5.3 Inverse Functions: Introduction

(Additional practice for 5.2: p. 338 #13, 15, 17, 41)

HW: 5.3: p. 347 #1, 5, 29, 33, 37, 41, 47, 51, 59, 77, 79, 23-27

Consider and compare:

$f(x) = -4x + 3$ and $g(x) = \dfrac{x-3}{-4}$

Start with x. Then:

Note the inverse operations

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Intro

5.3 Inverse Functions: Introduction

(Additional practice for 5.2: p. 338 #13, 15, 17, 41)

HW: 5.3: p. 347 #1, 5, 29, 33, 37, 41, 47, 51, 59, 77, 79, 23-27

Consider and compare:

$f(x) = -4x + 3$ and $g(x) = \dfrac{x-3}{-4}$

Start with x. Then:

1. Mult by -4	1. Subtr. 3
2. Add 3	2. Divide by -4

Note the inverse operations

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Intro

5.3 Inverse Functions: Introduction

Consider and compare:

$f(x) = \sqrt{(2x - 3)}$ and $g(x) = \dfrac{x^2 + 3}{2}$

$x \geq 3/2$ $g \geq 3/2$

Start with x. Then:

These appear to be inverse functions, but what about x=0?
Are they inverse functions for all x? all y?

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Intro

5.3 Inverse Functions: Introduction

Consider and compare:

$f(x) = \sqrt{(2x - 3)}$ and $g(x) = \dfrac{x^2 + 3}{2}$

$x \geq 3/2$ $g \geq 3/2$

Start with x. Then:

1. Mult. by 2	1. Square
2. Subt. 3	2. Add 3
3. Sqrt.	3. Div. by 2

These appear to be inverse functions, but what about x=0?
Are they inverse functions for all x? all y?

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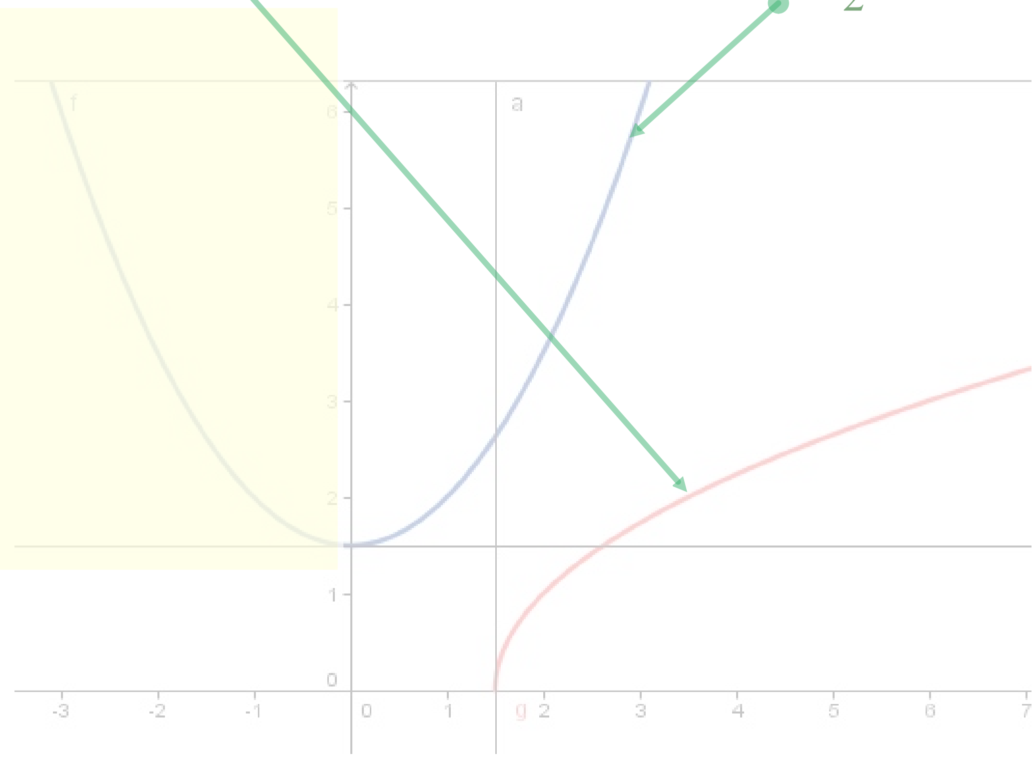
Intro

