5.1 The Natural Logarithm
Introduction

Study 5.1; p. 331 # 7, 9, 11, 15, 19-27, 31-43, 49, 51, 57-63, 69-73, 77, 80

Background: Integrate the following.

$$\int x^4 dx =$$

$$\int x^2 dx =$$

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Homework Part 1

Homework Part 2

5.1 The Natural Logarithm

$$\int \frac{1}{x^3}\,dx$$

$$\int \frac{1}{x^2}\,dx$$

5.1 The Natural Logarithm

Introduction

$$\int x^4 dx = \frac{x^5}{5} + c$$

$$\int x^2 dx = \frac{x^3}{3} + c$$

$$\int \frac{1}{x^3} dx = \int x^{-3} dx = \frac{x^{-2}}{-2} + c = -\frac{1}{2x^2} + c$$

$$\int \frac{1}{x^2} dx = \int x^{-2} dx = \frac{x^{-1}}{-1} + c = -\frac{1}{x} + c$$

5.1 The Natural Logarithm

What about $\int \frac{1}{x}\,dx$?

5.1 The Natural Logarithm

What about $\int \frac{1}{x}\,dx$?

$$\int \frac{1}{x}\,dx = \frac{x^0}{0} + c \quad \textit{not defined!}$$

Think:

What function has a derivative = 1/x?

Remember the 2nd FTC?

From Calc 1, Section 4.4

2nd FTC

If f is continuous on open interval I containing a, then, for every x on the interval

$$\frac{d}{dx}\left[\int_a^x f(t)\,dt\right] = f(x)$$

5.1 The Natural Logarithm

$$\frac{d}{dx}\left[\int_a^x f(t)\,dt\right] = f(x)$$

OR:

$$\frac{d}{dx}\left[\int_a^x \frac{1}{t}\,dt\right] = \frac{1}{x}$$

So, we need to define a
function that has a derivative = 1/x.

5.1 The Natural Logarithm

Definition of the natural logarithm

$$\ln x = \int_1^x \frac{1}{t}\, dt$$

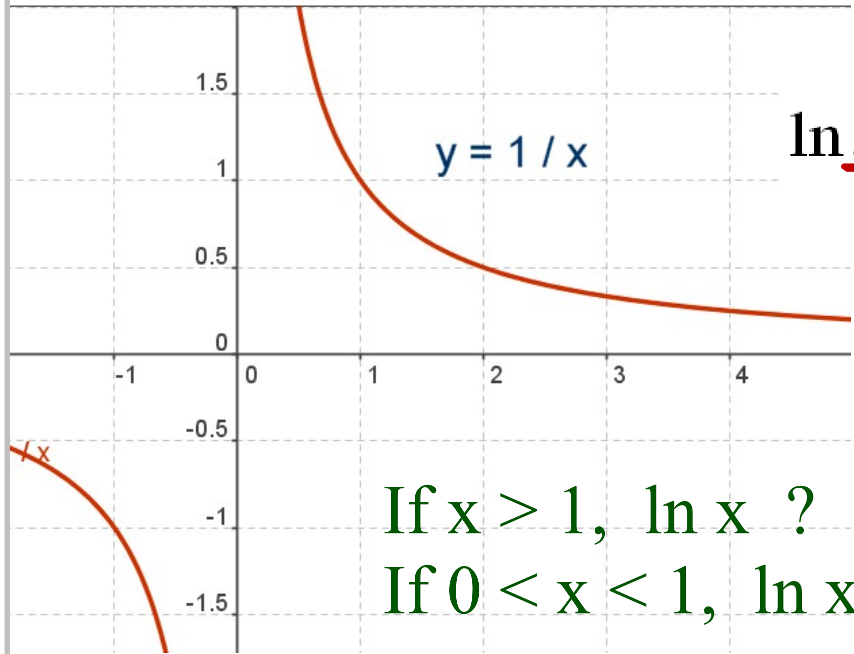
What is its domain? $x > 0$

What is its range? all reals

What does graph look like?

