# 4.4 Fundamental Theorem of Calculus

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## Start with Indefinite Integration: complete the following

1. $\int (x^{3/2} + 2x - 1)\,dx =$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$$

2. $\int (x+1)(3x-2)\,dx = \int (3x^2 + x - 2)\,dx$

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## Start with Indefinite Integration: complete the following

1. $\int (x^{3/2} + 2x - 1)\,dx =$

$\dfrac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1} + \dfrac{2x^2}{2} - x + C$

$\dfrac{2}{5}x^{5/2} + x^2 - x + C$

$\int x^n\,dx = \dfrac{x^{n+1}}{n+1} + C$

$\dfrac{3}{2} + 1 = \dfrac{3}{2} + \dfrac{2}{2} = \dfrac{5}{2}$

2. $\int (x+1)(3x-2)\,dx = \int (3x^2 + x - 2)\,dx$

$-2x^0$

$\dfrac{3x^3}{3} + \dfrac{x^2}{2} - 2x + C$

$= x^3 + \dfrac{1}{2}x^2 - 2x + C$

3. $\frac{dy}{dx} = 3x^2$ F: solve for $y$

Start with the differential of y,

$dy = \frac{dy}{dx} dx$

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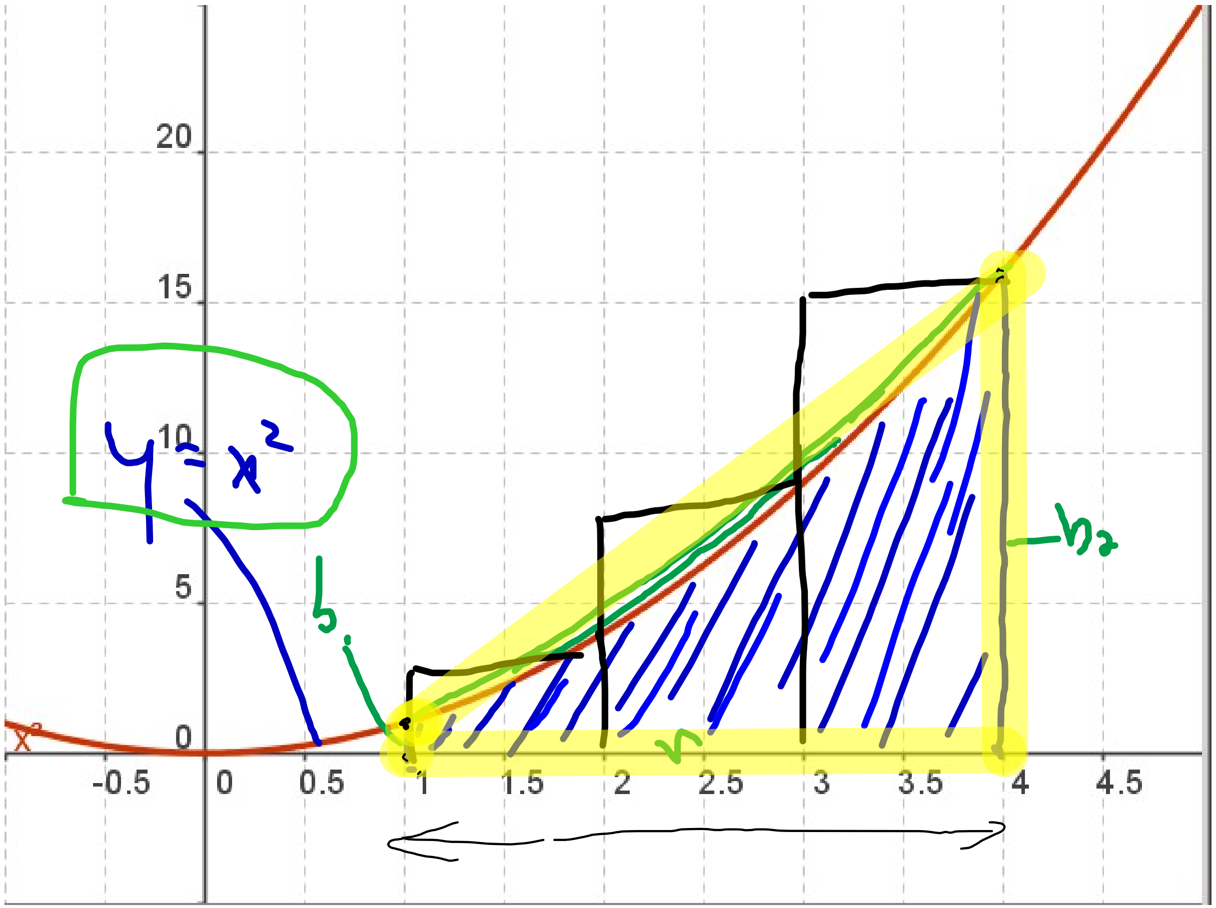
$$\int (\sec^2\theta - \sin\theta)\, d\theta$$

