4.5 Integration by Substitution

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4.5 Integration by Substitution

$$y = (x^2+2)^{10} \qquad F: \frac{dy}{dx} = 10(x^2+2)^9(2x)$$

Remember the Chain Rule

Complete the table

$y=(2x+1)^3$	$\frac{dy}{dx} =$
$y=\sin(x^2)$	$\frac{dy}{dx} =$
$y=$	$\frac{dy}{dx} = 2\cos(2x+1)$
$y=\tan(x^2)$	$\frac{dy}{dx} =$
$y=$	$\frac{dy}{dx} = 3x^2\sec^2(x^3)$

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$y=(2x+1)^3$	$\frac{dy}{dx} = 6(2x+1)^2$
$y=\sin(x^2)$	$\frac{dy}{dx} = 2x\cos(x^2)$
$y=$	$\frac{dy}{dx} = 2\cos(2x+1)$
$y=\tan(x^2)$	$\frac{dy}{dx} = 2x\sec^2(x^2)$
$y=$	$\frac{dy}{dx} = 3x^2\sec^2(x^3)$

1. All derivatives here use the Chain Rule to find the derivative of composite functions.

$y=(2x+1)^3$	$\frac{dy}{dx} = 6(2x+1)^2$
$y=\sin(x^2)$	$\frac{dy}{dx} = \underline{\underline{2x}}\,\underline{\cos(x^2)}$
$y=\sin(2x+1)+c$	$\frac{dy}{dx} = \underline{\underline{2}}\,\underline{\cos(2x+1)}$
$y=\tan(x^2)$	$\frac{dy}{dx} = \underline{\underline{2x}}\,\underline{\sec^2(x^2)}$
$y=\tan(x^3)+c$	$\frac{dy}{dx} = \underline{\underline{3x^2}}\,\underline{\sec^2(x^3)}$

Derivative of the primary function.

Derivative of the nested function.

2. To find the integrals of functions that are the derivatives of composite functions, **the integrand requires the presence of the derivative of the nested function as a factor.** This is the reverse process of the Chain Rule.

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2. $\int x^2 \sqrt{x^3+1}\, dx$

Think:
1. This integrand is the result of the chain rule.
2. How do I reverse the operation of the chain rule?

$$\boxed{\begin{array}{c}\underline{\text{Chain Rule}} \\ \dfrac{dy}{dx} = \dfrac{dy}{du} \cdot \dfrac{du}{dx}\end{array}}$$

The chain rule involves multiplication.

1. Look for factors of the integrand.
2. Find the factors such that one factor is the derivative of the other factor.
3. Identify the factor u.
4. Is the remaining portion of the integrand the correct du?
5. If not, can you correct it by scalar multiplication?

$u =$

$du =$

2. $\int ?\,x^2\sqrt{x^3+1}\,dx$ $\qquad u = x^3+1\ ;\ du = 3x^2\,dx$

du is not complete

Use the multiplication property of one:
multiply by 3/3

$$2.\ \frac{1}{3}\int 3x^2\sqrt{x^3+1}\,dx \qquad u = x^3+1\ ;\ du = 3x^2\,dx$$

$$= \frac{1}{3}\int u^{1/2}\,du = \frac{1}{3}\,\frac{u^{3/2}}{3/2} + C$$

$$= \frac{2}{9}\left(x^3+1\right)^{3/2} + C$$

4. $\int \sec 2x \tan 2x \, dx$

4. $\frac{1}{2}\int \sec 2x \tan 2x \, 2\,dx$ $\qquad u = 2x$; $du = 2\,dx$

$= \frac{1}{2}\int \sec u \tan u \, du = \frac{1}{2}\sec u + c = \frac{1}{2}\sec 2x + c$

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