3.9 Differentials

3.9 # 1, 7-9, 11-19, 20, 27, 31, 37, 41

Definition of Differentials:
Do these pages.

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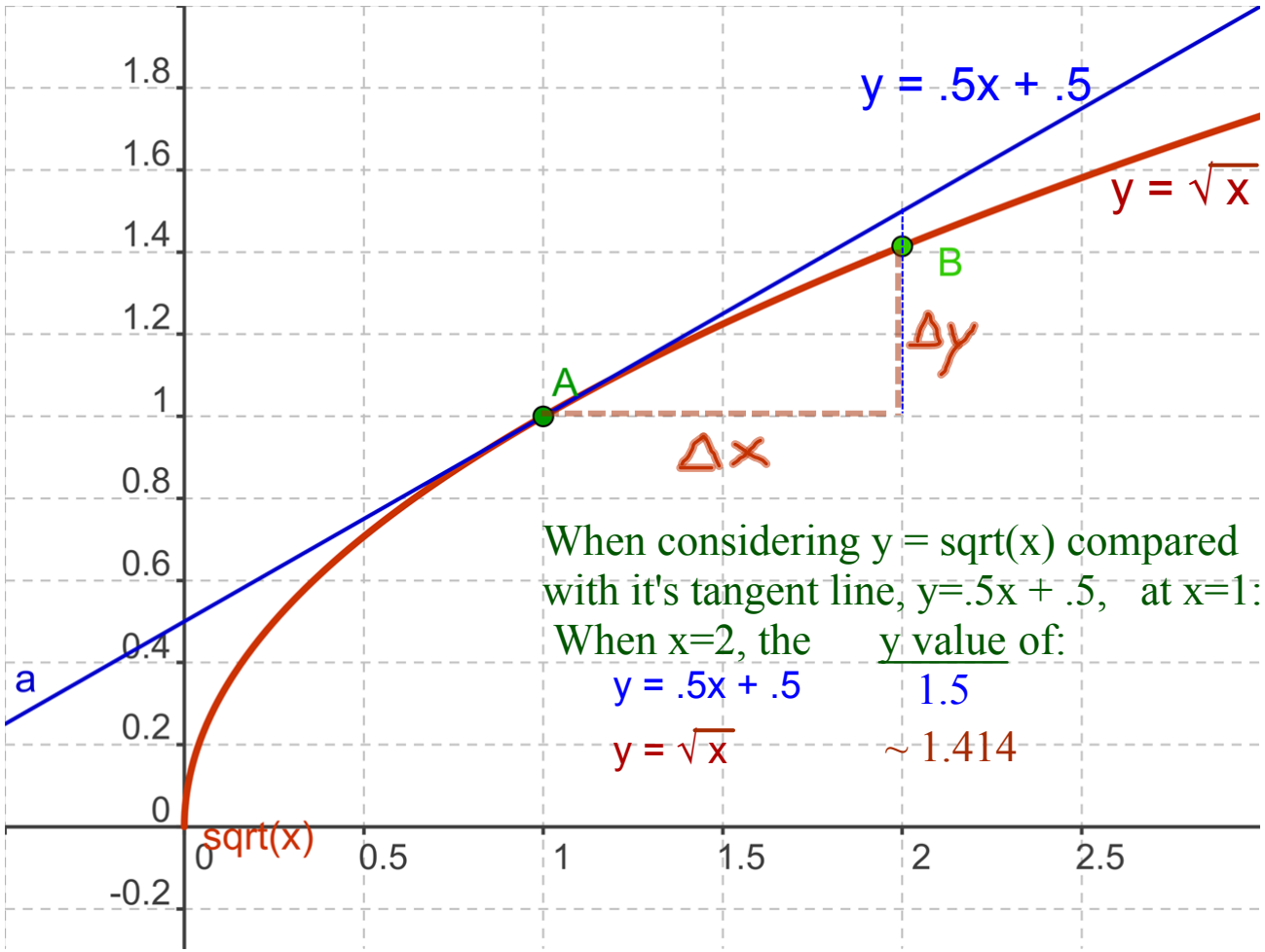
3.9 Differentials



When considering y = sqrt(x) compared with it's tangent line, y=.5x + .5, at x=1:

When x=2, the y value of:

y = .5x + .5	1.5
y = √x	~ 1.414

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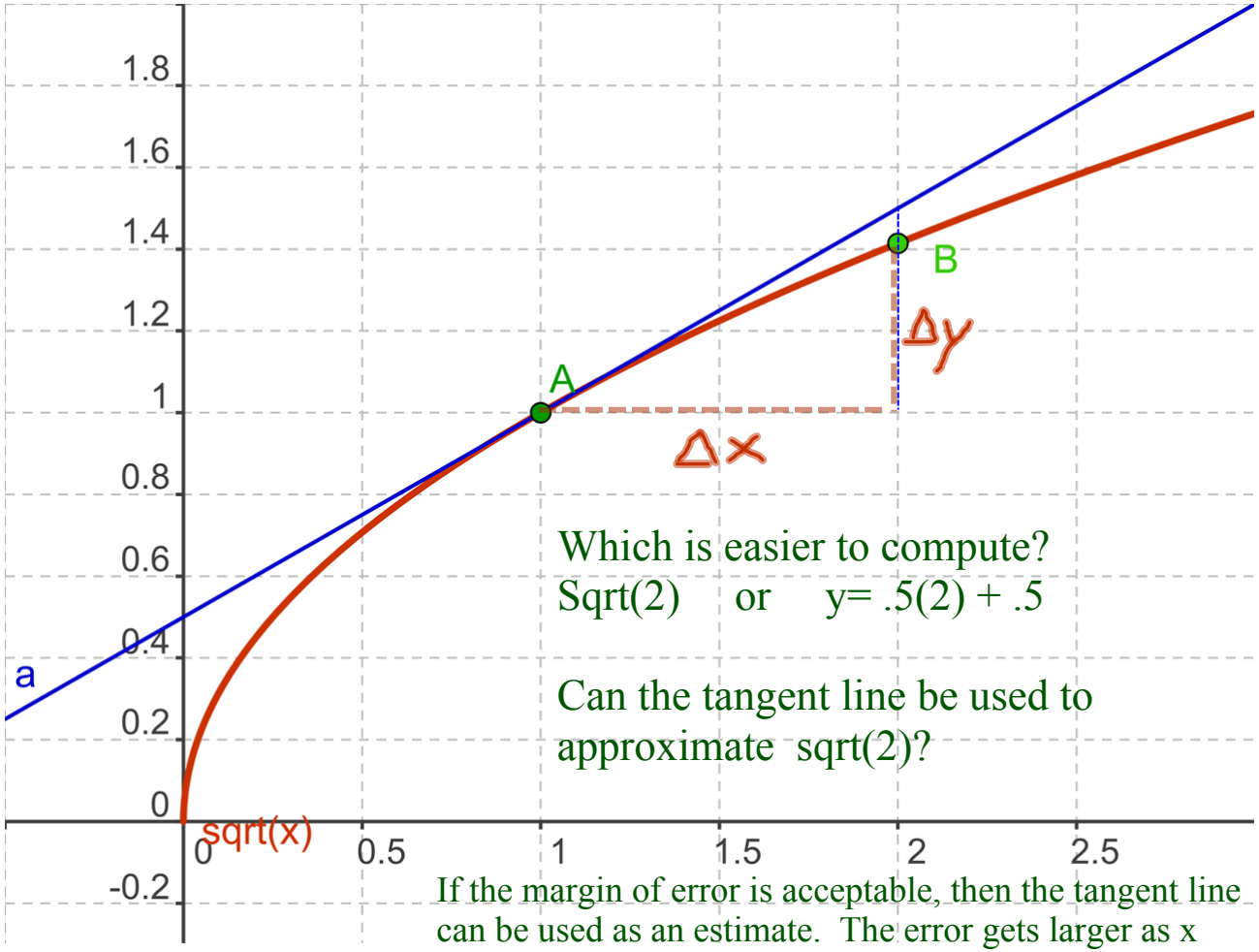
3.9 Differentials



Which is easier to compute?
Sqrt(2) or y= .5(2) + .5

Can the tangent line be used to approximate sqrt(2)?

If the margin of error is acceptable, then the tangent line can be used as an estimate. The error gets larger as x gets further from the original value.

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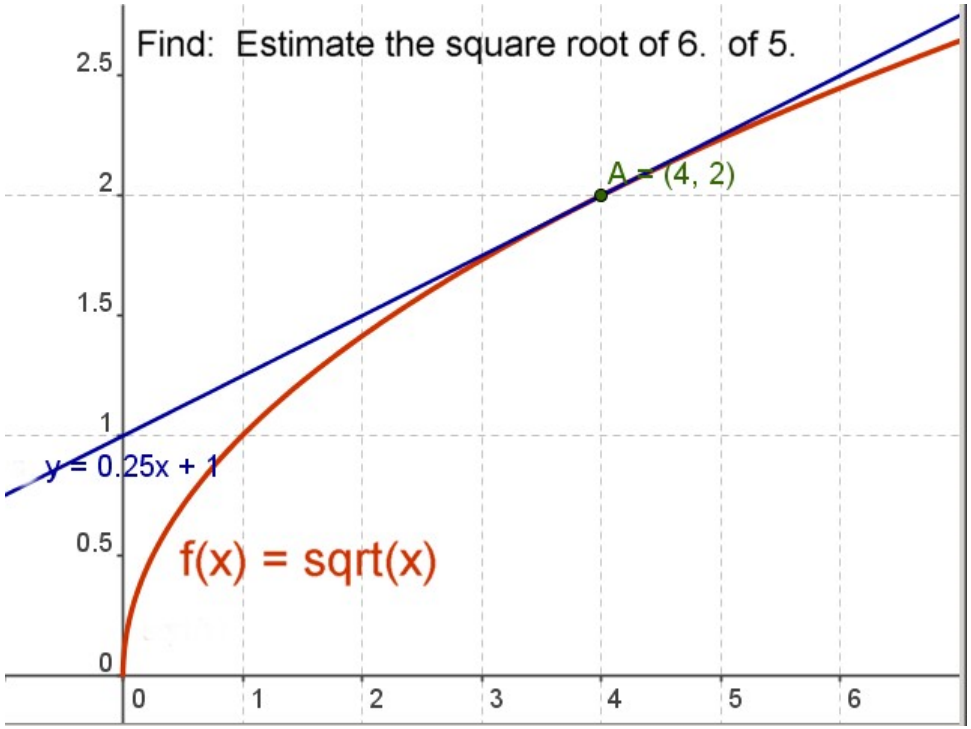
3.9 Differentials