5.3 Inverse Functions: Introduction

$f(x) = \sqrt{(2x - 3)}\ , x \geq 3/2$ and $g(x) = \dfrac{x^2 + 3}{2}\ , x \geq 0$



But, *f* and *g* are inverses ONLY when $x \geq 0$ for $g(x)$

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Graph Preview

5.3 Inverse Functions: Definition

A function g is the inverse of a function f if

$f(g(x)) = x$ **for each** x **in the domain of** g

AND $g(f(x)) = x$ **for each** x **in the domain of** f

(f^{-1} notation means "f inverse", NOT exponent)

and $g(x) = f^{-1}(x)$

Notes

1. If $g(x) = f^{-1}(x)$, then $f(x) = g^{-1}(x)$
2. Domain of f^{-1} is the range of f and the range of f^{-1} is the domain of f
3. If f^{-1} exists, then it is unique (only one)
4. goes in both directions

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definition

5.3 Inverse Functions

p. 347 #2.

G: $f(x) = 3 - 4x$, $g(x) = \frac{3-x}{4}$

F: show $g(x) = f^{-1}(x)$

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example

5.3 Inverse Functions

p.347 #2.

G: $f(x) = 3-4x$, $g(x) = \dfrac{3-x}{4}$

F: show that $g(x) = f^{-1}(x)$

$$\boxed{f(g(x)) = g(f(x)) = x}$$

$f(\ ) = 3-4(\ )$

$f(g(x)) = 3 - \cancel{4}\left(\frac{3-x}{\cancel{4}}\right) = 3-(3-x)$

$= 3-3+x = x$

$g(\ ) = \dfrac{3-(\ )}{4}$

$g(f(x)) = \dfrac{3-(3-4x)}{4} = \dfrac{3-3+4x}{4} = \dfrac{4x}{4} = x$

$\therefore f(x)$ is inv. of $g(x)$

$g(x)$ " " $f(x)$

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example

5.3 Inverse Functions

p 347 # 4.

G: $f(x) = 1 - x^3$, $g(x) = \sqrt[3]{1-x}$

F: Show that $g(x) = f^{-1}(x)$

$f(g(x)) = g(f(x)) = x$

$f(\quad) = 1 - (\quad)^3$

$f(g(x)) = 1 - (\sqrt[3]{1-x})^3$

$= 1 - (1-x)$

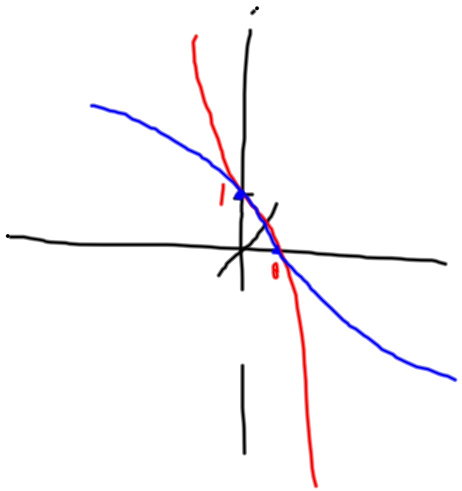
$= x$ ✓

$g(\quad) = \sqrt[3]{1-(\quad)}$

$g(f(x)) = \sqrt[3]{1-(1-x^3)}$

$= \sqrt[3]{x^3} = x$ ✓

$\therefore$ are inverses.



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example

5.3 Inverse Functions

Explore relationships between a function and its inverse, using Geogebra. Click on the globe.
Uses a quadratic function.

Explore relationships between a function and its inverse, using Geogebra. Click on the globe.
Uses *ln x*.

