4.4 Fundamental Theorem of Calculus

Study 4.4 #1-23, 27-35,39-49, 51, 55, 75-87,91*

Goals:
1. Recognize and understand the Fundamental Theorem of Calculus.
2. Use the Fundamental Theorum of Calculus to evaluate Definite Integrals.
3. Recognize and understand the Mean Value Theorem for Integrals.
4. Find the average value of a function on [a,b].
5. Understand the significance of the Second Fundamental Theorem of Calculus.

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4.4 Fundamental Theorem of Calculus

Start with Indefinite Integration: complete the following

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$$

1. $\int (x^{3/2} + 2x - 1)\,dx =$

2. $\int (x+1)(3x-2)\,dx = \int (3x^2 + x - 2)\,dx$

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4.4 Fundamental Theorem of Calculus

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$$

Start with Indefinite Integration: complete the following

1. $$\int \left(x^{3/2} + 2x - 1\right)dx = \frac{x^{5/2}}{\frac{5}{2}} + \frac{2x^2}{2} - x + C$$

$$= \frac{2}{5}x^{5/2} + x^2 - x + C$$

2. $$\int (x+1)(3x-2)\,dx = \int \left(3x^2 + x - 2\right)dx$$

$$x^3 + \frac{x^2}{2} - 2x + C$$

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4.4 Fundamental Theorem of Calculus

G: $\frac{dy}{dx} = 3x^2$ F: solve for y

$$dy = \frac{dy}{dx}\,dx$$

Start with the differential of y dy

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4.4 Fundamental Theorem of Calculus

G: $\frac{dy}{dx} = 3x^2$

F: solve for y

$$dy = \boxed{\frac{dy}{dx}}\, dx$$

$$\int dy = \int 3x^2\, dx$$

$$y = \frac{3x^3}{3} + c = x^3 + c$$

Start with the differential of y*dy*

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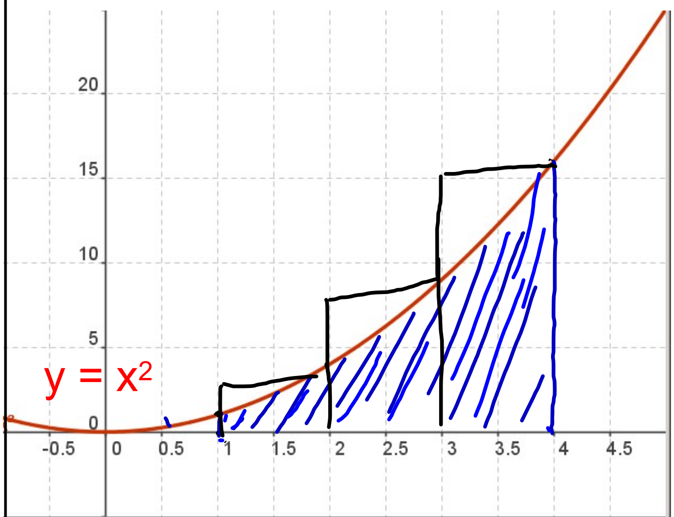
4.4 Fundamental Theorem of Calculus

Problem: Find the area under $y = x^2$ from x=1 to x=4



Estimates:

Sum of rectangles:
1(4) + 1(9) + 1(16)
= 29

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4.4 Fundamental Theorem of Calculus

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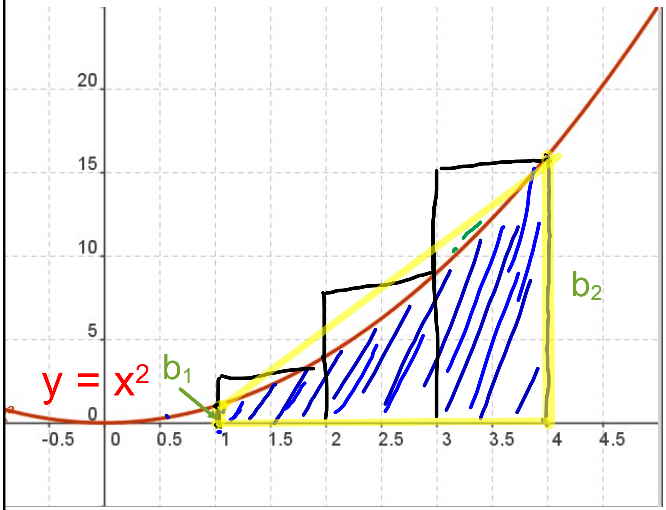