Need to investigate y = 1/x

5.1 The Natural Logarithm

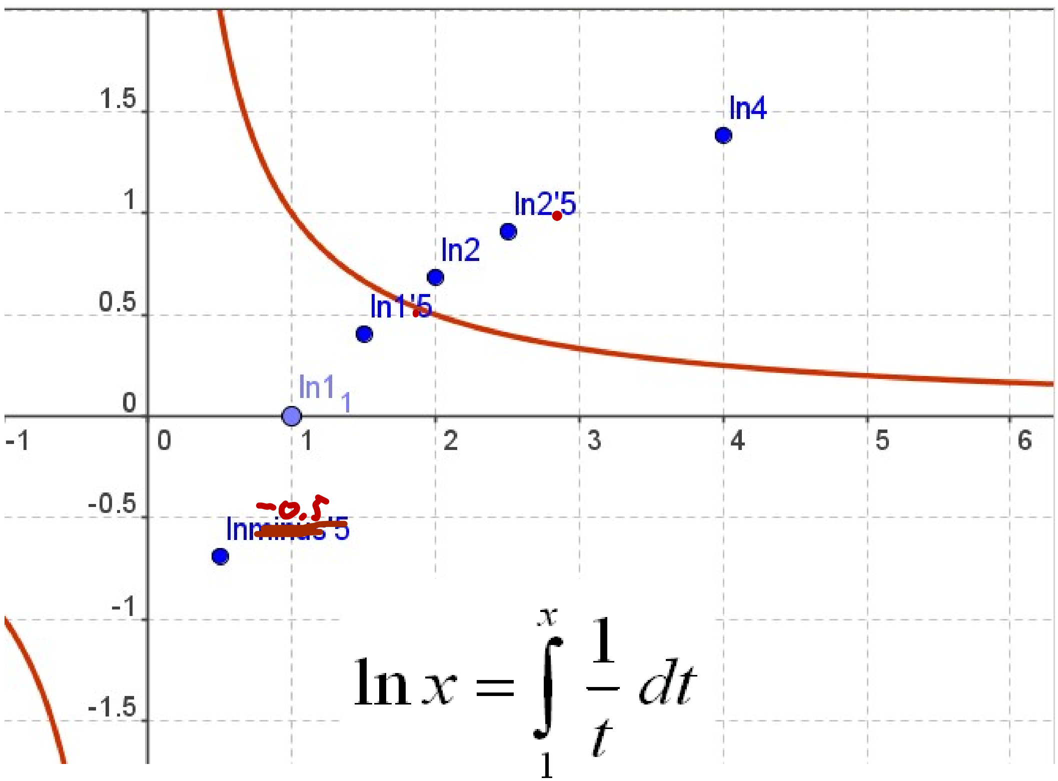


$$\ln x = \int_1^x \frac{1}{t}\,dt$$

If x > 1, ln x ? 0
If 0 < x < 1, ln x ? 0

Because of Vertical Asymptote,
limit domain to x > 0

5.1 The Natural Logarithm



$$\ln x = \int_1^x \frac{1}{t}\,dt$$

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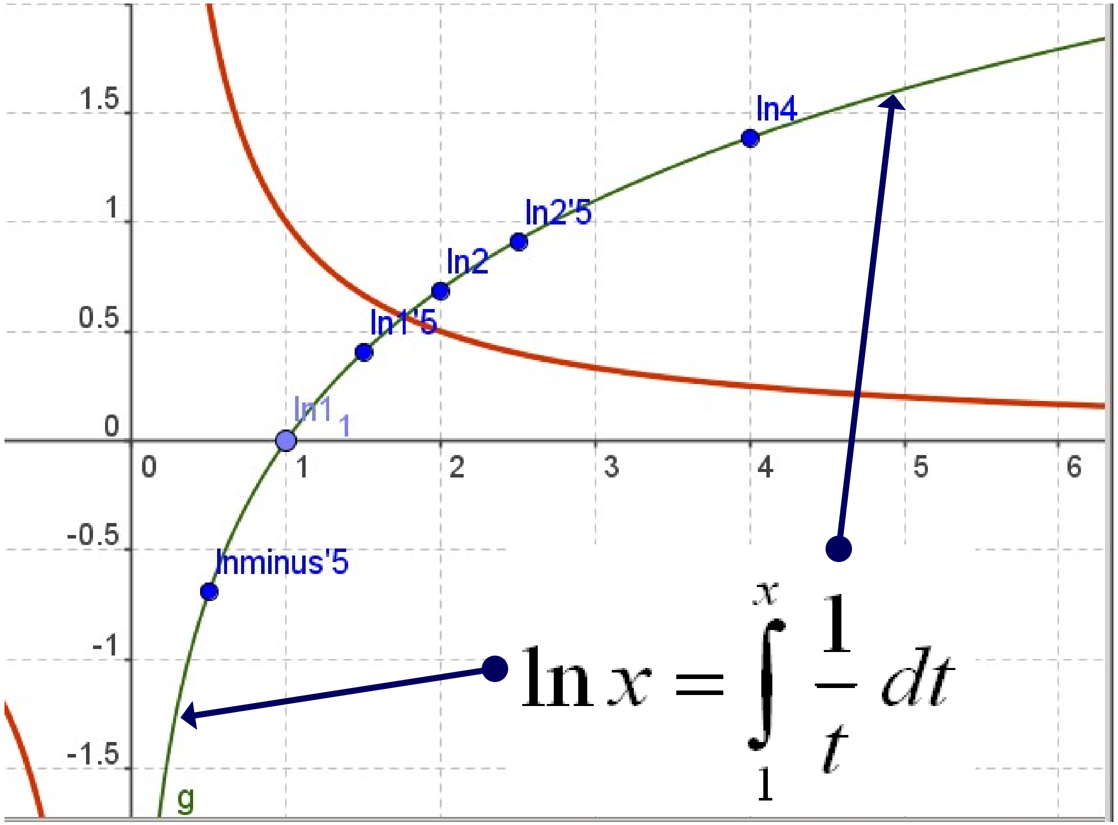
Homework Part 1

