3.9 Differentials

3.9 # 1, 7-15, 19, 27, 31, 37, 41

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3.9 Differentials

Two Objectives:

1. Approximations

2. Definition of Differential - needed for Integration

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3.9 Differentials

1. Approximations

Without using a calculator, find:

$\sqrt{2}$

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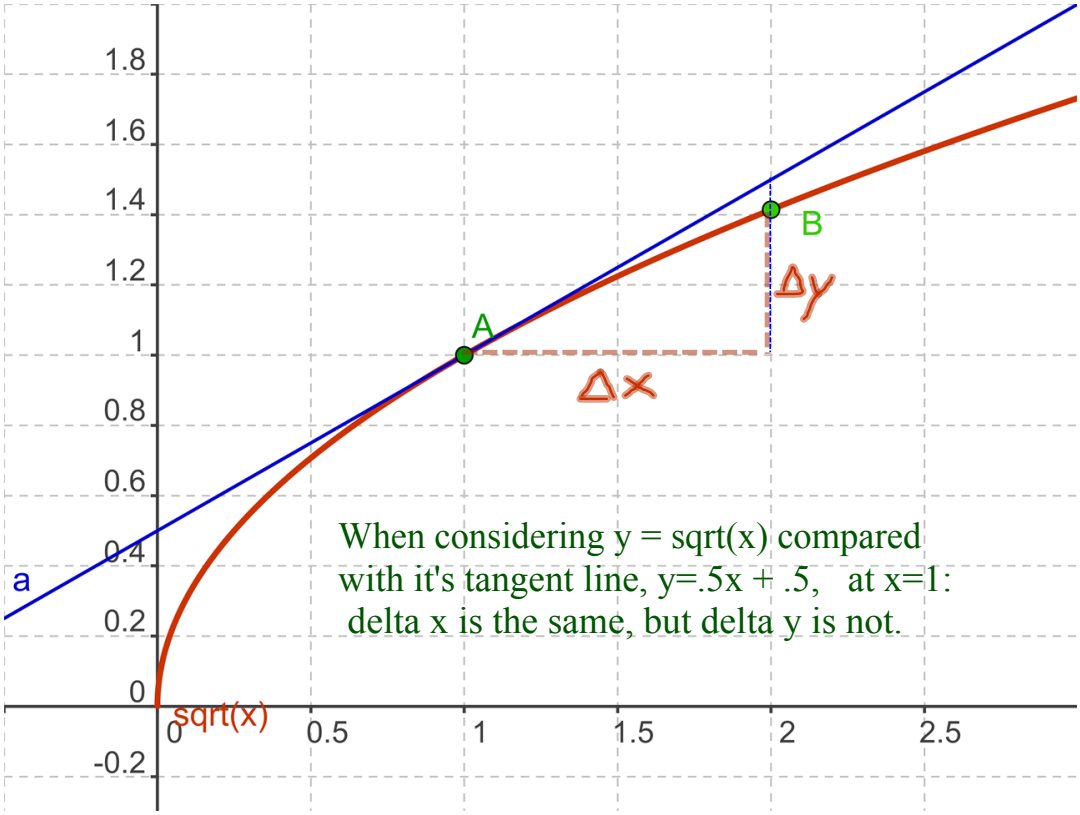
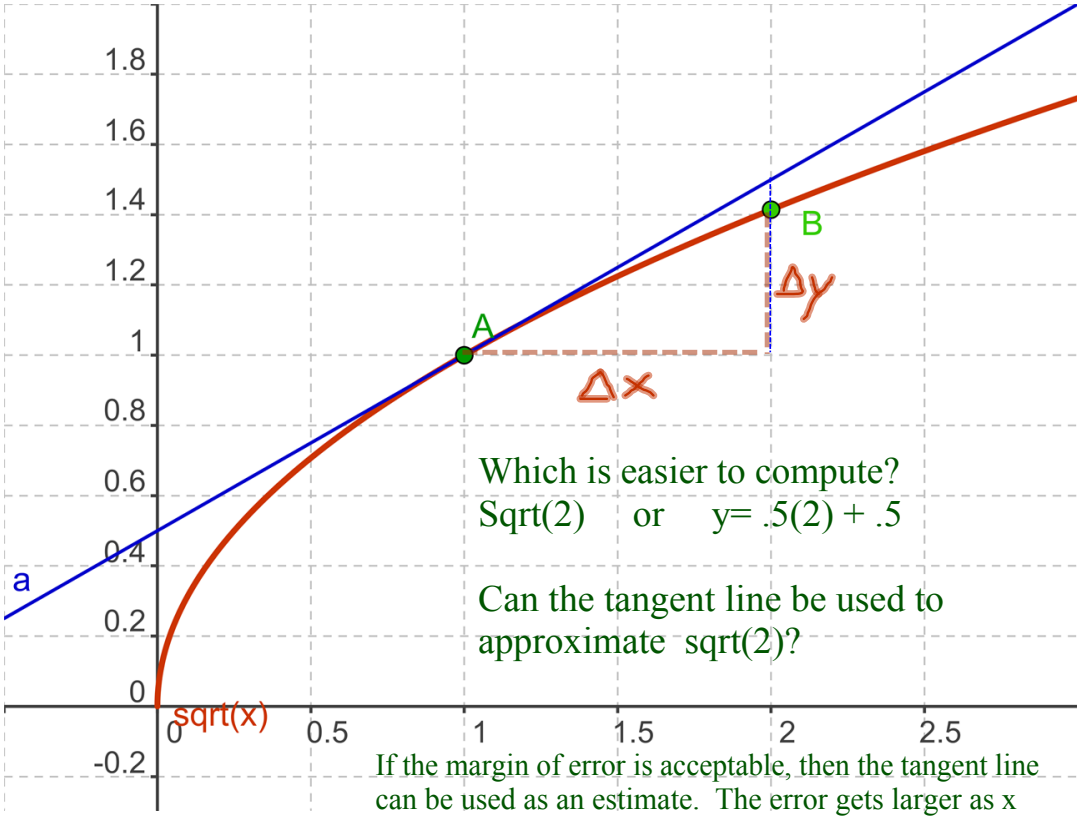
3.9 Differentials