$$= \tan\theta + \cos\theta + C$$

$\rightarrow A_T = \frac{1}{2}(b_1+b_2)h = \frac{1}{2}(1+16)\,3 = \frac{1}{2}(17)(3)$

$= \frac{51}{2}$

$= 25\frac{1}{2}$




F: area under $y = x^2$ from $x=1$ to $x=4$

Estimating Area under a Curve, since y is non-negative.
Also estimating the definite integral.

Use multiple trapazoids for better estimate.

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# Fundamental Th. of Calculus FTC

If a function $f$ is continuous on $[a, b]$

and $F$ is an antiderivative of $f$ on the interval, then

$$\boxed{\int_a^b f(x)\,dx = F(b) - F(a)}$$

# Fundamental Th. of Calculus

If a function $f$ is continuous on $[a,b]$ and $F$ is an antiderivative of $f$ on the interval, then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

Do not need *c*, the constant of integration. It gets added and subtracted to add to 0.

$$\int_1^4 x^2\,dx = \left.\frac{x^3}{3} + \cancel{c}\right]_1^4 = \left[\frac{4^3}{3} + c\right] - \left[\frac{1}{3} + c\right]$$

$y = x^2$

$$= \frac{4^3}{3} + c - \frac{1}{3} - c = \frac{63}{3} = 21$$

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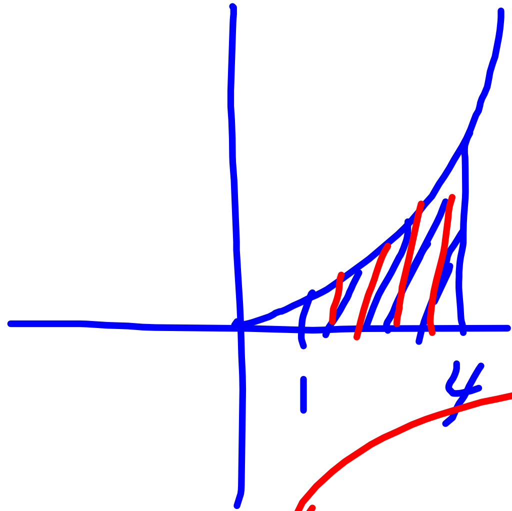
$$\int_1^4 x^2\,dx$$

$$= \frac{x^3}{3}\Big]_1^4$$




Area =

$$= \frac{4^3}{3} - \frac{1^3}{3} = \frac{64}{3} - \frac{1}{3} = \frac{63}{3} = 21$$

Compare to approximation
using area of trapazoid.

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6.G: $\int_2^7 3\,dx$ F: Evaluate

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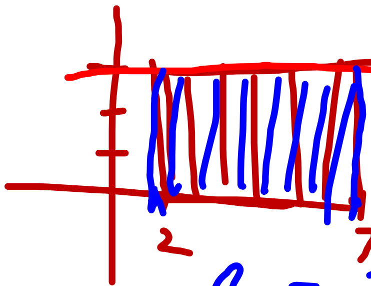
6.G: $\int_2^7 3\,dx$

