4.3 Reimann Sums and the Definite Integral

video

Study 4.3 #9 - 43,
#48 a), $\int_1^2 f(x)dx$, $\int_0^2 f(x)dx$, $\int_0^2 |f(x)|dx$

Goals:
1. Recognize that Riemann Sums use Δx_i that are not all equal.
2. Learn the definition of the Definite Integral.
3. Understand how the Definite Integral relates to Area under a curve.
4. Learn basic properties of definite integrals.

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area and definite integral

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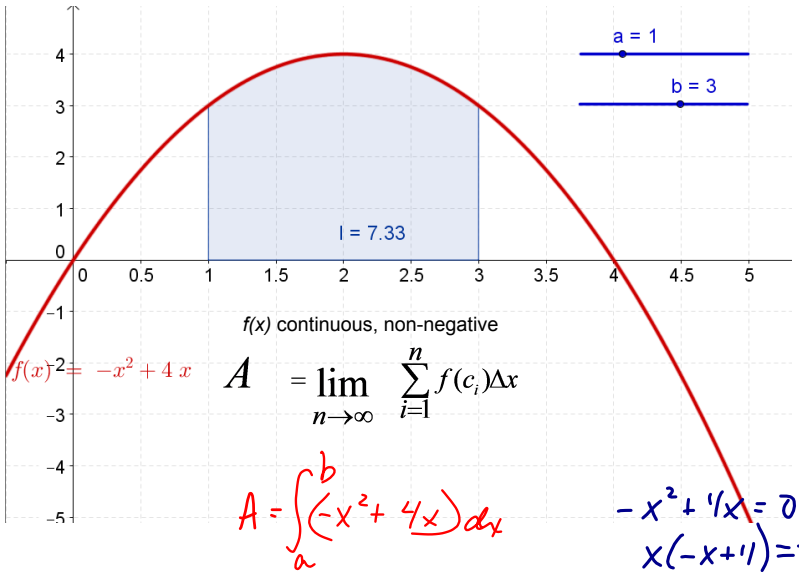

area and definite integral

Using the sliders, **find the area under the curve, y = -x² + 4x, from a=0 to b=4**. Then **find the definite integral from a=4 to b=5.**
How does the sum of these two values
compare to the integral of the the function from a=0 to a=5? _____

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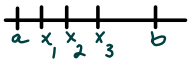
Definition of Riemann Sum

Let f be defined on [a,b], and let
Δ be a partition of [a,b], given by
$a = x_0 < x_1 < x_2 < \ldots < x_{n-1} < x_n = b$
where Δx_i is the width of the ith sub-interval
If c_i is any point on the ith sub-interval, then
the sum

$$\sum_{i=1}^{n} f(c_i)\,\Delta x_i \quad , \qquad x_{i-1} < c_i < x_i$$

is called a Riemann Sum of f for the partition Δ

Δx_i not all equal



norm of the partition ||Δ|| =
width of longest subinterval

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} f(c_i)\Delta x$$

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Definition of Definite Integral

If *f* is defined on [a,b], and the following limit exists

$$\lim_{\|\Delta\|\to 0} \sum_{i=1}^{n} f(c_i)\,\Delta x_i$$

Then *f* is integrable on [a,b] and the limit is denoted as:

$$\lim_{\|\Delta\|\to 0} \sum_{i=1}^{n} f(c_i)\,\Delta x_i = \int_a^b f(x)\,dx$$

This is called the Definite Integral of *f* from a to b

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$$\int x^3\,dx = \frac{x^4}{4} + C$$

Indefinite

$$\int_0^1 x^3\,dx = \left.\frac{x^4}{4}\right]_0^1$$

Definite

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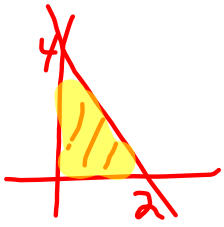
G: $f(x) = 4 - 2x$

