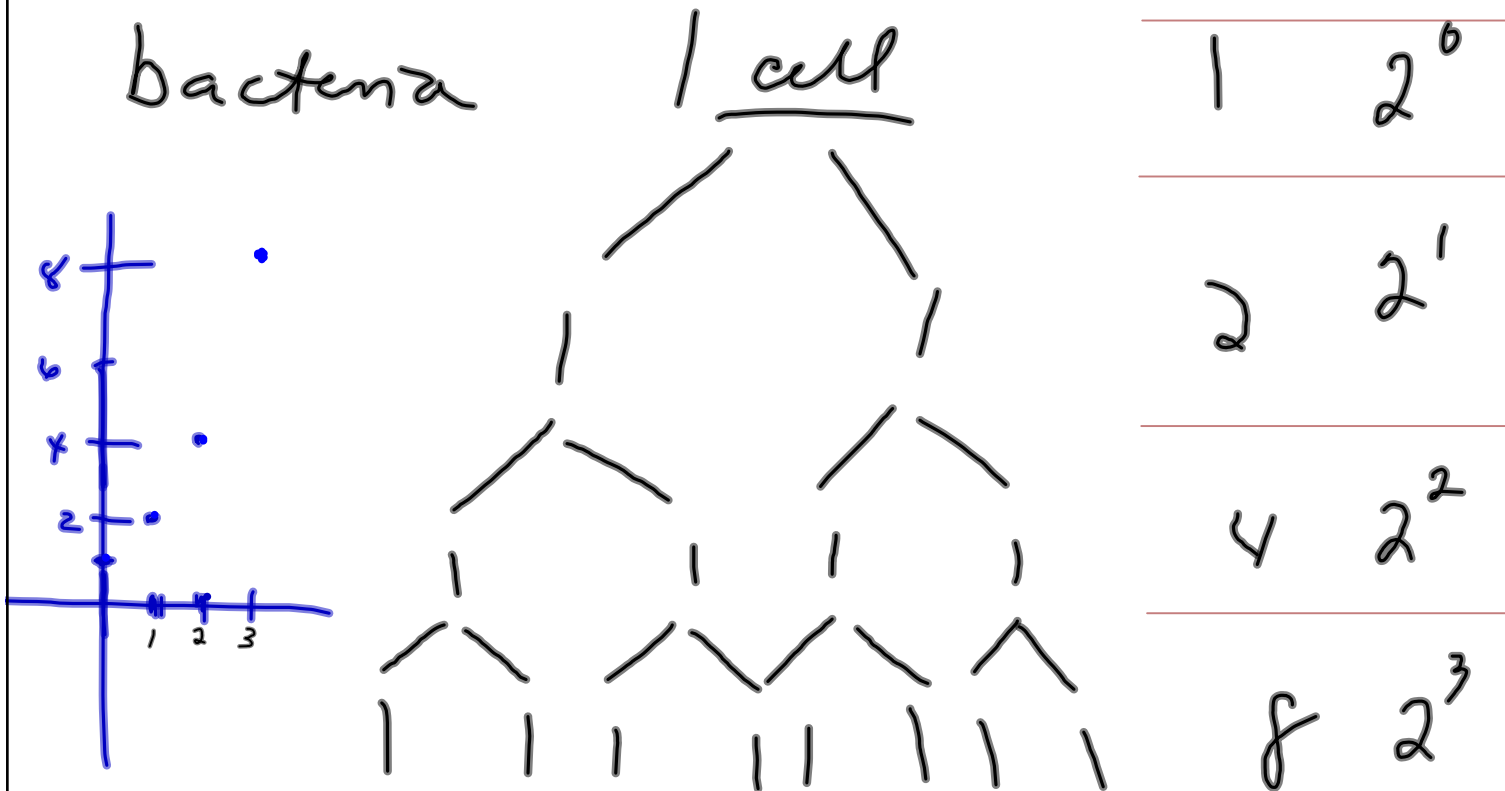
### Assignment:

5.4 # 1-15, 25-28 all, 29, 37, 39;  
41, 45, 49, 51, 53, 57, 63-39,  
73, 79; 99, 103, 107, 111,  
115, 117, 121

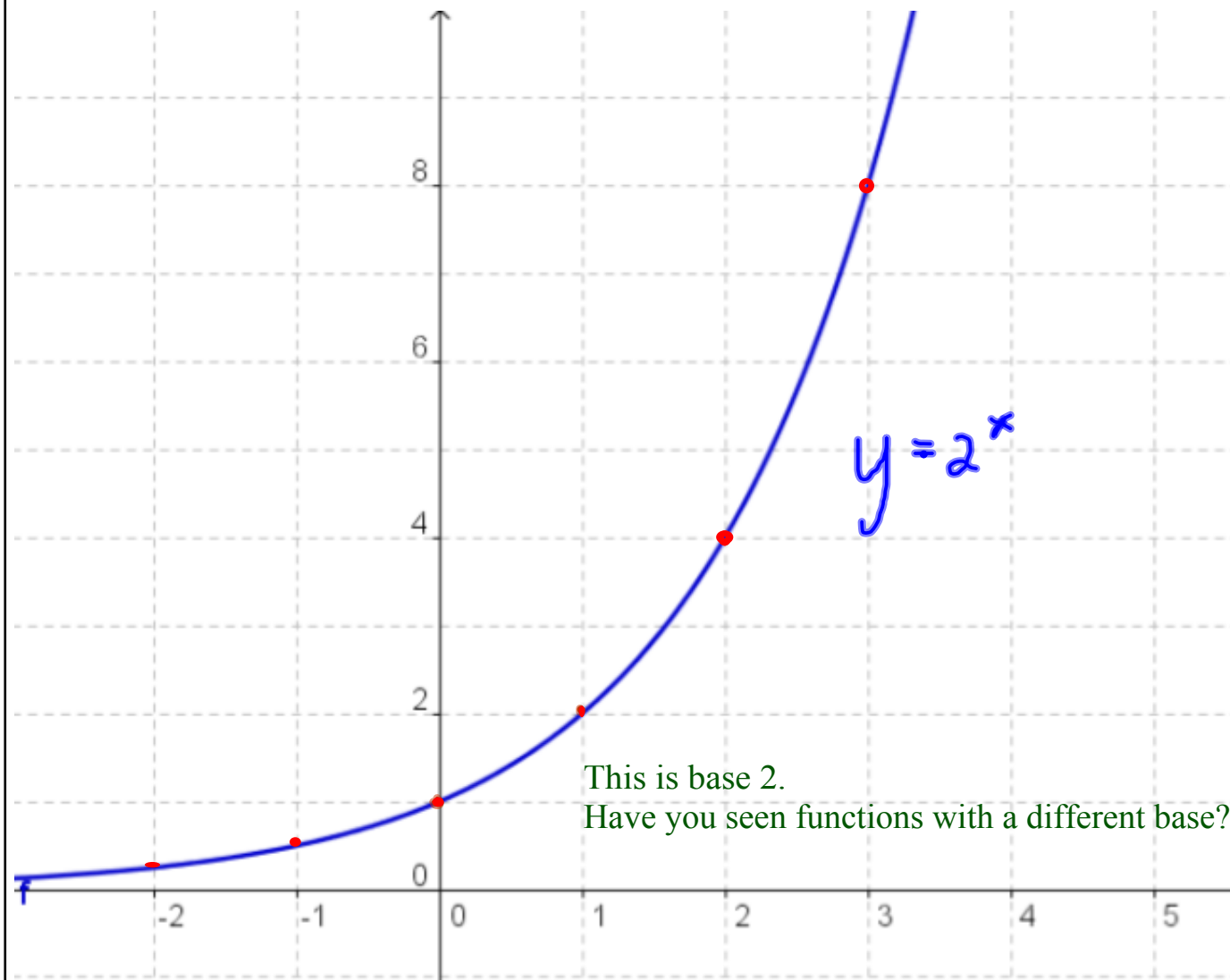
## 5.4 Exponential Functions: Differentiation and Integration

Consider a single cell bacteria:

exponential  
 $2^?$





# Exponential Function



This is base 2.  
Have you seen functions with a different base?