Approximations using Tangent Lines

Estimate $\sqrt{6}$ **Estimate $\sqrt{5}$**



One method is to **use the tangent line**
This can be a good estimate when the value of x is close to a value that is easy to compute.

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Estimate w Tangent

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Another method is to use differentials:

Definition:

Let $y = f(x)$ represent a function that is differentiable on an open interval containing x.

The differential of x, dx, is any nonzero real number.

The differential of y, dy, is:

$$dy = f'(x)\,dx$$

3.9 Differentials

Approximations using Differentials

Find $\sqrt{5}$ Use $y = \sqrt{x}$. Let $x = 4$ and $dx = +1$

$$dy = f'(x)\,dx$$

$$dy = 1/(2\sqrt{x})\,dx$$

$$dy = 1/(2\sqrt{4})\,(1)$$

$$dy = 1/4$$

Therefore, $\sqrt{5} \approx \sqrt{4} + dy = 2 + 1/4 = 2.25$

Calculator ~ 2.24

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Estimate w differentials

p. 240 #31.

F: a) ~ % error in A

b) max allowable error in s of error in A ≤ 2.5%

$s = 15\text{cm} \pm 0.05\text{cm}.$

$A = s^2$

$\frac{dA}{ds} = 2s$

$dA = \frac{dA}{ds}\,ds.$

$dA = 2s\,ds$

a) $dA = 2(15\text{cm})(0.05\text{cm}) = 1.5\text{cm}^2$

$\%\text{error}A = \frac{1.5\text{cm}^2}{15^2\text{cm}^2} = \frac{\cancel{1.5}^{1}}{(15)(\cancel{15}^{10})} = \frac{1}{100}$

$= (.007)100 = 0.7\%$

$\%\text{error}A = \frac{dA}{A} \le 0.025$

$dA = 0.025(15\text{cm})^2 =$

$= 0.025(225) = 5.625\text{cm}^2$

$dA = 2s\,ds$

$5.625\text{cm}^2 = 2(15\text{cm})\,ds$

$ds = \frac{5.625\text{cm}^2}{30\text{cm}} = 0.1875\text{ cm}.$

0.1875cm

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Example 1

3.9 Differentials

Finding Differentials of Functions using Definition of the Differential

G: $y = 3\,x^{2/3}$ F: differential dy

$$dy = f'(x)\ dx$$

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3.9 Differentials

Finding Differentials of Functions using Definition of the Differential

G: $y = 3\,x^{2/3}$ F: differential dy

$$dy = f'(x)\ dx$$

$$dy = 3\,(2/3)\,x^{-1/3}\ dx$$

$$dy = \frac{2}{x^{1/3}}\ dx$$

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3.9 Differentials

Finding Differentials of Functions using Definition of the Differential

G: $y = x\,cosx$ F: differential dy

$$dy = \frac{dy}{dx}\,dx$$

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3.9 Differentials

Finding Differentials of Functions using Definition of the Differential

G: $y = x \cos x$ F: differential dy

$$dy = \frac{dy}{dx}\, dx$$

$$dy = (-x \sin x + \cos x)\, dx$$

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3.9 Differentials

Finding Differentials of Functions using Definition of the Differential

G: $y = 2 - x^4$ F: differential dy

$$dy = \frac{dy}{dx}\, dx$$

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3.9 Differentials

Finding Differentials of Functions using Definition of the Differential

G: $y = 2 - x^4$ F: differential dy

$$dy = \frac{dy}{dx}\, dx$$

$$dy = 4x^3\, dx$$

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3.9 Differentials

Finding Differentials of Functions using Definition of the Differential

G: $y = 3x^5 - 2x^2 + 1$

F: differential dy

$$dy = \frac{dy}{dx}\, dx$$

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3.9 Differentials

Finding Differentials of Functions using Definition of the Differential

G: $y = 3x^5 - 2x^2 + 1$ F: differential dy

$$dy = \frac{dy}{dx}\, dx$$

$$dy = (15x^4 - 4x)\, dx$$

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