F: Set up the definite integral that yields the area of the shaded region.



$$\lim_{\|\Delta\|\to 0} \sum_{i=1}^{n} f(c_i)\,\Delta x_i = \int_a^b f(x)\,dx$$

Need to rewrite with *a*, *b*, and *f(x)* for this problem.

$$\int_a^b f(x)\,dx$$

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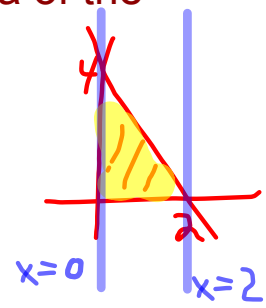

$$\lim_{||\Delta||\to 0} \sum_{i=1}^{n} f(c_i)\,\Delta x_i = \int_a^b f(x)\,dx$$

Need to rewrite with *a*, *b*, and *f(x)* for this problem.

$$\int_a^b f(x)\,dx$$

$$\int_0^2 (4-2x)\,dx$$

4.3 Reimann Sums and the Definite Integral

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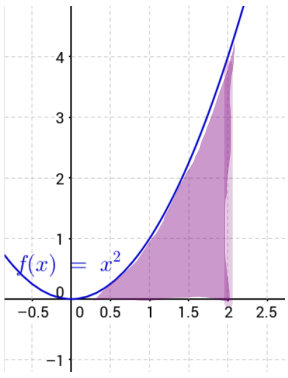

Need to rewrite with *a*, *b*, and *f(x)* for this problem.

$$\int_a^b f(x)\,dx$$

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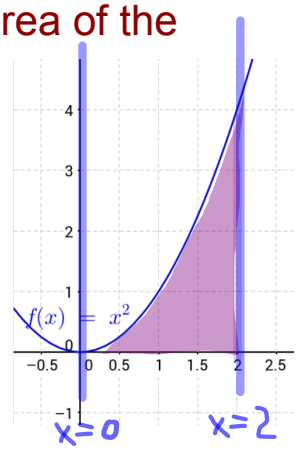

Need to rewrite with *a*, *b*, and *f(x)* for this problem.

$\int_a^b f(x)\,dx$

$\int_0^2 x^2\,dx$

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G: $\int_0^4 \frac{x}{2}\,dx$

F: 1. Sketch region
2. Use geometry to find the area

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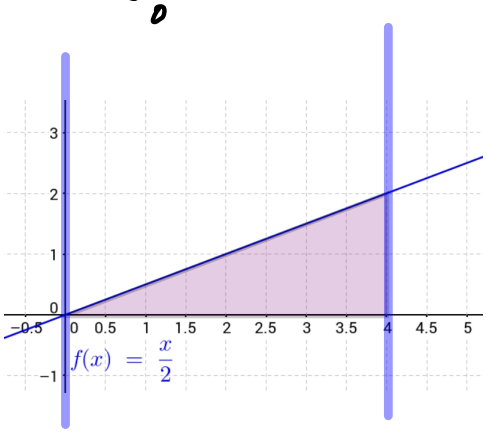

x=0 x=4

$A = \frac{1}{2}bh$

$= \frac{1}{2}(4)(2) = 4$ sq. units

$\int_0^4 \frac{x}{2}dx = \frac{1}{2}\frac{x^2}{2}\Big]_0^4 = \frac{1}{4}x^2\Big]_0^4 = \frac{1}{4}\left[4^2-0^2\right] = 4$ sq. units

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G: $\int_0^8 (8-x)\,dx$

F: 1. Sketch region
2. Use geometry to find the area

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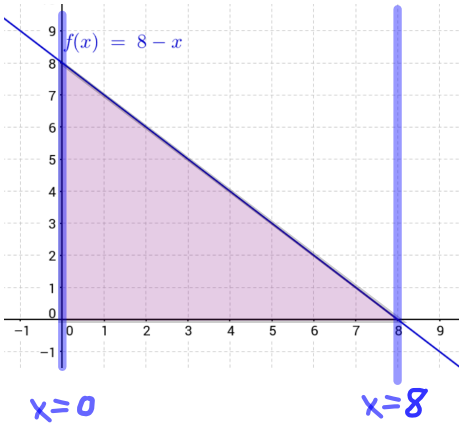
G: $\int_0^8 (8-x)\,dx$