Homework Part 2

5.1 The Natural Logarithm



$$\ln x = \int_1^x \frac{1}{t}\,dt$$

5.1 The Natural Logarithm

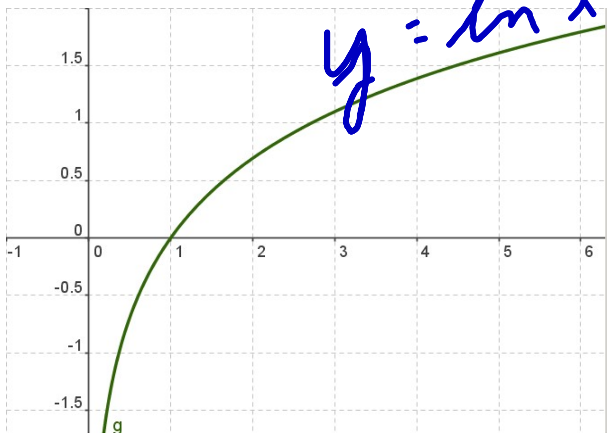
Properties of y = ln *x*

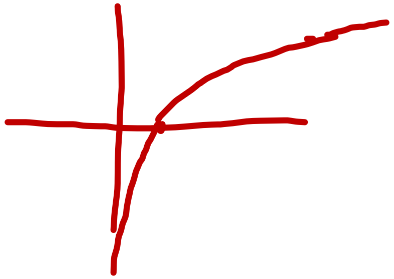
1. Domain: $x > 0$ Range: all y

2. continuous, increasing over all domain

3. concave down

$y = \ln x$





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Homework Part 1

Homework Part 2

22. $\ln(xyz) = \ln x + \ln y + \ln z$

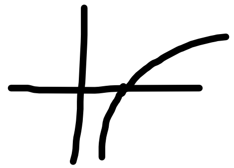
$$\ln \sqrt{xy} = \frac{1}{2}(\ln xy)$$

$$= \frac{1}{2}[\ln x + \ln y]$$

26. $\ln(3e^2) = \ln 3 + \ln e^2$

$= \ln 3 + 2\underline{\ln_e} e$

$= \ln 3 + 2$



Study 5.1 p. 329 # 7, 9, 11, $\overline{\underline{15}}$, 19–27, 29–33, 37, 39, 47, 51–57, 41, 63–67, 77