3.9 Differentials

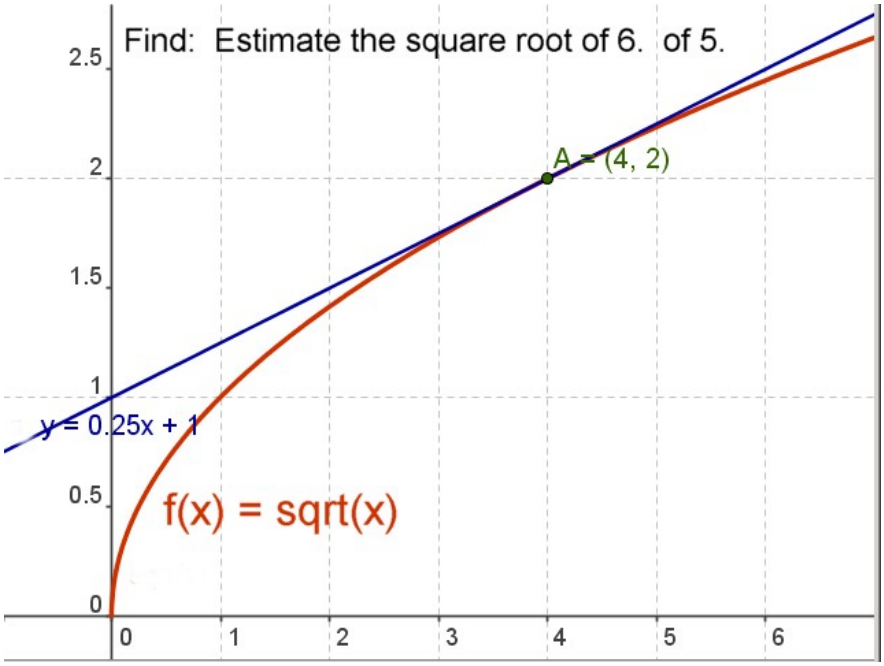


Which is easier to compute?
Sqrt(2) or y= .5(2) + .5

Can the tangent line be used to approximate sqrt(2)?

If the margin of error is acceptable, then the tangent line can be used as an estimate. The error gets larger as x gets further from the original value.

Estimate $\sqrt{6}$ **Estimate $\sqrt{5}$**



One method is to **use the tangent line**.
This can be a good estimate when the value of x is close to a value that is easy to compute.

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3.9 Differentials

Another method is to use differentials:

Definition:

Let $y = f(x)$ represent a function that is differentiable on an open interval containing x.

The differential of x (dx) is any nonzero real number.

The differential of y (dy) is:

$$dy = f'(x)\,dx$$

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Another method is to use differentials:

Find $\sqrt{5}$ Use $y = \sqrt{x}$. Let $x = 4$ and $dx = +1$

$$dy = f'(x)\,dx$$
$$dy = 1/(2\sqrt{x})\,dx$$
$$dy = 1/(2\sqrt{4})\,(1)$$
$$dy = 1/4$$

Therefore, $\sqrt{5} \approx \sqrt{4} + dy = 2 + 1/4 = 2.25$

Calculator ~ 2.24

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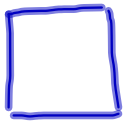
p. 240 #31.

F: a) ~ % error in A

b) max allowable error in s if error in A ≤ 2.5%



$s = 15\text{cm} \pm 0.05\text{cm}$.

$A = s^2$

$dA = \frac{dA}{ds} ds$.

$\frac{dA}{ds} = 2s$

$dA = 2s\,ds$

a) $dA = 2(15\text{cm})(0.05\text{cm}) = 1.5\text{cm}^2$

$\%\text{ error } A = \frac{1.5\text{cm}^2}{15^2\text{cm}^2} = \frac{1.5}{(15)(15)} = \frac{1}{150}$

$= (.007)100 = 0.7\%$

$\%\text{ err } A = \frac{dA}{A} \leq 0.025$

$dA = 0.025(15\text{cm})^2 =$

$= 0.025(225) = 5.625\text{cm}^2$

$dA = 2s\,ds$

$5.625\text{cm}^2 = 2(15\text{cm})\,ds$

$ds = \frac{5.625\text{cm}^2}{30\text{cm}} = .1875\text{ cm}.$

0.1875cm.