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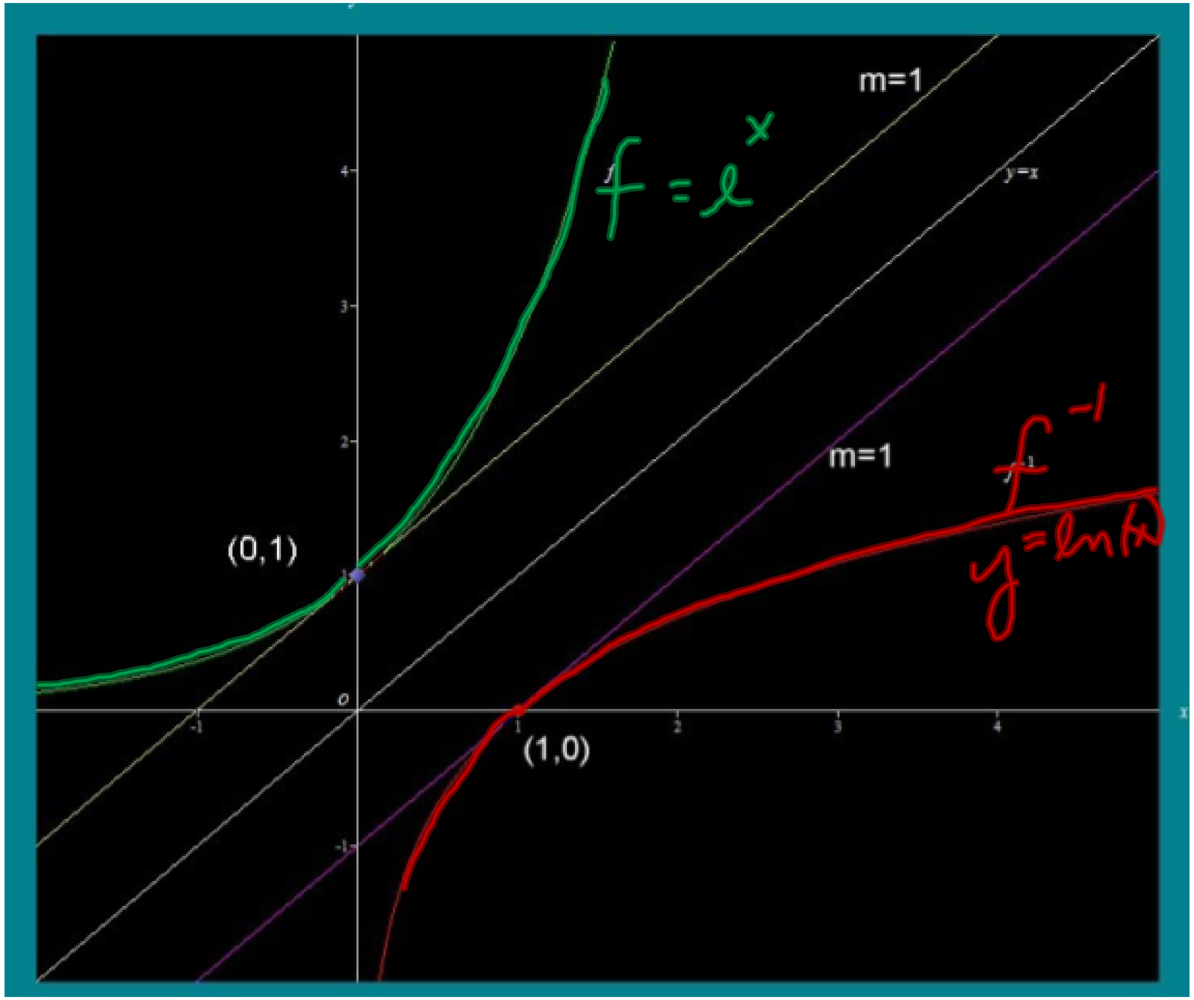
example