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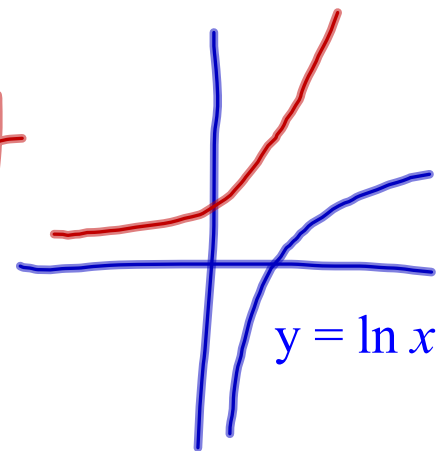
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## 5.4 Exponential Functions: Differentiation and Integration

### Natural Exponential Function


$$y = e^x \text{ IFF } x = \ln y$$


$$\ln y = x$$



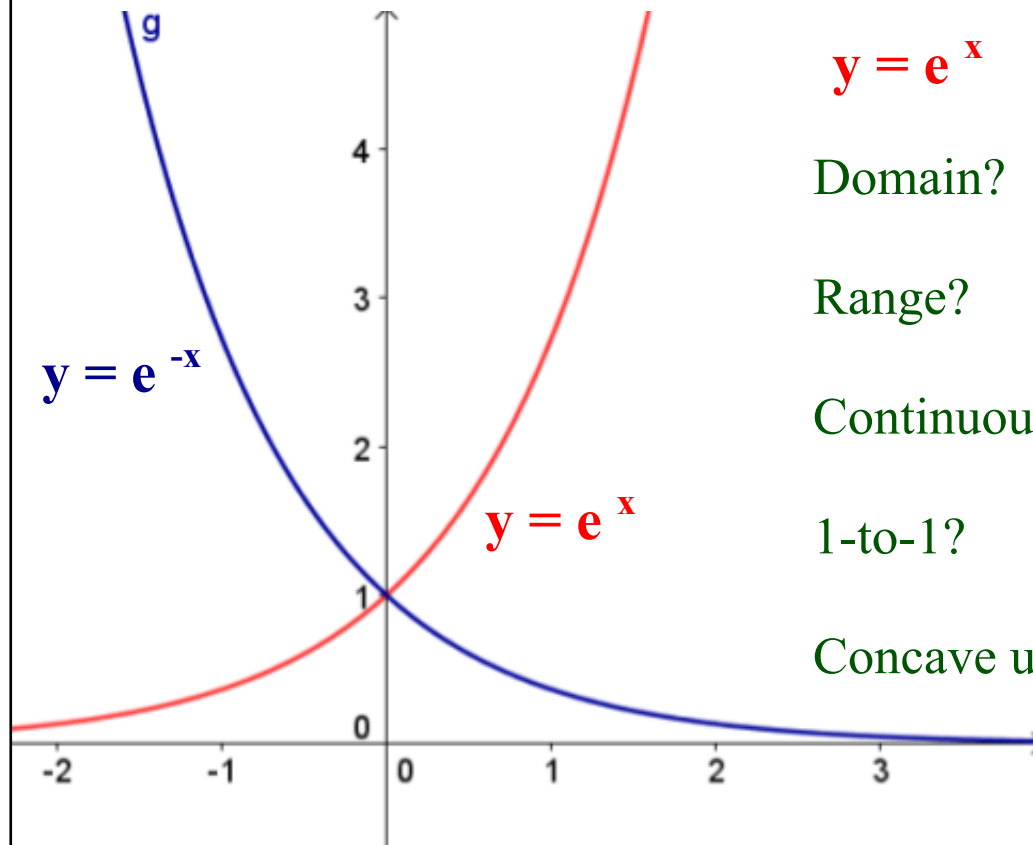
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## Properties of the Natural Exponential Function



$$y = e^x$$

Domain?

