4.3 Reimann Sums and the Definite Integral

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Homework Part 1

Def. of Riemann Sum

Let f be defined on $[a,b]$, and let Δ be a partition of $[a,b]$, given by

$$a = x_0 < x_1 < x_2 < \dots < x_{n-1} < x_n = b$$

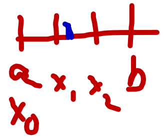
where Δx_i is width of ith subinterval

If c_i is any pt. in the ith subinterval

then the sum $\sum_{i=1}^{n} f(c_i)\Delta x_i$, $x_{i-1} \le c_i \le x_i$

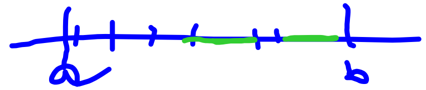
is called a Riemann Sum of f for the partition Δ.

Δx_i not all equal



norm of the partition $\|\Delta\|$ = width of largest subinterval

$\|\Delta\|$



* Def. of Definite Integral * **

If (1) f defined on [a, b] and the limit

(2) $\lim_{\|\Delta\| \to 0} \sum_{i=1}^{n} f(c_i)\Delta x_i$ exists

then f is integrable on [a, b] and the limit is denoted as:

$$\lim_{\|\Delta\| \to 0} \sum_{i=1}^{n} f(c_i)\Delta x_i = \int_a^b f(x)\,dx$$

This is called the Definite integral of f from a to b

Indefinite

$$\int x^3\,dx = \frac{x^4}{4} + C$$

$$\int_0^1 x^3\,dx = \left.\frac{x^4}{4}\right]_0^1$$

Definite

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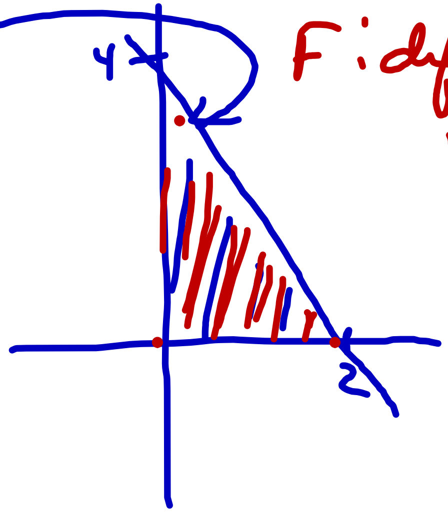
Homework Part 1

14. G: $f(x) = 4 - 2x$



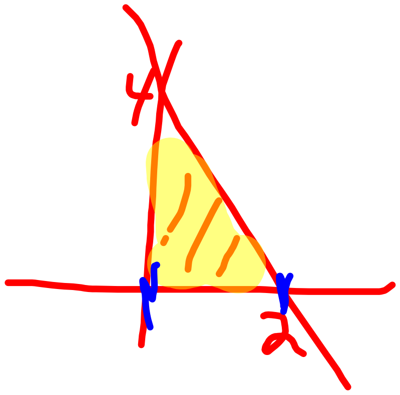
F: definite integral.