5.3 Inverse Functions: Properties

skip if have done
geogebra for *ln x*



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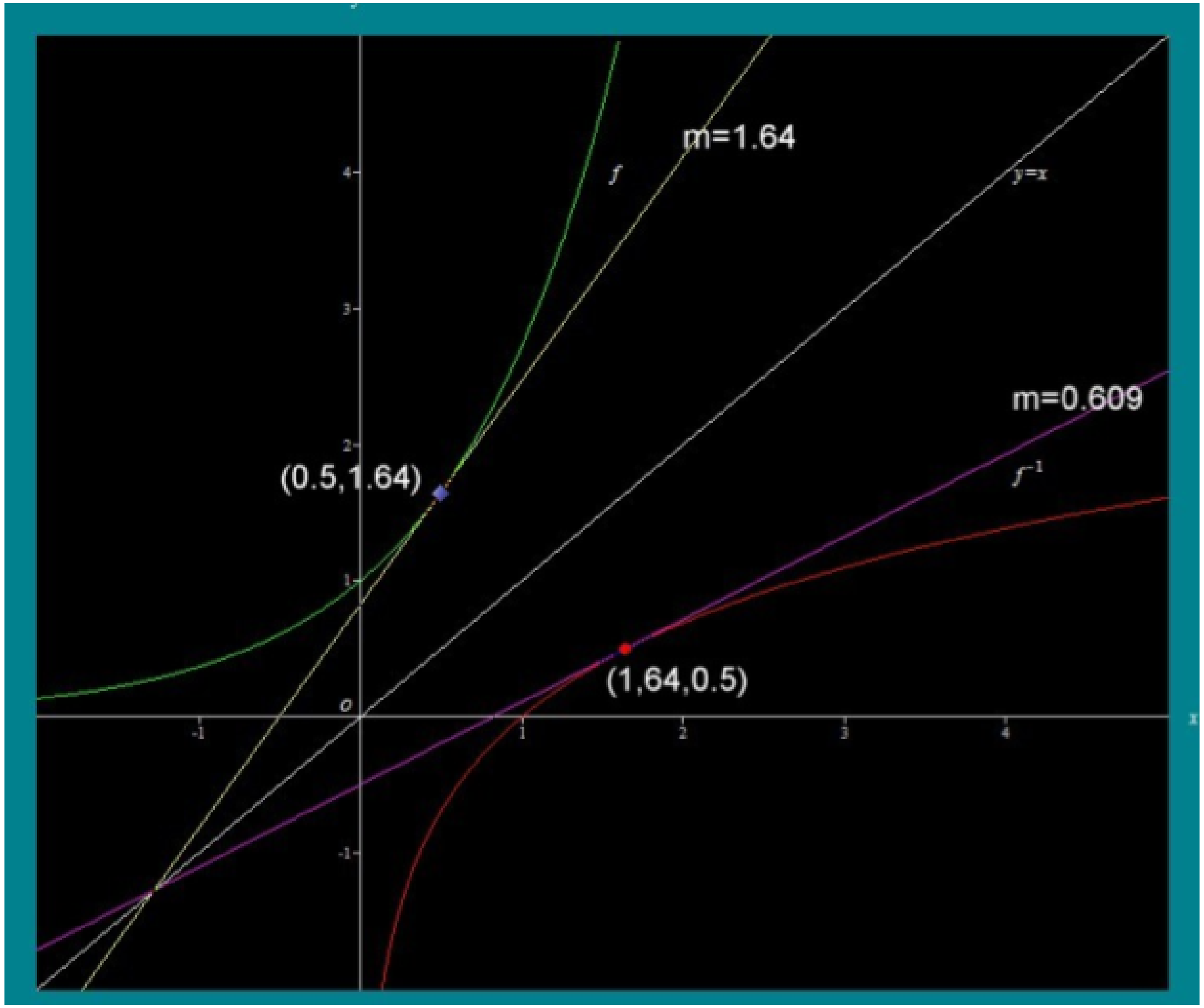
properties