30. G: $3\ln x + 2\ln y - 4\ln z$

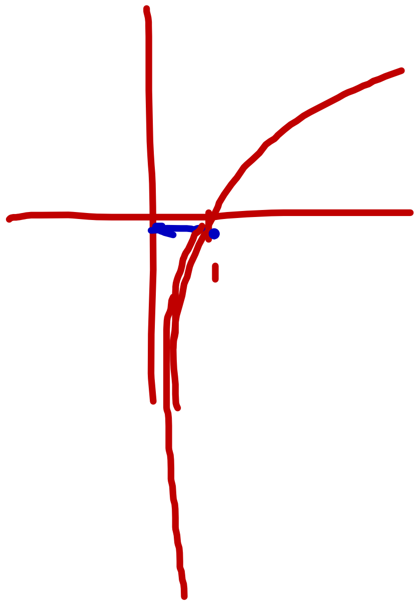
$\ln x^3 + \ln y^2 - \ln z^4$

$\ln \dfrac{x^3 y^2}{z^4}$

(6-x) approaches 0 from the right

38. $\lim_{x \to 6^-} \ln(\underline{6-x}) \to -\infty$

X is approaching 6
from the left



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$$\ln x = \int_1^x \frac{1}{t}\,dt$$

$u = f(x)$

$$\frac{d(\ln x)}{dx} = \frac{1}{x}$$

$$\boxed{\frac{d(\ln u)}{dx} = \frac{1}{u}\frac{du}{dx}}$$

$$= \frac{u'}{u}$$

Title: derivative of ln x (17 of 29)

$y = \ln x \qquad \frac{dy}{dx} = \frac{1}{x}$

$y = \ln 2x$

$= \underline{\ln 2} + \ln x$

$\frac{dy}{dx} = \frac{1}{x}$

$\frac{dy}{dx} = \frac{1}{2x} \cdot 2 = \frac{2}{2x} = \frac{1}{x}$

G: $h(x) = \ln(2x^2+1)$ F: $h'(x)$

$u = 2x^2+1 \qquad \frac{du}{dx} = 4x$

$$h'(x) = \frac{1}{u}\frac{du}{dx} = \frac{1}{2x^2+1} \cdot 4x$$

$$= \frac{4x}{2x^2+1}$$

48: G: $y = x\ln x$ F: dy/dx

$$\frac{dy}{dx} = x\cdot\frac{1}{x} + \ln x = 1 + \ln x$$

50. G: $y = \ln\sqrt{x^2-4}$ F: dy/dx

$$y = \ln(x^2-4)^{1/2} = \frac{1}{2}\ln(\overset{u}{x^2-4})$$

$u = x^2-4$

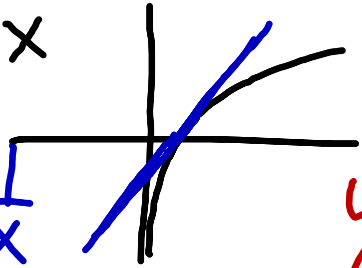
$\frac{du}{dx} = 2x$

$$\frac{dy}{dx} = \frac{1}{2}\left[\frac{1}{u}\frac{du}{dx}\right] = \frac{1}{2}\left[\frac{1}{x^2-4}\cdot 2x\right]$$

$$= \frac{x}{x^2-4}$$

42. G: $y = \ln x^{3/2}$ F: eq. tang. $(1,0)$

$y = \frac{3}{2} \ln x$



$m = \frac{dy}{dx} = \frac{3}{2} \cdot \frac{1}{x}$

at $(1,0) = \frac{3}{2} \cdot \frac{1}{1} = \frac{3}{2}$

$y = mx + b$

$y - y_0 = m_T (x - x_0)$

$y - 0 = \frac{3}{2}(x - 1)$

$y = \frac{3}{2}x - \frac{3}{2}$

Point slope form of straight line

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$y_2 - y_1 = m(x_2 - x_1)$

$y - y_1 = m(x - x_1)$

5.1 The Natural Logarithm

The Derivative of the Natural Logarithm of the Absolute Value

$$\frac{d\left[\ln|u|\right]}{dx} = \frac{du/dx}{u}$$

since:

$$|u| = \begin{cases} u, & \text{if } u \geq 0 \\ -u, & \text{if } u < 0 \end{cases}$$

and:

$$\ln|u| = \begin{cases} \ln u, & \text{if } u > 0 \\ \ln(-u), & \text{if } u < 0 \end{cases}$$