Estimates:

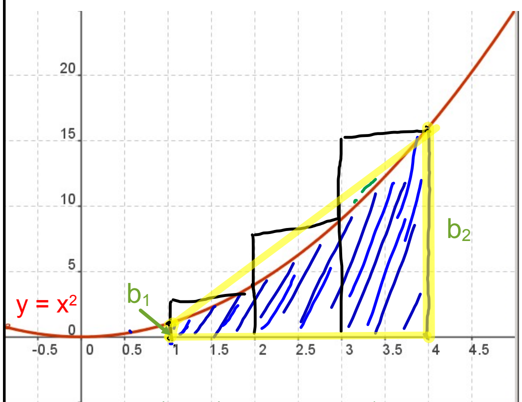
Sum of rectangles:
$1(4) + 1(9) + 1(16) = 29$

a single trapazoid:
$(1/2)(b_1+b_2)h = (1/2)(1+16)(3) = 25.5$

Use multiple trapazoids for better estimate.

4.4 Fundamental Theorem of Calculus

Problem: Find the area under $y = x^2$ from $x=1$ to $x=4$



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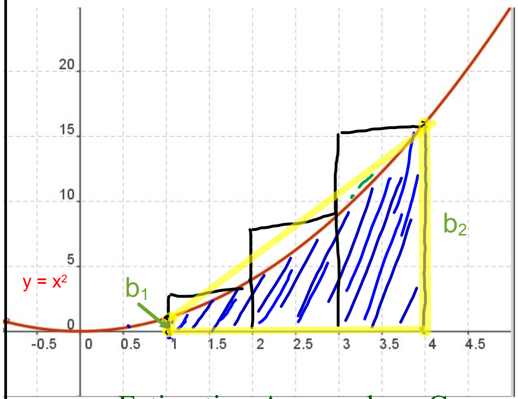
Use multiple trapazoids for better estimate.

$$\lim_{||\Delta||\to 0} \sum_{i=1}^{n} f(c_i)\,\Delta x_i = \int_a^b f(x)\,dx$$

Estimating Area under a Curve, since y is non-negative.
Also estimating the definite integral.
Therefore, use the definite integral $\int_1^4 x^2\,dx$

4.4 Fundamental Theorem of Calculus

Problem: Find the area under $y = x^2$ from x=1 to x=4



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25.5

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Estimating Area under a Curve, since y is non-negative.
Also estimating the definite integral.
Therefore, use the definite integral $\int_1^4 x^2\,dx$

How do we use the definite integral to actually compute the area?

4.4 Fundamental Theorem of Calculus

Fundamental Theorem of Calculus (FTC)

If :

1. a function ***f* is continuous on [a, b]** and

2. ***F* is an antiderivative of f** on the interval,

then:

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

The integral of *f* from *a* to *b* is the difference:
(antiderivative of *f* evaluated at *x=b*) - (antiderivative of *f* evaluated at *x=a*.)