5.3 Inverse Functions: Properties



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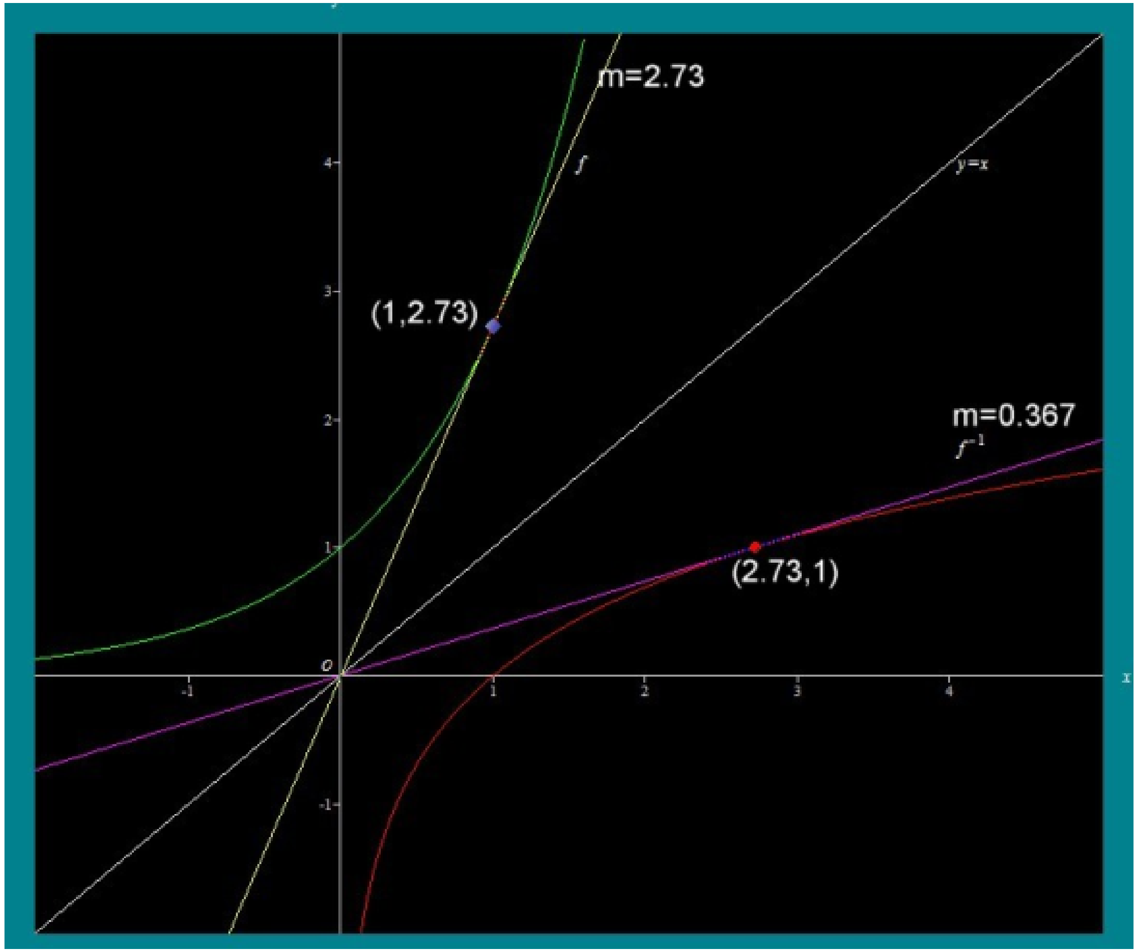
5.3 Inverse Functions: Properties