Range?

Continuous?

1-to-1?


Concave up or down?


$$\lim_{x \rightarrow -\infty} e^x =$$

$$\lim_{x \rightarrow +\infty} e^x =$$

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## 5.4 Exponential Functions: Differentiation and Integration

Solving equations with variable exponents

$$4. \quad 4e^x = 83 \quad F: \text{solve for } x$$
$$e^x = 83/4$$

$$\ln e^x = \ln \frac{83}{4}$$

$$x \ln e = \ln \frac{83}{4} \quad \square \quad x = \ln \frac{83}{4}$$

$$x \ln e = \ln \frac{83}{4}$$

$$x = \ln \left( \frac{83}{4} \right)$$

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$$\begin{aligned}\#2. \quad G: \quad e^{\ln 2x} &= 12 \\ \ln e^{(\ln 2x)} &= \ln 12 \\ (\ln 2x) \cancel{\ln e} &= \ln 12 \\ \ln 2x &= \ln 12 \\ 2x &= 12 \\ x &= 6\end{aligned}$$

## 5.4 Exponential Functions: Differentiation and Integration

$$12. \ln 4x = 1$$

log is exp.

$$e^1 = 4x$$

$$x = e/4$$

$$\ln D = a \Rightarrow e^a = D$$

$$F: x$$

$$\log 4x = 1$$

$$10^1 = 4x$$

$$x = 10/4 = 5/2$$

The log is the exponent.

$$6. \quad -6 + 3e^x = 8$$

Solve for  $x$

$$3e^x = 14$$

$$e^x = 14/3$$

$$\ln e^x = \ln(14/3)$$

$$x \ln e = \ln(14/3)$$

$$x = \ln(14/3)$$

## 5.4 Exponential Functions: Differentiation and Integration

$$10. \ln x^2 = 10$$

$$F: X$$

$$2 \ln x = 10$$

$$\ln x = 5$$

$$e^5 = x$$

either  $e^{10} = x^2$

$$x = \sqrt{e^{10}}$$

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Finding the derivative.....

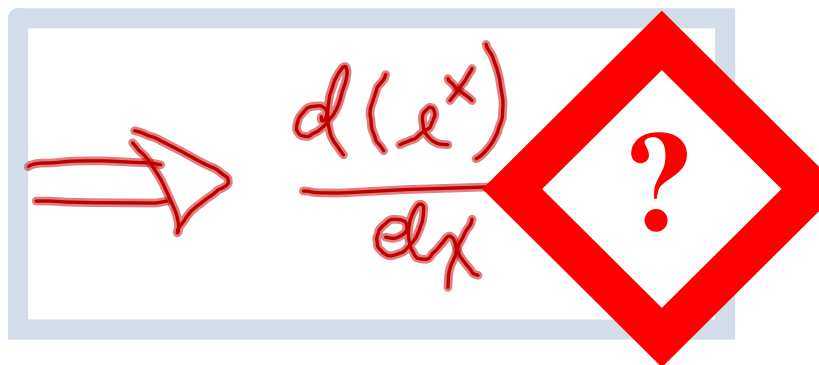
$$y = e^x$$

$$\ln y = \ln e^x = x \ln e = x$$

$$\frac{d(\ln y)}{dx} = \frac{dx}{dx} = 1$$

$$\frac{1}{y} \frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = y = e^x$$


$$\frac{d(e^x)}{dx}$$

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$$\frac{d(e^u)}{dx} = e^u \frac{du}{dx}$$

