3.5 Limits at Infinity

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\# 1, 21-29, 33, 3-7, 15, 17, 41, 49, 51, 55-59, 63

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Homework Part 1

Consider: Sketch

$$y = \frac{1}{x} = x^{-1}$$

At a quick inspection,
what do you know about this function?

$x \neq 0$

also VA: $x = 0$

What information do you need
to sketch this function?

Rel. extr IP concavity

Also, behavior at extreme values of x.

Consider: Sketch

$y = \frac{1}{x} = x^{-1}$ $\quad x \neq 0$

VA: $x = 0$ → $\frac{\text{num} \neq 0}{\text{den} = 0}$

$\frac{dy}{dx} = -x^{-2} = \frac{-1}{x^2}$

No CNs. Only possible CN is x=0, but not in Domain. -> No rel. extr.

$\frac{d^2y}{dx^2} = 2x^{-3} = \frac{2}{x^3}$

IP: where $\frac{d^2y}{dx^2}$ change sign

~~$x = 0$: poss IP~~

$x < 0$, $\frac{d^2y}{dx^2} < 0$ c.d.

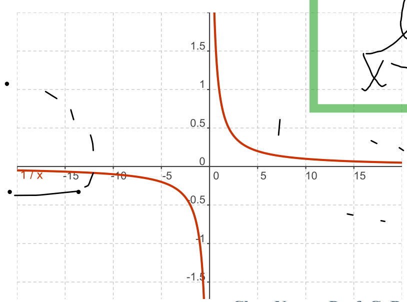
$x > 0$, $\frac{d^2y}{dx^2} > 0$ c.u.

No IPs. Only possible IP is at x=0, but not in Domain. -> No IP

Limits at Infinity

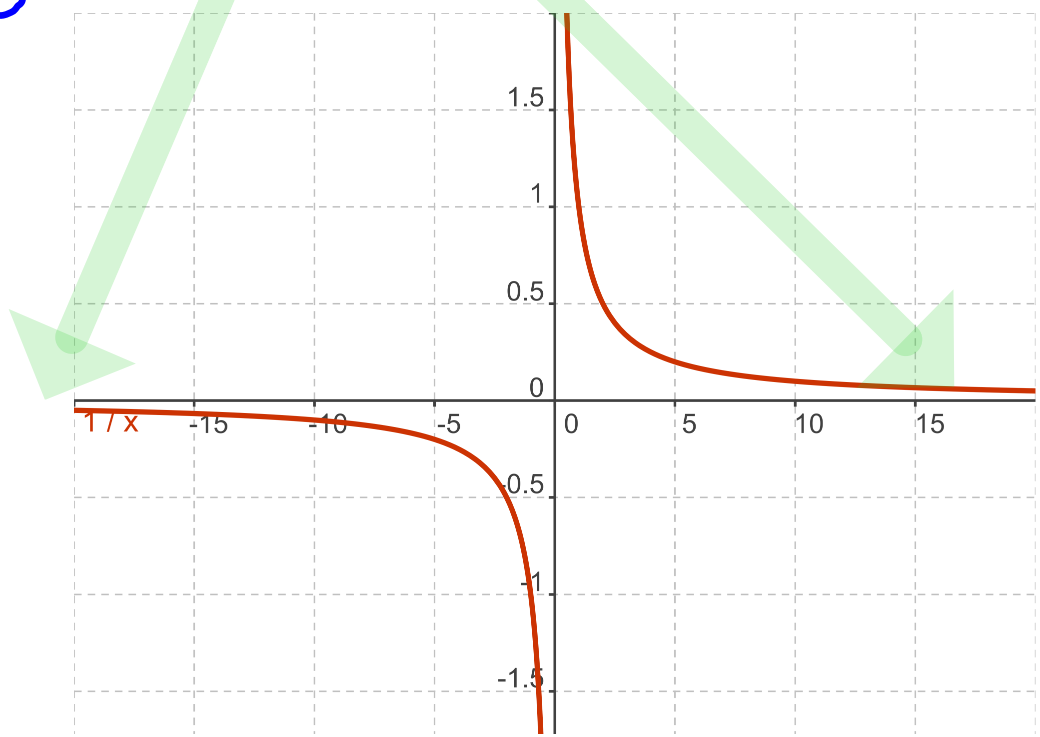
$\lim_{x \to +\infty} \frac{1}{x}$ $\qquad$ $\lim_{x \to -\infty} \frac{1}{x}$