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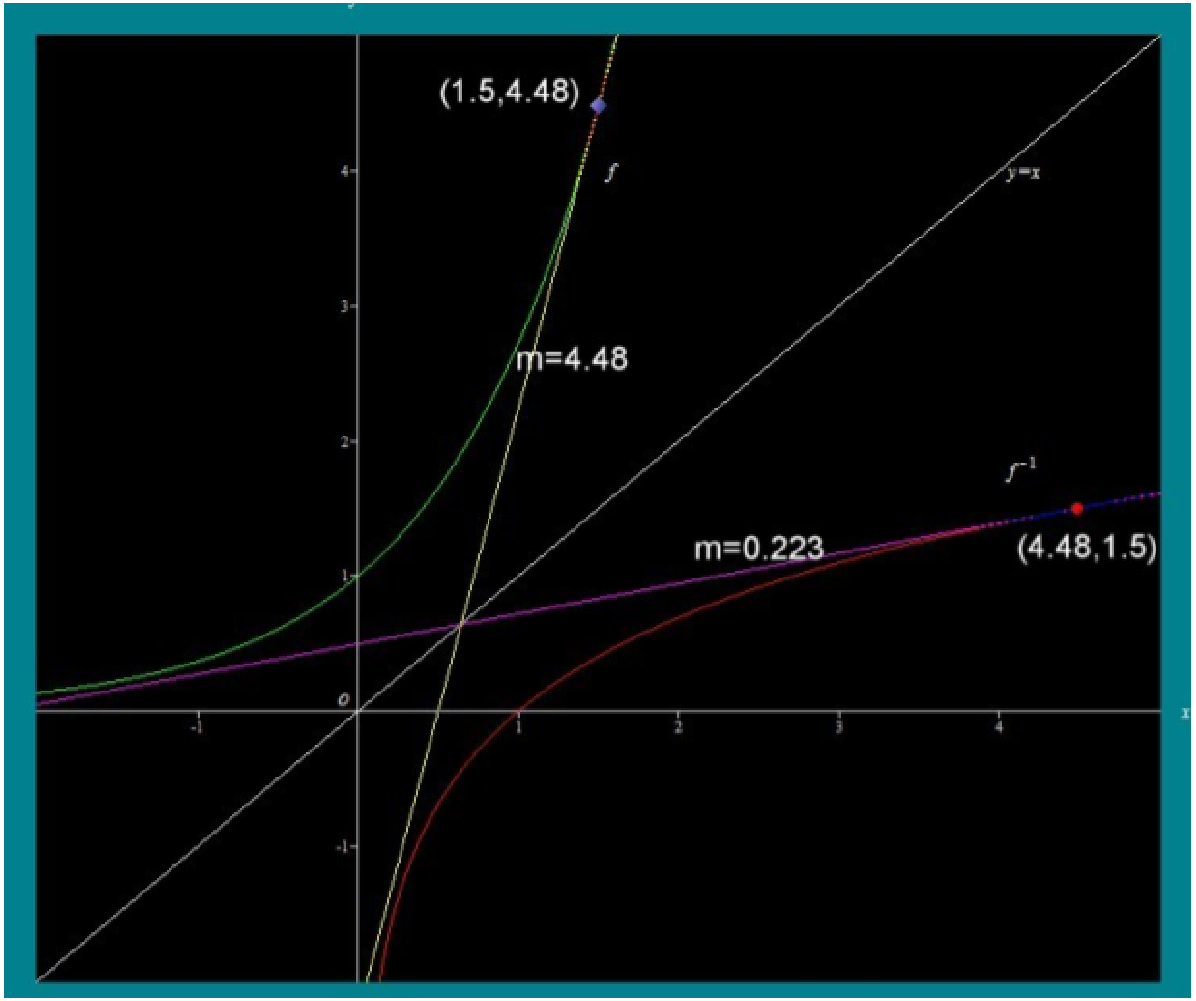
5.3 Inverse Functions: Properties



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properties

5.3 Inverse Functions: Properties

(Insert images from APCD or demo the APCD)

(a,b) on f	slope of f	(b,a) on f^{-1}	slope of f^{-1}
(0,1)	1	(1,0)	1
	1.64		0.609
	2.73		0.369
	4.48		.223

(a,b) is on f

(b,a) on f^{-1}

points and slopes

5.3 Inverse Functions:

Reflective Property of Inverse Function

The graph of f contains the point (a,b) IFF

the graph of f^{-1} contains the point (b,a)