4.4 Fundamental Theorem of Calculus

$$\int_1^4 x^2\,dx = ?$$

Fundamental Theorem of Calculus (FTC)

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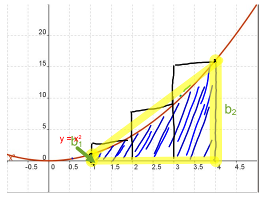

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4.4 Fundamental Theorem of Calculus

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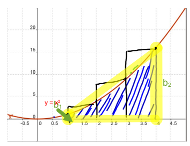

$$\int_1^4 x^2\,dx = \frac{x^3}{3} + c\,\Big]_1^4$$

Do not need *c*, the constant of integration. It gets added and subtracted to add to 0.

$$= \left[\frac{4^3}{3}+c\right] - \left[\frac{1^3}{3}+c\right]$$

Since y is non-negative, the **area = 21 sq. units**

$$= \frac{4^3}{3} + c - \frac{1}{3} - c = \frac{63}{3} = 21$$

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4.4 Fundamental Theorem of Calculus

G: $\int_2^7 3\,dx$ F: Evaluate

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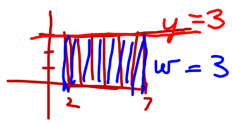
4.4 Fundamental Theorem of Calculus

G: $\int_2^7 3\,dx$ F: Evaluate

$= 3x\Big]_2^7$

$= 3(7) - 3(2)$

$= 21 - 6 = 15$



$l = 7 - 2 = 5$

$A = 15$

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4.4 Fundamental Theorem of Calculus

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

10. $\int_1^3 (3x^2 + 5x - 4)\,dx$

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4.4 Fundamental Theorem of Calculus

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

10. $\int_1^3 (3x^2 + 5x - 4)\,dx$

$$= \frac{3x^3}{3} + \frac{5x^2}{2} - 4x\Big]_1^3$$

$$= x^3 + \frac{5}{2}x^2 - 4x\Big]_1^3$$

$F(b) - F(a)$
$= F(3) - F(1)$

$$= 3^3 + \frac{5}{2}\cdot 3^2 - 4(3) - \left[1 + \frac{5}{2} - 4\right]$$

$$= 27 + \frac{45}{2} - 12 - 1 - \frac{5}{2} + 4 \quad = 27 + 4 - 13 + \frac{45-5}{2}$$

$$= 31 - 13 + \frac{40}{2} = 18 + 20 = \boxed{38}$$

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4.4 Fundamental Theorem of Calculus

$$\int_1^8 \sqrt{\frac{2}{x}}\,dx =$$

4.4 Fundamental Theorem of Calculus

$$\sqrt{\frac{b}{a}} = \frac{\sqrt{b}}{\sqrt{a}}$$

$$\frac{\sqrt{2}}{\sqrt{x}} = \frac{\sqrt{2}}{x^{1/2}}$$

$$\int_1^8 \sqrt{\frac{2}{x}}\,dx = \int_1^8 \sqrt{2}\,x^{-1/2}\,dx$$

$$= \sqrt{2}\int_1^8 x^{-1/2}\,dx$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$$

$$= \sqrt{2}\left.\frac{x^{1/2}}{\frac{1}{2}}\right]_1^8 = \left.2\sqrt{2}\sqrt{x}\right]_1^8$$

$$F(b) - F(a) = F(8) - F(1)$$

$$= 2\sqrt{2}\sqrt{8} - 2\sqrt{2}\sqrt{1}$$

$$= 2\sqrt{2}\,2\sqrt{2} - 2\sqrt{2}$$

$$= 4\cdot 2 - 2\sqrt{2}$$

$$= 8 - 2\sqrt{2}$$

~ 5.17

4.4 Fundamental Theorem of Calculus

$$\int_{-8}^{-1} \frac{x - x^2}{2\sqrt[3]{x}}\,dx =$$

4.4 Fundamental Theorem of Calculus

$$\int_{-8}^{-1} \frac{x - x^2}{2\sqrt[3]{x}}\,dx = \frac{1}{2}\int_{-8}^{-1} \frac{x - x^2}{x^{1/3}}\,dx$$

$$= \frac{1}{2}\int_{-8}^{-1} \left(x^{2/3} - x^{5/3}\right) dx = \frac{1}{2}\left[\frac{x^{5/3}}{\frac{5}{3}} - \frac{x^{8/3}}{\frac{8}{3}}\right]_{-8}^{-1}$$

$$= \frac{1}{2}\left[\frac{3}{5}x^{5/3} - \frac{3}{8}x^{8/3}\right]_{-8}^{-1}$$

$$= \frac{1}{2}\left[\left[\frac{3}{5}(-1)^{5/3} - \frac{3}{8}(-1)^{8/3}\right] - \left[\frac{3}{5}(-8)^{5/3} - \frac{3}{8}(-8)^{8/3}\right]\right]$$