F: Evaluate




$y=3$

$w=3$

$l=7-2=5$

$A=15$

$= 3x\Big]_2^7$

$= 3(7) - 3(2)$

$= 21-6 = 15$

10. $\int_1^3 (3x^2+5x-4)\,dx$

10. $\int_1^3 (3x^2+5x-4)\,dx$

$$\frac{3x^3}{3} + \frac{5x^2}{2} - 4x\Big]_1^3 = x^3 + \frac{5}{2}x^2 - 4x\Big]_1^3$$

$$= 3^3 + \frac{5}{2}\cdot 3^2 - 4(3) - \left[1^3 + \frac{5}{2}\cdot 1^2 - 4(1)\right]$$

$$= F(b) - F(a)$$

$$= 27 + \frac{5}{2}\cdot 9 - 12 - \left(1 + \frac{5}{2} - 4\right)$$

$$15 + \frac{45}{2} + 3 - \frac{5}{2} = 18 + \frac{40}{2} = 38$$

18 $\int_1^8 \sqrt{\frac{2}{x}}\,dx =$

18 $\int_1^8 \sqrt{\frac{2}{x}}\,dx = \sqrt{2}\int_1^8 x^{-1/2}\,dx$

$$= \sqrt{2}\,\frac{x^{1/2}}{\frac{1}{2}}\Bigg]_1^8$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$$

$$= 2\sqrt{2}\,\sqrt{x}\Big]_1^8$$

$$= 2\sqrt{2}\left[\sqrt{8} - \sqrt{1}\right] = 2\sqrt{2}\left[2\sqrt{2} - 1\right] = 4(2) - 2\sqrt{2}$$
$$= 8 - 2\sqrt{2}$$

22. $\displaystyle\int_{-8}^{-1} \frac{x-x^2}{2\sqrt[3]{x}}\,dx = \frac{1}{2}\int_{-8}^{-1} \frac{x^1-x^2}{x^{1/3}}\,dx$

$$= \frac{1}{2}\int_{-8}^{-1}\left(x^{2/3}-x^{5/3}\right)dx = \frac{1}{2}\left[\frac{x^{5/3}}{\frac{5}{3}} - \frac{x^{8/3}}{\frac{8}{3}}\right]_{-8}^{-1}$$

$$= \frac{1}{2}\left[\frac{3}{5}x^{5/3} - \frac{3}{8}x^{8/3}\right]_{-8}^{-1}$$

$$\frac{x-x^2}{2\sqrt[3]{x}} = \frac{1}{2}\cdot\frac{x-x^2}{\sqrt[3]{x}} = \frac{1}{2}\cdot\frac{x-x^2}{x^{1/3}}.$$

$$\frac{a^3}{a} = a^2.$$

$$= \frac{1}{2}\left[\frac{x^1}{x^{1/3}} - \frac{x^2}{x^{1/3}}\right]$$

$$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$$

$$= \frac{1}{2}\left[x^{\left(1-\frac{1}{3}\right)} - x^{\left(2-\frac{1}{3}\right)}\right] \qquad 2 = \frac{6}{3}$$

$$\frac{1}{2}\left[x^{2/3} - x^{5/3}\right]$$

$$\frac{x^{10}}{x^6} = x^4$$

22. $\displaystyle\int_{-8}^{-1} \frac{x-x^2}{2\sqrt[3]{x}}\,dx = \frac{1}{2}\int_{-8}^{-1} \frac{x^1-x^2}{x^{1/3}}\,dx$

$\displaystyle = \frac{1}{2}\int_{-8}^{-1} \left(x^{2/3} - x^{5/3}\right)dx = \frac{1}{2}\left[\frac{x^{5/3}}{\frac{5}{3}} - \frac{x^{8/3}}{\frac{8}{3}}\right]_{-8}^{-1}$

$\displaystyle = \frac{1}{2}\left[\frac{3}{5}x^{5/3} - \frac{3}{8}x^{8/3}\right]_{-8}^{-1}$

$\displaystyle = \frac{1}{2}\left\{\left[\frac{3}{5}(-1)^{5/3} - \frac{3}{8}(-1)^{8/3}\right] - \left[\frac{3}{5}(-8)^{5/3} - \frac{3}{8}(-8)^{8/3}\right]\right\}$

$\displaystyle = \frac{1}{2}\left\{\left[\frac{3}{5}(-1) - \frac{3}{8}(1)\right] - \left[\frac{3}{5}(-32) - \frac{3}{8}(+256)\right]\right\}$

$\displaystyle = \frac{1}{2}\left\{\left[-\frac{3}{5} - \frac{3}{8}\right] - \left[-\frac{96}{5} - 96\right]\right\} = \frac{1}{2}\left[\cancel{-\frac{3}{5} + \frac{96}{5}} - \frac{3}{8} + 96\right]$

$\displaystyle = \frac{1}{2}\left(\frac{93}{5} - \frac{3}{8} + 96\right) = \frac{1}{2}\left(\frac{744-15}{40} + 96\right)$

$\dfrac{729}{40}$ $\quad 57.1125$

24. $\int_1^4 (3 - |x-3|)\,dx$

Presents some problems.