Note: u $\neq 0$

25 $\ln\left(\frac{x^2-1}{x^3}\right)^3$

$3\ln\left(\frac{x^2-1}{x^3}\right)$

$= 3\left[\ln(x^2-1) - \ln x^3\right]$

$= 3\left[\ln(x^2-1) - 3\ln x\right]$

$= 3\left[\ln[(x+1)(x-1)] - 3\ln x\right]$

$= 3\left[\ln(x+1) + \ln(x-1) - 3\ln x\right]$

29 $2\ln(x-2) - \ln(x+2)$

$\cancel{\ln\left(\frac{x-2}{x+2}\right)}$

$\ln(x-2)^2 -$

$a^m a^n$

$\frac{a^m}{a^n}$

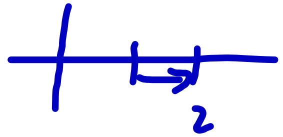
53. G: $g(t) = \dfrac{\ln t}{t^2}$ F: $g'(t)$

$$g'(t) = \frac{t^2 \frac{d(\ln t)}{dt} - (\ln t)(2t)}{t^4}$$

$$= \frac{t^2 \cdot \frac{1}{t} - 2t\ln t}{t^4} = \frac{t - 2t\ln t}{t^4}$$

$$= \frac{t(1-2\ln t)}{t^4} = \frac{1-2\ln t}{t^3}$$

$$\lim_{x \to 2^-} \ln\left[x^2(3-x)\right]$$



$$\longrightarrow \underline{\ln\left[4(1)\right]} = \ln 4$$

57. $y = \ln\sqrt{\dfrac{x+1}{x-1}}$ F: dy/dx

$$= \ln\left(\frac{x+1}{x-1}\right)^{1/2} = \frac{1}{2}\ln\left(\frac{x+1}{x-1}\right)$$

$$= \frac{1}{2}\left[\ln(x+1) - \ln(x-1)\right]$$

$$\frac{dy}{dx} = \frac{1}{2}\left[\frac{1}{x+1} - \frac{1}{x-1}\right] = \frac{1}{2}\left[\frac{x-1-(x+1)}{(x+1)(x-1)}\right]$$

$$= \frac{1}{2}\left[\frac{x-1-x-1}{(x+1)(x-1)}\right] = \frac{1}{2}\left[\frac{-2}{(x+1)(x-1)}\right]$$

$$= \frac{-1}{(x+1)(x-1)} = \frac{1}{1-x^2}$$

77. G: $x^2 - 3\ln y + y^2 = 10$ F: dy/dx

$$2x - 3 \cdot \frac{1}{y} \cdot \frac{dy}{dx} + 2y\frac{dy}{dx} = 0$$

$$\left[\frac{-3}{y} + 2y\right]\frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = \frac{-2x}{\frac{-3}{y} + 2y} \cdot \frac{y}{y} = \frac{-2xy}{-3 + 2y^2}$$

$$= \frac{2xy}{3 - 2y^2}$$

G: $y = \ln x^3 = 3\ln x$

$$\frac{dy}{dx} = 3\,\frac{1}{x} = \frac{3}{\cancel{x}}$$

$$y - 0 = \frac{3}{1}(x-1)$$

$$y = 3x - 3$$

F: eq. tang. line at $(1,0)$

$$y = mx + b$$

$$\boxed{y - y_1 = m_T (x - x_1)}$$

g'

67. G: $y = \ln\left|\dfrac{-1+\sin x}{2+\sin x}\right|$ F: $\dfrac{dy}{dx}$

$$= \ln|-1+\sin x| - \ln|2+\sin x|$$

$$\frac{dy}{dx} = \frac{1}{-1+\sin x}\cdot \cos x - \frac{1}{2+\sin x}\cdot \cos x$$

$$= \cos x\left[\frac{1}{-1+\sin x} - \frac{1}{2+\sin x}\right]$$

LCD = $(-1+\sin x)(2+\sin x)$

$$= \cos x\left[\frac{1}{-1+\sin x}\cdot\frac{2+\sin x}{2+\sin x} - \frac{1}{2+\sin x}\cdot\frac{-1+\sin x}{-1+\sin x}\right]$$

$$= \cos x\left[\frac{2+\sin x - (-1+\sin x)}{(-1+\sin x)(2+\sin x)}\right]$$

$$= \cos x\left[\frac{2+\cancel{\sin x}+1-\cancel{\sin x}}{(-1+\sin x)(2+\sin x)}\right] = \boxed{\frac{3\cos x}{(-1+\sin x)(2+\sin x)}}$$

$-2+\sin x+\sin^2 x$