Can use calculator after substitution.

$$= \frac{1}{2}\left[\left[\frac{3}{5}(-1) - \frac{3}{8}(1)\right] - \left[\frac{3}{5}(-32) - \frac{3}{8}(+256)\right]\right]$$

$$= \frac{1}{2}\left[\left[-\frac{3}{5} - \frac{3}{8}\right] - \left[-\frac{96}{5} - 96\right]\right] = \frac{1}{2}\left[-\frac{3}{5} + \frac{96}{5} - \frac{3}{8} + 96\right]$$

$$= \frac{1}{2}\left(\frac{93}{5} - \frac{3}{8} + 96\right) = \frac{1}{2}\left(\frac{744 - 15}{40} + 96\right)$$

$$= \frac{1}{2} \cdot 114.225 = 57.1125$$

$\frac{729}{40} = 18.225$

4.4 Fundamental Theorem of Calculus

$$\int_{-2}^{-1} \left(u - \frac{1}{u^2}\right) du$$

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4.4 Fundamental Theorem of Calculus

$$\int_{-2}^{-1} \left(u - \frac{1}{u^2}\right) du$$

$$= \int_{-2}^{-1} (u - u^{-2})\, du = \frac{u^2}{2} - \frac{u^{-1}}{-1}\Bigg]_{-2}^{-1}$$

$$= \frac{u^2}{2} + \frac{1}{u}\Bigg]_{-2}^{-1}$$

$$= \frac{(-1)^2}{2} + \frac{1}{-1} - \left[\frac{(-2)^2}{2} + \frac{1}{-2}\right]$$

$$= \frac{1}{2} - 1 - \left[\frac{4}{2} - \frac{1}{2}\right] = -\frac{1}{2} - \left[\frac{3}{2}\right] = -\frac{1}{2} - \frac{3}{2} = \frac{-4}{2} = -2$$

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4.4 Fundamental Theorem of Calculus

$$\int_0^{\pi/4} (\sec^2\theta - \sin\theta)\, d\theta$$

$f(x)$	$f'(x)$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\csc^2 x$
$\csc x$	$-\csc x \cot x$

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4.4 Fundamental Theorem of Calculus

$$\int_0^{\pi/4} (\sec^2\theta - \sin\theta)\, d\theta$$

$$= \tan\theta + \cos\theta \Big]_0^{\pi/4}$$

$$= \left[1 + \frac{1}{\sqrt{2}}\right] - \left[0 + 1\right]$$

$$= \frac{1}{\sqrt{2}}$$

$f(x)$	$f'(x)$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\csc^2 x$
$\csc x$	$-\csc x \cot x$

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4.4 Fundamental Theorem of Calculus

$$\int_1^4 (3-|x-3|)\,dx$$

Presents some problems.
1. Start with definiton of absolute value and
2. consider what this means regarding the interval from lower to upper limits.

Step #1: Absolute value:

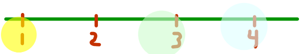
Step #2: About the interval

4.4 Fundamental Theorem of Calculus

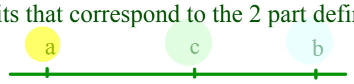
$$\int_1^4 (3-|x-3|)\,dx \qquad |a| = \begin{cases} a, & a \geq 0 \\ -a, & a < 0 \end{cases}$$

$$|x-3| = \begin{cases} x-3, & x-3 \geq 0, x \geq 3 \\ -(x-3), & x-3<0, x<3 \end{cases}$$

$-x+3$



The integrand is defined differently on the interval.
Since definite integrals are defined as limits of sums, we replace the original integral with the sum of 2 integrals which have integrands and limits that correspond to the 2 part definition of the original:



$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$

Rewrite the original integral as the sum of 2 integrals.