Derivative of an Inverse Function

Let f be a function differentiable on I, and $g(x) = f^{-1}(x)$

1. Then g is differentiable at any x for which $f'(g(x)) \neq 0$

2. $g'(x) = \dfrac{1}{f'(g(x))}$ **and** $f'(g(x)) \neq 0$

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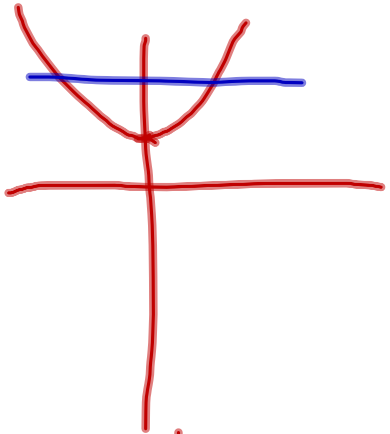
Reflective Prop & Derivative

5.3 Inverse Functions:

$y = x^2 + 1$

not strictly monotonic

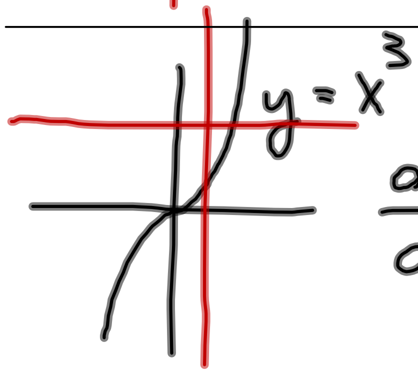


not 1-to-1

Cannot have inverse over whole domain.

$$\frac{dy}{dx} = 2x$$

$$\begin{cases} x > 0 & \frac{dy}{dx} > 0 \\ x < 0 & \quad " \quad < 0 \end{cases}$$



$$\frac{dy}{dx} = 3x^2 \geq 0$$

Is strictly monotonic.
Is 1-to-1.
Can have inverse over whole domain.

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Need 1-to-1

5.3 Inverse Functions:

30. $f(x) = 3x$ F: f^{-1}

$y = 3x$

$x = 3y$ Exchange variables, and solve for y.

$y = \frac{x}{3} = f^{-1}(x) = g(x)$

$f(g(x)) = g(f(x)) = x$?

Can check to be sure, using definition.

$f(\quad) = 3(\quad)$

$f(g(x)) = 3\left(\frac{x}{3}\right) = x$ ✓

$g(f(x)) = x$?

$g(\quad) = \frac{(\quad)}{3}$

$g(f(x)) = \frac{3x}{3} = x$ ✓

example

5.3 Inverse Functions:

36. $f(x) = \sqrt{x^2-4}, \; x \geq 2$ F: f^{-1}

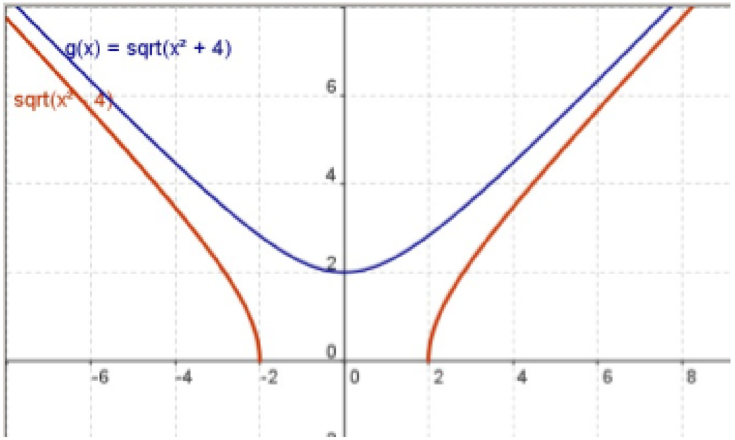
$y = \sqrt{x^2-4}$

$x = \sqrt{y^2-4}$

$x^2 = y^2-4$

$y^2 = x^2+4$

$y = \sqrt{x^2+4}$



How do we restrict the domain for these to be inverse functions?

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example

5.3 Inverse Functions:

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example