3.9 Differentials

3.9 # 1, 7-9, 11-19, 20, 27, 31, 37, 41

Definition of Differentials
Do these pages.
3.9 Differentials

When considering $y = \sqrt{x}$ compared with its tangent line, $y = 0.5x + 0.5$, at $x = 1$:

When $x = 2$, the $y$ value of:

- $y = 0.5x + 0.5 \quad \rightarrow \quad 1.5$
- $y = \sqrt{x} \quad \rightarrow \quad 1.414$

Homework
3.9 Differentials

Which is easier to compute?
\[ \sqrt{2} \quad \text{or} \quad y = 0.5(2) + 0.5 \]

Can the tangent line be used to approximate \( \sqrt{2} \)?

If the margin of error is acceptable, then the tangent line can be used as an estimate. The error gets larger as \( x \) gets further from the original value.

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3.9 Differentials

Approximations using Tangent Lines

Estimate $\sqrt{6}$

Estimate $\sqrt{5}$

Find: Estimate the square root of 6. of 5.

One method is to use the tangent line

This can be a good estimate when the value of x is close to a value that is easy to compute.
3.9 Differentials

Another method is to use differentials:

Definition:

Let \( y = f(x) \) represent a function that is differentiable on an open interval containing \( x \).

The differential of \( x \), \( dx \), is any nonzero real number.

The differential of \( y \), \( dy \), is:

\[
dy = f'(x) \, dx
\]
3.9 Differentials

Approximations using Differentials

Find $\sqrt{5}$ Use $y = \sqrt{x}$. Let $x = 4$ and $dx = +1$

$$dy = f'(x) \, dx$$
$$dy = \frac{1}{(2\sqrt{x})} \, dx$$
$$dy = \frac{1}{(2\sqrt{4})} \, (1)$$
$$dy = \frac{1}{4}$$

Therefore, $\sqrt{5} \approx \sqrt{4} + dy = 2 + 1/4 = 2.25$

Calculator $\sim 2.24$
Example 1

F: a) $\% \text{error in } A$

\[ A = s^2 \]
\[ \frac{dA}{ds} = 2s \]
\[ dA = 2sd comedian \]
\[ dA = 2(15\text{ cm})(0.05\text{ cm}) = 1.5\text{ cm}^2 \]

\[ \% \text{error in } A = \left( \frac{1}{15^2\text{ cm}^2} \right) \left( \frac{0.05\text{ cm}}{15\text{ cm}} \right) \]
\[ = \left( \frac{1}{225\text{ cm}^2} \right) \left( \frac{0.05}{15} \right) \]
\[ = 0.00718 = 0.718 \%
\]

\[ \% \text{error in } A = \frac{dA}{A} \leq 0.025 \]

\[ dA = 0.025(15\text{ cm})^2 = 0.025(225) = 5.625\text{ cm}^2 \]
\[ dA = 2s ds \]
\[ 5.625\text{ cm}^2 = 2(15\text{ cm}) ds \]
\[ ds = \frac{5.625\text{ cm}^2}{30\text{ cm}} = 0.1875\text{ cm} \]

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_Homework_
3.9 Differentials

Finding Differentials of Functions using Definition of the Differential

\[ G: \quad y = 3 \ x^{2/3} \quad \text{F: differential } dy \]

\[ dy = f'(x) \ dx \]
3.9 Differentials

Finding Differentials of Functions using Definition of the Differential

G: \( y = 3 \, x^{2/3} \)  
F: differential \( dy \)

\[
dy = f'(x) \, dx \\
y = 3 \, (2/3) \, x^{-1/3} \, dx \\
y = \frac{2}{x^{1/3}} \, dx
\]
3.9 Differentials

Finding Differentials of Functions using Definition of the Differential

G: \( y = x \cos x \)  

F: differential \( dy \)

\[
dy = \frac{dy}{dx} \, dx
\]
3.9 Differentials

Finding Differentials of Functions using Definition of the Differential

G: \( y = x \cos x \)

F: differential \( dy \)

\[
\frac{dy}{dx} = dx
\]

\[
dy = (-x \sin x + \cos x) \, dx
\]
3.9 Differentials

Finding Differentials of Functions using Definition of the Differential

G: \( y = 2 - x^4 \) \nF: differential \( dy \)

\[ dy = \frac{dy}{dx} \, dx \]
3.9 Differentials

\[ dy = \frac{dy}{dx} \, dx \]

\[ dy = 4x^3 \, dx \]
3.9 Differentials

Finding Differentials of Functions using Definition of the Differential

G: \[ y = 3x^5 - 2x^2 + 1 \]

F: differential \( dy \)

\[ dy = \frac{dy}{dx} \, dx \]
3.9 Differentials

Finding Differentials of Functions using Definition of the Differential

G: \( y = 3x^5 - 2x^2 + 1 \)

F: differential \( dy \)

\[ dy = \frac{dy}{dx} \, dx \]

\[ dy = (15x^4 - 4x) \, dx \]

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