? ? ?



$$\lim_{x \to \infty} \frac{1}{x} = 0$$

$$\lim_{x \to -\infty} \frac{1}{x} = 0$$



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Homework Part 1

Theorems

If r is pos. rational #
and c is any real #,
then $\lim_{x \to \infty} \frac{c}{x^r} = 0$

If x^r is defined when $x < 0$,
then $\lim_{x \to -\infty} \frac{c}{x^r} = 0$

24. $\lim_{x \to \infty} \left(4 + \frac{3}{x}\right) = 4$

$= \lim_{x \to \infty} 4 + \lim_{x \to \infty} \frac{3}{x}$

$= 4 + 0 = 4$

26. $\lim_{x \to -\infty} \left(\frac{1}{2}x - \frac{4}{x^2} \right)$

$= -\infty - 0 = -\infty$

DNE

22. $\lim_{x\to\infty} \frac{3x^3+2x^2}{9x^3-2x^2+7} \cdot \frac{\frac{1}{x^3}}{\frac{1}{x^3}} \to \frac{\infty}{\infty}$

indeterminate

$$\frac{\frac{3x^3}{x^3}+\frac{2x^2}{x^3}}{\frac{9x^3}{x^3}-\frac{2x^2}{x^3}+\frac{7}{x^3}} = \frac{3+\frac{2}{x}}{9-\frac{2}{x}+\frac{7}{x^3}}$$

$$\lim_{x\to\infty} \frac{3+\frac{2}{x}}{9-\frac{2}{x}+\frac{7}{x^3}} = \frac{3+0}{9-0+0} = \frac{1}{3}$$

$$\lim_{x \to \infty} \frac{3x^3+2}{9x^3-2x^2+7} = \frac{1}{3}$$

bec. same order

exp. 3 = 3

$$y = \frac{1}{3} \qquad \lim_{x \to -\infty} \frac{x^2 - x + 3}{3x^3 + 2x} = -\frac{1}{3}$$

$$\lim_{x \to -\infty}$$

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Homework Part 1

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$$\lim_{x\to -\infty} \frac{x}{\sqrt{x^2+1}} \cdot \frac{x}{(x^2+1)^{1/2}} \quad \sqrt{x^2+1} \quad \sqrt{\frac{x^2+1}{x^2}} \quad \sqrt{x^2}$$

$$\frac{x}{\sqrt{x^2+1}} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} = \frac{1}{\frac{\sqrt{x^2+1}}{-\sqrt{x^2}}} = \frac{1}{-\sqrt{\frac{x^2+1}{x^2}}}$$

$$= \frac{1}{-\sqrt{1+\frac{1}{x^2}}}$$

$$\lim_{x\to -\infty} \frac{x}{\sqrt{x^2+1}} = \lim_{x\to -\infty} -\frac{1}{\sqrt{1+\frac{1}{x^2}}}$$

$$= -1$$

$= -1$ since $\sqrt{x^2+1} \approx x$

$y = -1$

Title: example w. radical (10 of 11)

4) $\lim_{x\to\infty} \left(2x - \sqrt{4x^2+1}\right) \to \infty - \infty$

$$\frac{\left(2x - \sqrt{4x^2+1}\right)}{1} \cdot \frac{\left(2x + \sqrt{4x^2+1}\right)}{2x + \sqrt{4x^2+1}}$$

$$\frac{4x^2 - (4x^2+1)}{2x + \sqrt{4x^2+1}} = \frac{-1}{2x + \sqrt{4x^2+1}}$$

$$\lim_{x\to\infty} \left(2x - \sqrt{4x^2+1}\right) = \lim_{x\to\infty} \frac{-1}{2x + \sqrt{4x^2+1}} = 0$$