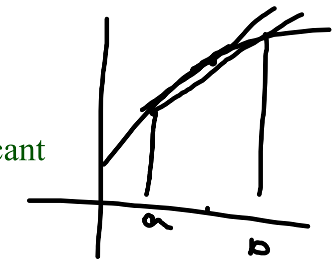
Fundamental Th. of Calculus

If a function f is continuous on $[a,b]$ and F is an antiderivative of f on the interval, then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

MVT for deriv.

(equal slopes: slope of tangent at (c,f(c))=slope of secant line connecting endpoints of interval)



for Integral

(equal areas: area of rectangle, f(c)*(b-a)= area under curve from a to b)

$f(c)(b-a)$

if f is continuous on $[a,b]$

then $\exists$ a $c \in [a,b] \ni$

$$\int_a^b f(x)\,dx = f(c)(b-a)$$

Average Value of a Function on [a,b]

From MVT: $\int_a^b f(x)\,dx = f(c)(b-a)$

Multiply both members of the equation by 1/(b-a)

$$\frac{1}{b-a}\int_a^b f(x)\,dx = \frac{f(c)(b-a)}{(b-a)}$$

Results in an equation for f(c), the **average value** of the function f(x) on the interval [a,b]

$$\boxed{\underline{f(c)} = \frac{1}{b-a}\int_a^b f(x)\,dx}$$

Ave. Value of f on [a, b]

p.291 #44.

G: $f(x) = \frac{9}{x^3}$

F: $c \in [1,3] \ni$ MVT applies.

$$\int_1^3 9x^{-3}\,dx = \frac{9x^{-2}}{-2}\Big]_1^3$$

$$-\frac{9}{2x^2}\Big]_1^3 = -\frac{9}{2\cdot 3^2} - \left[-\frac{9}{2\cdot 1^2}\right]$$

$$= -\frac{1}{2} + \frac{9}{2} = \frac{8}{2} = 4$$

$$\int_a^b f(x)\,dx = f(c)(b-a)$$

$$\int_1^3 9x^{-3}\,dx = f(c)(3-1)$$

$$4 = f(\cancel{x})(3-1)$$

$$f(\cancel{x}) = 2 = \frac{9}{x^3}$$

$$\frac{1}{2} = \frac{x^3}{9} \qquad x^3 = \frac{9}{2}$$

$$x = \sqrt[3]{\frac{9}{2}} = \sqrt[3]{4.5}$$

291/#48.

G: $f(x) = \dfrac{4(x^2+1)}{x^2}$ F: Ave.val. on $[1,3]$

$$\text{Ave.val} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

$$\text{Ave} = \frac{1}{3-1}\int_1^3 \frac{4(x^2+1)}{x^2}\,dx$$

$$\frac{x^2+1}{x^2} = \frac{x^2}{x^2} + \frac{1}{x^2} = 1 + x^{-2}$$

$$= 2\int_1^3 (1+x^{-2})\,dx$$

$$= 2\left[x + \frac{x^{-1}}{-1}\right]_1^3 = 2\left[x - \frac{1}{x}\right]_1^3 = 2\left[\left(3-\frac{1}{3}\right) - \left(1-\frac{1}{1}\right)\right]$$

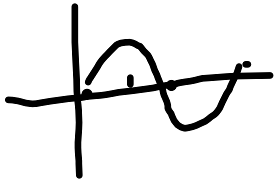
$$= 2\left[3 - \frac{1}{3}\right] = 2\left(\frac{8}{3}\right) = \frac{16}{3}$$

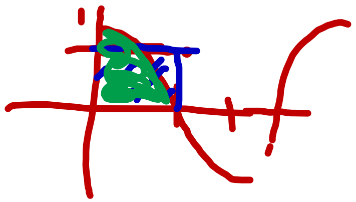
50. G: $f(x) = \cos x$ F: ave. value on $[0, \pi/2]$

$$\text{Ave} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

$$\text{Ave} = \frac{1}{\pi/2 - 0}\int_0^{\pi/2} \cos x\,dx$$

$$= \frac{2}{\pi}\sin x\Big]_0^{\pi/2} = \frac{2}{\pi}\left[\overset{1}{\sin\frac{\pi}{2}} - \overset{0}{\sin 0}\right] = \frac{2}{\pi}$$





2nd Fundamental Theorem of Calculus - Concept

$$\int_1^2 t\,dt = \frac{t^2}{2}\Big]_1^2 = \frac{4}{2} - \frac{1}{2} = \frac{3}{2}$$

$$\int_1^x t\,dt = \frac{t^2}{2}\Big]_1^x = \frac{x^2}{2} - \frac{1}{2}$$

$$\frac{d}{dx}\left[\frac{x^2}{2} - \frac{1}{2}\right] = \underline{x}$$

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$$\boxed{\frac{d}{dx}\left[\int_1^x \underline{t}\,dt\right] = \underline{x}}$$

Try that, again.....

$$\int_1^2 (t^2+2)\,dt = \frac{t^3}{3}+2t\Big]_1^2 = \frac{8}{3}+4-\left(\frac{1}{3}+2\right)$$

$$= \frac{8}{3}-\frac{1}{3}+2 = \frac{7}{3}+2 = \frac{13}{3}$$

$$\int_1^x (t^2+2)\,dt = \frac{t^3}{3}+2t\Big]_1^x = \frac{x^3}{3}+2x-\left(\frac{1}{3}+2\right)$$

$$= \frac{x^3}{3}+2x-\frac{7}{3}$$

$$\frac{d}{dx}\left[\int_1^x (t^2+2)\,dt\right] = x^2+2$$

2nd FTC

If f is continuous on open interval I containing a, then, for every x on the interval

$$\frac{d}{dx}\left[\int_a^x f(t)\,dt\right] = f(x)$$