$$\int_a^b f(x)\,dx$$

$$= \int_0^2 (4-2x)\,dx$$

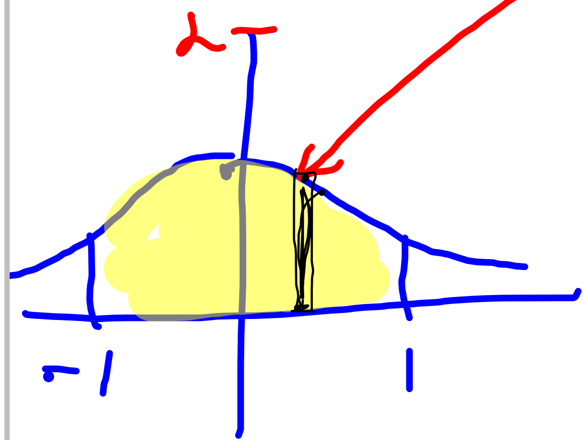
14. G: $f(x) = 4 - 2x$

F: set up definite integral ⇒ area



$$\int_0^2 (4-2x)\,dx$$

18. G : $f(x) = \frac{1}{x^2+1}$



$$\int_{-1}^{1} \frac{1}{x^2+1}\,dx$$

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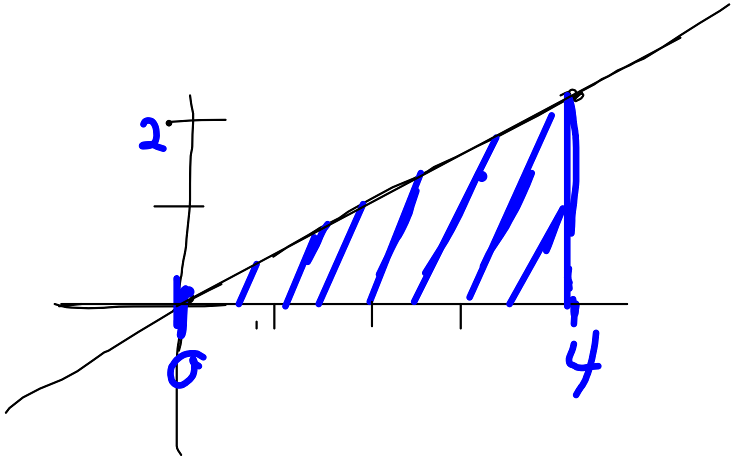
Homework Part 1

26. G: $\int_0^4 \frac{x}{2}\,dx$

F: ① Sketch region ② Use geometry to find area.



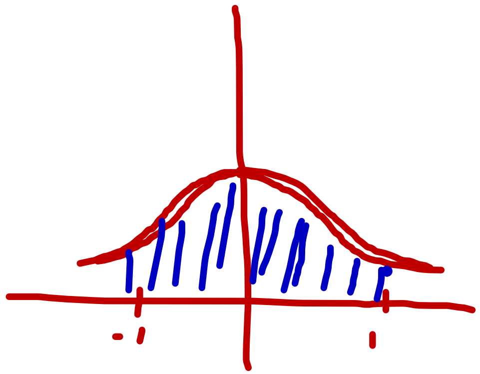
$A = \frac{1}{2}bh = \frac{1}{2}\cdot 4 \cdot 2$

$= 4$ sq. units

$$\int_0^4 \frac{x}{2}\,dx = \frac{1}{2}\,\frac{x^2}{2}\Big]_0^4 = \frac{1}{4}x^2\Big]_0^4 = \frac{1}{4}\left[4^2 - 0^2\right] = 4$$

