Integration by Substitution

$$\int \frac{x^2}{(1+x^3)^2}\,dx$$

Not division by monomial.
Cannot easily simplify.......
Can this problem be converted to one that fits the power rule?

$$\boxed{\int u^n\,du = \frac{u^{n+1}}{n+1} + C,\quad n \neq -1}$$

Let $u = 1+x^3$

$du = 3x^2\,dx$

3 is not a factor of the integrand:
Must complete the *du*, using the multiplication property of 1.
Multiply by $\frac{3}{3}$

$$\frac{1}{3}\int \frac{3x^2}{(1+x^3)^2}\,dx$$

$$= \frac{1}{3}\int \frac{1}{u^2}\,du$$

$$= \frac{1}{3}\int u^{-2}\,du = \frac{1}{3}\,\frac{u^{-1}}{-1} + C = -\frac{1}{3u} + C = -\frac{1}{3(1+x^3)} + C$$

9. $\int \sqrt{9-x^2}\,(-2x)\,dx$

$u = 9-x^2$

$du = -2x\,dx$

$= \int (9-x^2)^{1/2}(-2x)\,dx$

$= \int u^{1/2}\,du = \frac{2}{3}u^{3/2} + c = \frac{2}{3}(9-x^2)^{3/2} + c$

Check: Derivative of the answer should be the original integrand.

$y = \frac{2}{3}(9-x^2)^{3/2} + c$

$\frac{dy}{dx} = \frac{2}{3} \cdot \left[\frac{3}{2}(9-x^2)^{1/2}(-2x)\right] = -2x(9-x^2)^{\frac{1}{2}}$ ✓

25. $-\int \left(1+\frac{1}{t}\right)^3 \left(\frac{-1}{t^2}\right) dt$

$u = 1 + t^{-1}$

$du = -t^{-2}\,dt$

$= \frac{-1}{t^2}\,dt$

$= -\int u^3\,du$

$= -\frac{u^4}{4} + c = -\frac{1}{4}\left(1+\frac{1}{t}\right)^4 + c$

$= -\frac{1}{4}\left(1+\frac{1}{t}\right)^4 + c$

11. $\int 5x \sqrt[3]{1-x^2}\, dx$

$u = 1-x^2$

$du = -2x\, dx$

$= \frac{5}{-2} \int (1-x^2)^{1/3} (-2)x\, dx$

$= -\frac{5}{2} \int u^{1/3}\, du = -\frac{5}{2} \frac{u^{4/3}}{\frac{4}{3}} + c$

$= -\frac{15}{8} (1-x^2)^{4/3} + c$

28. $\int \frac{1}{2\sqrt{x}}\,dx = \frac{1}{2}\int x^{-1/2}\,dx$ ....

30. $\int \frac{t+2t^2}{\sqrt{t}}\,dt = \int\left(\frac{t}{t^{1/2}} + \frac{2t^2}{t^{1/2}}\right)dt$

$= \int \left(t^{1/2} + 2t^{3/2}\right)dt$

$= \frac{2}{3}t^{3/2} + \frac{2\cdot 2}{5}t^{5/2} + c$

$= \frac{2}{3}t^{3/2} + \frac{4}{5}t^{5/2} + c$

chk: $\frac{2}{3}\cdot\frac{3}{2}t^{1/2} + \frac{4}{5}\cdot\frac{5}{2}t^{3/2} = t^{1/2} + 2t^{3/2}$ ✓

$\int x^{11/12}\,dx$

$$\int \frac{x^2+3x+7}{\sqrt{x}}\,dx$$

$$k\int x^n\,du = \frac{kx^{n+1}}{n+1}+c$$

$$\int \frac{x^2+3x+7}{x^{1/2}}\,dx$$

$$\boxed{\frac{a+b+c}{d} = \frac{a}{d}+\frac{b}{d}+\frac{c}{d}}$$

$$\int \left[\frac{x^2}{x^{1/2}}+\frac{3x}{x^{1/2}}+\frac{7}{x^{1/2}}\right]dx$$

$$= \int \left(x^{3/2}+3x^{1/2}+7x^{-1/2}\right)dx$$

Some problems do not require substitution.

$$= \frac{x^{3/2+1}}{\frac{3}{2}+1} + 3\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + 7\frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + c$$

$$= \frac{2}{5}x^{5/2} + 3\left(\frac{2}{3}\right)x^{3/2} + 7(2)x^{1/2} + c$$

$$= \frac{2}{5}x^{5/2} + 2x^{3/2} + 14x^{1/2} + c$$

G: $\frac{dy}{dx} = 4x + \frac{4x}{\sqrt{16-x^2}}$

F: solve

Or: find y

$dy = \frac{dy}{dx}\, dx$

Given the differential equation, we use the definition of the differential dy.