4.4 Fundamental Theorem of Calculus

$$\int_1^4 (3-|x-3|)\,dx$$

$$|a| = \begin{cases} a, & a \ge 0 \\ -a, & a < 0 \end{cases}$$

$$|x-3| = \begin{cases} x-3, & x-3 \ge 0, x \ge 3 \\ -(x-3), & x-3<0, x<3 \end{cases}$$

$-x+3$

1 2 3 4

$$\int_1^4 (3-|x-3|)\,dx = \int_1^3 [3-(3-x)]\,dx + \int_3^4 [3-(x-3)]\,dx$$

a c b

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$

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4.4 Fundamental Theorem of Calculus

$$|a| = \begin{cases} a, & a \ge 0 \\ -a, & a < 0 \end{cases}$$

$$\int_1^4 (3-|x-3|)\,dx$$

$$|x-3| = \begin{cases} x-3, & x-3 \ge 0, x \ge 3 \\ -(x-3), & x-3<0, x<3 \end{cases}$$

$-x+3$

$$\int_1^4 (3-|x-3|)\,dx = \int_1^3 [3-(3-x)]\,dx + \int_3^4 [3-(x-3)]\,dx$$

$$= \int_1^3 x\,dx + \int_3^4 (6-x)\,dx$$

$$= \left[\frac{x^2}{2}\right]_1^3 + \left[6x - \frac{x^2}{2}\right]_3^4$$

$$= \left[\frac{9}{2} - \frac{1}{2}\right] + \left[24 - \frac{16}{2} - \left(18 - \frac{9}{2}\right)\right]$$

$$4 + 16 - 18 + \frac{9}{2}$$

$$4 - 2 + \frac{9}{2} = 2 + \frac{9}{2} = 13/2$$

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4.4 Fundamental Theorem of Calculus

$$\int_{\pi/4}^{\pi/2} (2 - \csc^2 x)\, dx = 2x + \cot x \Big]_{\pi/4}^{\pi/2}$$

$$2 - \frac{1}{\sin^2 x}$$

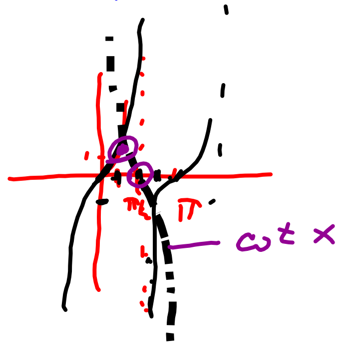
$$2\cdot\frac{\pi}{2} + \cot\frac{\pi}{2} - \left[2\cdot\frac{\pi}{4} + \cot\frac{\pi}{4}\right]$$

$$\pi + \cot\frac{\pi}{2} - \frac{\pi}{2} - \cot\frac{\pi}{4}$$

$$\frac{\pi}{2} + \cot\frac{\pi}{2} - \cot\frac{\pi}{4}$$

$$\frac{\pi}{2} + 0 - 1$$

$$\frac{\pi}{2} - 1 \sim 0.5$$



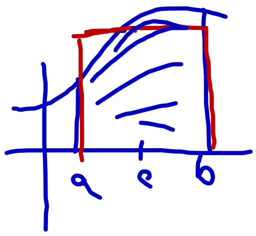
$$(\csc x)^2 = \frac{1}{(\sin x)^2}$$

$$\tan\frac{\pi}{4} = 1$$

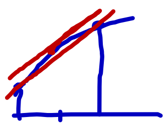
$$\tan\frac{3\pi}{4} = -1$$

4.4 Fundamental Theorem of Calculus

Mean Value Th. for Integrals



Area



Slope

MVT for Derivatives