sq units

18. $f(x) = \frac{1}{x^2+1}$



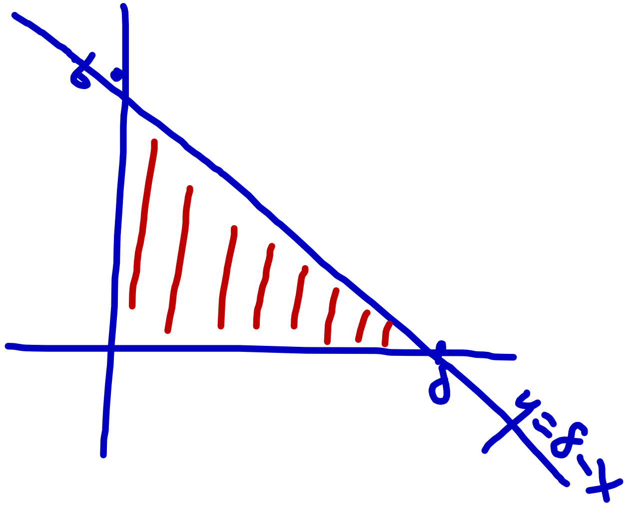
$$\int_a^b f(x)\,dx$$

$$= \int_{-1}^{1} \left(\frac{1}{x^2+1}\right) dx$$

or: symmetry @ y-axis

$$= 2\int_0^1 \left(\frac{1}{x^2+1}\right) dx$$

28. G: $\int_0^8 (8-x)\,dx$ F: Sketch, evaluate.



$A = \frac{1}{2} b h$

$= \frac{1}{2} \cdot 8 \cdot 8 = 32$

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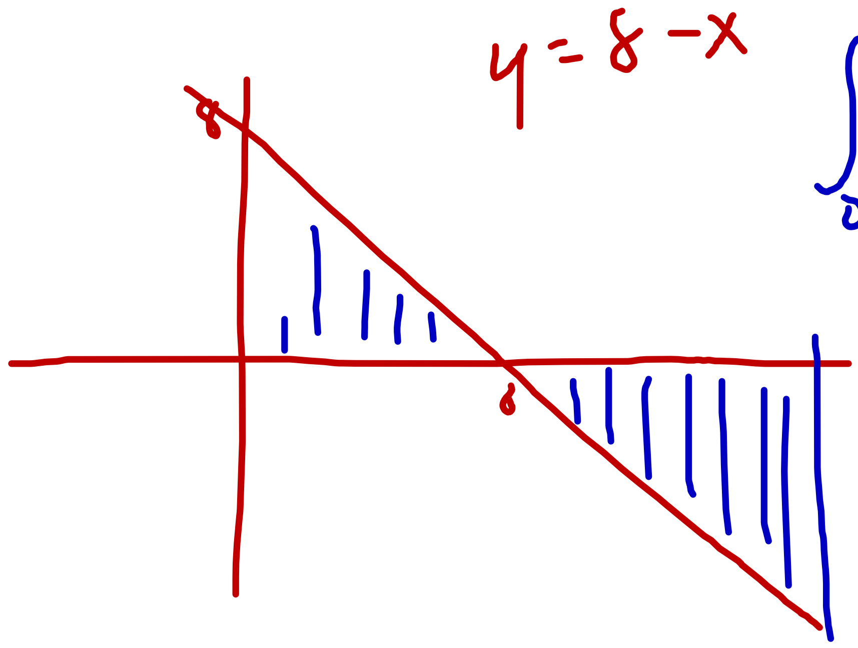
Homework Part 1



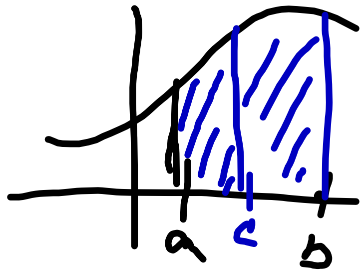
$$\int_0^{16} (8-x)\,dx = 0$$

Definite Integral as the Area of a Region

If f is cont. & nonnegative on $[a,b]$, then the area of the region bounded by the graph of f, the x-axis, and the vertical lines $x=a$ and $x=b$ is given by

$$\text{Area} = \int_a^b f(x)\,dx$$

Special Integrals



① $$\int_a^a f(x)\,dx = 0$$

② $$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

③ $$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$

Continuity Implies Integrability

If a funct f is cont on [a, b]
then f is integrable on [a, b]

Given:

$$\int_2^4 x^3\,dx = 60$$

$$\int_2^4 x\,dx = 6$$

$$\int_2^4 dx = 2$$

34. $\int_2^2 x^3\,dx = 0$

36. $\int_2^4 15\,dx = 15\int_2^4 dx$

$= 15(2) = 30$

38. $\int_2^4 (x^3+4)\,dx$

$= \int_2^4 x^3\,dx + \int_2^4 4\,dx = 60 + 8 = 68$

42 (279)

G: $\int_0^3 f(x)\,dx = 4$, $\int_3^6 f(x)\,dx = -1$

a) $\int_0^6 f(x)\,dx = \int_0^{3} f(x)\,dx + \int_{3}^6 f(x)\,dx = 4 - 1 = 3$

b) $\int_6^3 f(x)\,dx = -\int_3^6 f(x)\,dx = -(-1) = +1$

c) $\int_3^3 f(x)\,dx = 0$ d) $\int_3^6 -5 f(x)\,dx = (-5)(-1) = 5$