

5.2 The Natural Logarithm: Integration

Introduction

HW: 5.2 #1-13, 17, 21-33, 37, 39,
43-45,47,53-59,67,69

1. $y = \ln x$ 1. $dy/dx =$

2. $y = \ln 2x$ 2. $dy/dx =$

3. $y = \ln (x^2 + 2)$ 3. $dy/dx =$

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1. $y = \ln x$

1. $dy/dx = \frac{1}{x}$

2. $y = \ln 2x$

2. $dy/dx = \frac{1}{2x} \cdot 2 = \frac{1}{x}$

3. $y = \ln(x^2 + 2)$

3. $dy/dx =$
 $\frac{1}{x^2+2} \cdot 2x = \frac{2x}{x^2+2}$

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$$3. \quad y = \underline{\hspace{2cm}}? \quad 3. \quad dy/dx = \frac{2x+3}{x^2+3x}$$

$$u = x^2 + 3x$$
$$du = (2x+3)dx$$

$$dy = \frac{dy}{dx} \cdot dx$$

$$\int dy = \int \frac{dy}{dx} dx$$
$$= \int \frac{(2x+3) dx}{x^2+3x}$$

$$= \int \frac{du}{u} = \int \frac{1}{u} du$$

$$= \ln|u| + C$$
$$= \ln|x^2+3x| + C$$

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$$3. \quad y = \underline{\hspace{2cm}} \quad ? \quad 3. \quad dy/dx = \frac{2x+3}{x^2+3x}$$

Let $u = \underline{\hspace{2cm}}$

Then $du/dx = \underline{\hspace{2cm}}$

and remembering $\frac{d(\ln u)}{dx} = \frac{1}{u} \frac{du}{dx}$

we need to consider
the integral: $\int \frac{1}{u} \frac{du}{dx}$

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Log Rule for Integration

Let u be a differentiable function of x . Then:

$$\int \frac{1}{x} dx = \ln|x| + c \qquad \int \frac{1}{u} du = \ln|u| + c$$

$$x \neq 0$$

$$u \neq 0$$

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What about $\int \frac{2}{x} dx$?

$$= 2 \int \frac{1}{x} dx = 2 \ln|x| + C \quad x \neq 0$$

$$\int \frac{\sqrt{3}}{x} dx = \sqrt{3} \int \frac{1}{x} dx = \sqrt{3} \ln|x| + C$$

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$$\int \frac{1}{2x} dx = \frac{1}{2} \int \frac{1}{x} dx = \frac{1}{2} \ln|x| + c$$

4. $\int \frac{1}{x-5} dx$ $u = x - 5$
 $du = dx$ $\int \frac{1}{u} du = \int \frac{du}{u}$

$$\int \frac{1}{u} du = \ln|u| + c$$
$$= \ln|x-5| + c$$

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even problems

$$6. \frac{1}{3} \int \frac{3}{3x+2} dx$$

$$u = 3x+2$$
$$du = 3 dx$$

$$= \frac{1}{3} \int \frac{du}{u} = \frac{1}{3} \ln|u| + C = \frac{1}{3} \ln|3x+2| + C$$

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even problems

$$8. \quad -\frac{1}{3} \int \frac{(-3)x^2}{3-x^3} dx$$

$$u = 3 - x^3$$
$$du = -3x^2 dx$$

$$= -\frac{1}{3} \int \frac{1}{u} du = -\frac{1}{3} \ln|u| + c$$

$$= -\frac{1}{3} \ln|3 - x^3| + c$$

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even problems

$$10. \int \frac{x}{\sqrt{9-x^2}} dx = -\frac{1}{2} \int \underbrace{(9-x^2)^{-\frac{1}{2}}}_{(-2)x} dx$$

$$= -\frac{1}{2} \int u^{-\frac{1}{2}} du \quad \begin{array}{l} u = 9-x^2 \\ du = -2x dx \end{array}$$

$$= -\frac{1}{2} \frac{u^{\frac{1}{2}}}{\frac{1}{2}} + c = \underbrace{-\sqrt{9-x^2} + c}_{\text{circled}} = -(9-x^2)^{\frac{1}{2}} + c$$

$$\int u^n du = \frac{u^{n+1}}{n+1} + c$$

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even problems

$$14. \int \frac{2x^2 + 7x - 3}{x-2} dx$$

required to factor numerator

$$ac = -6$$

$$a + c = 7$$

but only helps if one factor is (x-2).
otherwise cannot reduce to lowest terms

$$\begin{array}{r} 2x + 11 \\ x-2 \overline{) 2x^2 + 7x - 3} \\ \underline{-2x^2 - 4x} \\ 11x - 3 \\ \underline{-11x - 22} \\ 19 \end{array} + \frac{19}{x-2}$$

But, can do long division

Or Synthetic Division:

[PowerPoint \(click Read Only\)](#)

[Step by Step Instructions](#)

$$= \int (2x+11) dx + \int \frac{19}{x-2} dx$$

$$\int \left(2x + 11 + \frac{19}{x-2} \right) dx$$

$$= \frac{2x^2}{2} + 11x + 19 \int \frac{1}{x-2} dx$$

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$$= x^2 + 11x + 19 \ln|x-2| + c$$

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$$\int \frac{x(x+2)}{x^3+3x^2-4} dx = \frac{1}{3} \int \frac{3(x^2+2x)}{x^3+3x^2-4} dx$$

$$\begin{aligned} u &= x^3+3x^2-4 \\ du &= (3x^2+6x)dx \\ &= 3(x^2+2x)dx \end{aligned}$$

$$= \frac{1}{3} \int \frac{du}{u}$$

$$= \frac{1}{3} \ln |u| + c$$

$$= \frac{1}{3} \ln |x^3+3x^2-4| + c$$

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even problems

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