1. Start with definiton of absolute value and
2. consider what this means regarding the interval from lower to upper limits.

Step #1: Absolute value:

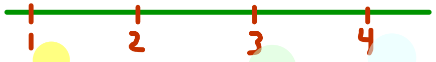
Step #2: About the interval

27. $\int_1^4 (3-|x-3|)\,dx$

$$|a| = \begin{cases} a, & a\ge 0 \\ -a, & a<0 \end{cases}$$

$$|x-3| = \begin{cases} x-3, & x-3\ge 0,\ x\ge 3 \\ -(x-3), & x-3<0,\ x<3 \end{cases}$$

$-x+3$




The integrand is defined differently on the interval.
Since definite integrals are defined as limits of sums, we replace the original integral with the sum of 2 integrals which have integrands and limits that correspond to the 2 part definition of the original:


$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$

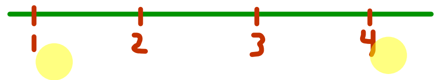
Rewrite the original integral as the sum of 2 integrals.

27. $\int_1^4 (3-|x-3|)\,dx$

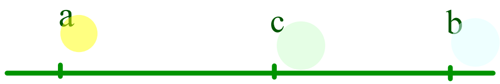
$$|a| = \begin{cases} a, & a \ge 0 \\ -a, & a < 0 \end{cases}$$

$$|x-3| = \begin{cases} x-3, & x-3 \ge 0,\ x \ge 3 \\ -(x-3), & x-3 < 0,\ x < 3 \end{cases}$$

$-x+3$




$$\int_1^4 (3-|x-3|)\,dx = \int_1^3 [3-(3-x)]\,dx + \int_3^4 [3-(x-3)]\,dx$$




$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$

27. $\int_1^4 (3-|x-3|)\,dx$

$(3-[3-x])\,dx$

$|a| = \begin{cases} a, & a \ge 0 \\ -a, & a < 0 \end{cases}$

$$|x-3| = \begin{cases} x-3, & x-3 \ge 0, \quad x \ge 3 \\ \underbrace{-(x-3)}_{=3-x}, & x-3<0, \quad x<3 \end{cases}$$

$$\int_1^4 (3-|x-3|)\,dx = \int_1^3 [3-(3-x)]\,dx + \int_3^4 [3-(x-3)]\,dx$$

$$= \int_1^3 x\,dx + \int_3^4 (6-x)\,dx$$

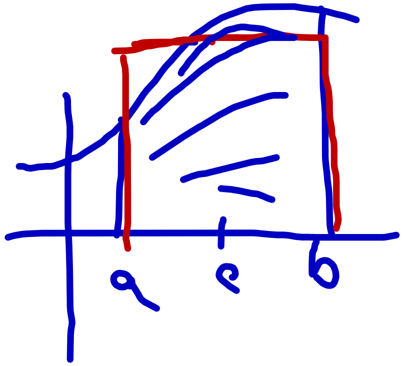
$$= \left[\frac{x^2}{2}\right]_1^3 + \left[6x - \frac{x^2}{2}\right]_3^4$$

$$= \left[\frac{9}{2} - \frac{1}{2}\right] + \left[24 - \frac{16}{2} - \left(18 - \frac{9}{2}\right)\right]$$

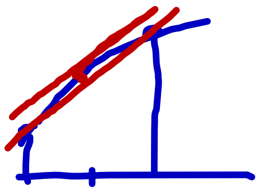
$4 \quad + \quad 16 - 18 + \frac{9}{2}$

$4 - 2 + \frac{9}{2} = 2 + \frac{9}{2} = 13/2$

# Mean Value Th. for Integrals




Area



Slope

MVT for Derivatives

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