$y = \int dy = \int \left(4x + \frac{4x}{\sqrt{16-x^2}}\right) dx$

$= 4\int x\,dx + \frac{4}{-2}\int -2x(16-x^2)^{-1/2}\,dx$

$u = 16-x^2$

$du = -2x\,dx$

$= \frac{4x^2}{2} - 2\int u^{-1/2}\,du$

$= 2x^2 - 2\frac{u^{1/2}}{\frac{1}{2}} + C$

$= 2x^2 - 4\sqrt{16-x^2} + C$

Chk: $4x - \frac{4}{2}(16-x^2)^{-1/2}(-2x)$

$4x + \frac{4x}{\sqrt{16-x^2}}$ ✓

Title: example 6 (7 of 12)

54. $\displaystyle\int \frac{\sin x}{\cos^3 x}\,dx$

$\cancel{u = \sin x}$

$\cancel{du = \cos x\,dx}$

$\cancel{dx = \frac{du}{\cos x}}$

$\displaystyle -- \int \cancel{\frac{u}{\cos^3 x}\,\frac{du}{\cos x}}$

Sometimes the first choice for u does not work. Try another, different choice.

$\sin x\,(\cos^3 x)^{-1}$

$\cancel{u = (\cos x)^3}$

$\cancel{du = -3(\cos x)^{-4}(-\sin x)\,dx}$

$u = \cos x$

$du = -\sin x\,dx$

$$-\int \frac{-\sin x}{\cos^3 x}\,dx$$

$$= -\int (\cos x)^{-3}\,(-\sin x)\,dx = -\int u^{-3}\,du = -\frac{u^{-2}}{-2} + C$$

$$= +\frac{1}{2(\cos x)^2} + C$$

$$\int \sqrt{\sin x}\ \cos x\,dx$$

$u = \sin x$

$du = \cos x\,dx$

$$= \frac{2}{3}(\sin x)^{3/2} + C$$

$$-\int \frac{-\sin x}{\sqrt{\cos x}}\,dx$$

$u = \cos x$

$du = -\sin x\,dx$

$$= -2\sqrt{\cos x} + C$$

$$\int \sin^2 x\ \cos x\,dx$$

$u = \sin x$

$du = \cos x\,dx$

$$= \int u^2\,du$$

$$= \frac{1}{3}\sin^3 x + C$$

4.5 Integration by Substitution: Examples

$$\int \sqrt{\sin x}\ \cos^2 x\, dx$$

$u = \cos x$

$du = -\sin x\, dx$

$$\cancel{\int u^2 \frac{du}{-\sin x}}$$

$u = \sin x$

$du = \cos x\, dx$

$$\cancel{\int u^{1/2} \cos x\, du}$$

Sometimes the given integrand is simply not integrable by the methods we know so far.

64. $\frac{1}{2}\int x\sqrt{2x+1}\,(2)dx$

$u = 2x+1$

$du = 2\,dx$

$u-1 = 2x$

$x = \frac{u-1}{2}$

$\frac{1}{2}\int x\, u^{1/2}\, du$

$\frac{1}{4}\int (u-1)u^{1/2}\,du = \frac{1}{4}\int \left(u^{3/2} - u^{1/2}\right)du$

$= \frac{1}{4}\cdot\frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2} + C$

$= \frac{1}{10}(2x+1)^{5/2} - \frac{2}{3}(2x+1)^{3/2} + C$

58. G: $f'(x) = \pi \sec \pi x \tan \pi x$ F: eq. f

$\left(\frac{1}{3}, 1\right)$

$$\int \pi \sec \pi x \tan \pi x \, dx$$

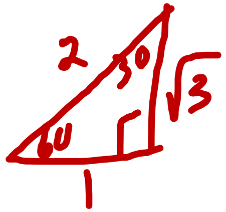
$u = \pi x$

$du = \pi \, dx$

$$\int \sec u \tan u \, du = \sec u + c$$

$f(x) = \sec \pi x + c$

$1 = \sec \frac{\pi}{3} + c = \frac{1}{\cos \frac{\pi}{3}} + c$



$1 = 2 + c$

$\therefore c = -1$

$$f(x) = \sec \pi x - 1$$