F: 1. Sketch region
2. Use geometry to find the area

$A = \frac{1}{2} b h$

$= \frac{1}{2} \cdot 8 \cdot 8 = 32$ sq units



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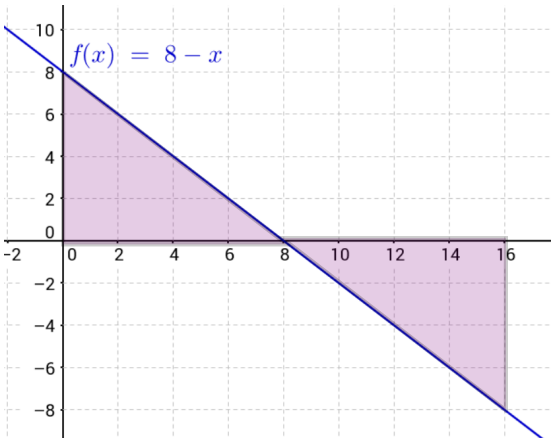
What about $\int_0^{16} (8-x)\,dx$?



NOT non-negative

$\int_0^{16} (8-x)\,dx = 0$

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Definite Integral as the Area of a Region

If ***f*** is continuous and non-negative on [a,b],
then the **area of the region** bounded by:
the graph of ***f***,
the **x-axis**, and
the vertical lines **x = a and x = b**
is given by:

$$Area = \int_a^b f(x)\,dx$$

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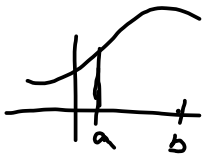
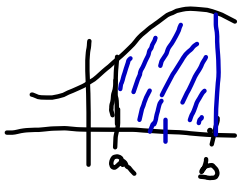
Special Integrals

① $\int_a^a f(x)\,dx = 0$



② $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$



③ $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$

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Continuity Implies Integrability

If a function f is continuous on [a,b], then f is integrable on [a,b].

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$\int_2^2 x^3\,dx$

$\int_2^4 15\,dx$

$\int_2^4 (x^3+4)\,dx$

Given:

$\int_2^4 x^3\,dx = 60$

$\int_2^4 x\,dx = 6$

$\int_2^4 dx = 2$

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$$\int_2^2 x^3\,dx = 0$$

$$\int_2^4 15\,dx = 15\int_2^4 dx$$
$$= 15(2) = 30$$

$$\int_2^4 (x^3+4)\,dx$$
$$= \int_2^4 x^3\,dx + \int_2^4 4\,dx = 60 + 8 = 68$$

Given:

$$\int_2^4 x^3\,dx = 60$$

$$\int_2^4 x\,dx = 6$$

$$\int_2^4 dx = 2$$

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G: $\int_0^3 f(x)\,dx = 4$ $\int_3^6 f(x)\,dx = -1$

$$\int_0^6 f(x)\,dx$$

$$\int_6^3 f(x)\,dx$$

$$\int_3^3 f(x)\,dx$$

$$\int_3^6 -5f(x)\,dx$$

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G: $\int_0^3 f(x)\,dx = 4$ $\int_3^6 f(x)\,dx = -1$

$\int_0^6 f(x)\,dx = \int_0^3 f(x)\,dx + \int_3^6 f(x)\,dx = 4-1=3$

$\int_6^3 f(x)\,dx = -\int_3^6 f(x)\,dx = -(-1) = +1$

$\int_3^3 f(x)\,dx = 0$

$\int_3^6 -5 f(x)\,dx = (-5)(-1) = 5$

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