36. G:  $y = e^{-x^2}$


$$\frac{dy}{dx} = e^{-x^2} (-2x)$$
$$= -2x e^{-x^2}$$


F:  $\frac{dy}{dx}$

$$u = -x^2$$
$$\frac{du}{dx} = -2x$$

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$$G: y = e^{-x^2}$$

$$F: \frac{dy}{dx}$$

$$\frac{d(e^u)}{dx} = e^u \frac{du}{dx}$$

$u = f(x)$

$$\frac{dy}{dx} = e^{-x^2} (-2x) = -2x e^{-x^2}$$

## 5.4 Exponential Functions: Differentiation and Integration

38.  $y = x^2 e^{-x}$  F:  $\frac{dy}{dx}$

$$\frac{d(e^u)}{dx} = e^u \frac{du}{dx}$$


$$\frac{dy}{dx} = x^2 \left[ \frac{d(e^{-x})}{dx} \right] + e^{-x} \frac{d(x^2)}{dx}$$


$$= x^2 e^{-x} (-1) + e^{-x} (2x)$$

$$= \left( -x^2 e^{-x} + 2x e^{-x} \right) = x e^{-x} (2 - x)$$

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## 5.4 Exponential Functions: Differentiation and Integration

$$\text{Ex. } y = \ln\left(\frac{1+e^x}{1-e^x}\right) = \ln(1+e^x) - \ln(1-e^x)$$


$$\frac{dy}{dx} = \frac{e^x}{1+e^x} - \frac{1}{1-e^x}(-e^x)$$


$$= \frac{e^x}{1+e^x} + \frac{e^x}{1-e^x} = \frac{e^x(1-e^x) + e^x(1+e^x)}{(1-e^{2x})}$$

$$= \frac{e^x(1-e^x+1+e^x)}{(1-e^{2x})} = \frac{2e^x}{1-e^{2x}}$$

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$$G: y = e^{-2x+x^2}$$

$$\frac{d(e^u)}{dx} = e^u \frac{du}{dx}$$

F: eq. tang. line  
at (2, 1)

↓

$$\frac{dy}{dx} = (e^{-2x+x^2})(-2+2x)$$

$$\left. \frac{dy}{dx} \right|_{(2,1)} = \left[ e^{(-4+4)} \right] [-2+4]$$

$$(2,1) = 2$$

$$y - y_1 = m_T(x - x_1)$$

$$y - 1 = (m_T)(x - 2)$$

$$y - 1 = 2(x - 2)$$

$$y - 1 = 2x - 4$$

$$y = 2x - 3$$

$$e^{xy} + x^2 - y^2 = 10$$

$$F: \frac{dy}{dx}$$

$$\frac{d(e^u)}{dx} = e^u \frac{du}{dx}$$

$$e^{xy} \left( x \frac{dy}{dx} + y(1) \right) + 2x - 2y \frac{dy}{dx} = 0$$

$$x e^{xy} \frac{dy}{dx} + e^{xy} y + 2x - 2y \frac{dy}{dx} = 0$$

$$(x e^{xy} - 2y) \frac{dy}{dx} = 0 - e^{xy} y - 2x$$

$$\frac{dy}{dx} = - \frac{(2x + e^{xy} y)}{x e^{xy} - 2y}$$

$$= \frac{2x + e^{xy} y}{2y - x e^{xy}}$$

## 5.4 Exponential Functions: Differentiation and Integration

Example of Exponential Function used in Statistics

### Standard Normal Probability Density Function

has mean of 0 and Inflection point at  $\pm 1\sigma$

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-x^2/2\sigma^2}$$

Find the inflection points if  $\sigma=1$

(See textbook to check your solution)

## 5.4 Exponential Functions: Differentiation and Integration


$$\text{If: } \frac{d(e^u)}{dx} = e^u \frac{du}{dx}$$

Then

$$\int e^x dx = ? \quad \int e^u du = ?$$


## 5.4 Exponential Functions: Differentiation and Integration

$$\int e^x dx = e^x + c \quad \int e^u du = e^u + c$$

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