## 3.3 Increasing & Decreasing Functions and the 1st Derivative Test

9e: Study 3.3 #1,5,9,13,...,37,39,31,35,51,59-69

10e: Study 3.3#1,5,9,13,...,37,31,35,41,49,57,61

Goals:
1. Understand how the sign of the derivative of a function relates to the behavior of the function, re: increasing or decreasing.
2. Use the First Derivative Test to determine relative extrema.

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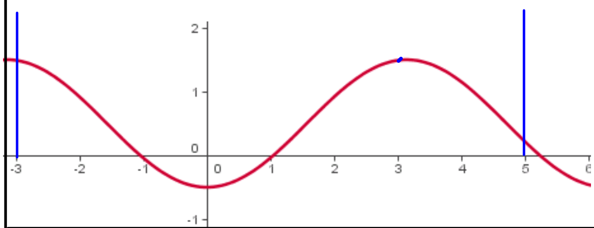
Problems for 3.3

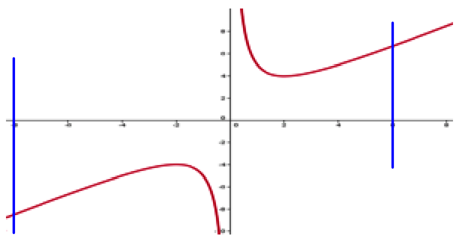
### 3.3 Increasing & Decreasing Functions and the 1st Derivative Test

Find: Intervals where function is increasing or decreasing.



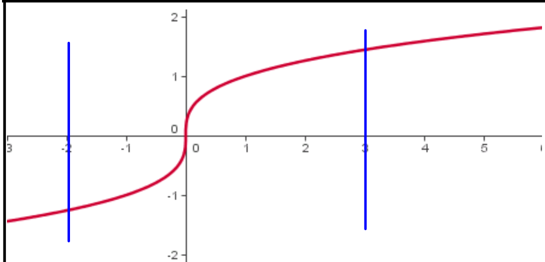

Consider [-3,5]

| Increasing | Decreasing |
|---|---|



Consider [-8,6]

| Increasing | Decreasing |
|---|---|




Consider [-2,3]

| Increasing | Decreasing |
|---|---|

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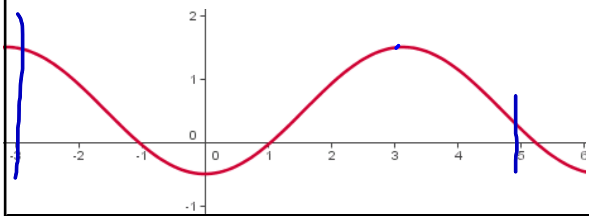
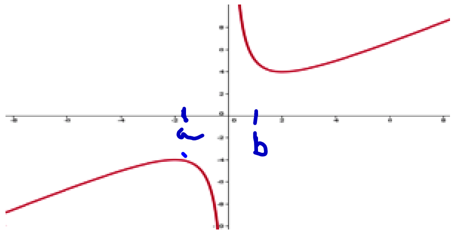
**3.3 Increasing & Decreasing Functions and the 1st Derivative Test**

Find: Intervals where function is increasing or decreasing.

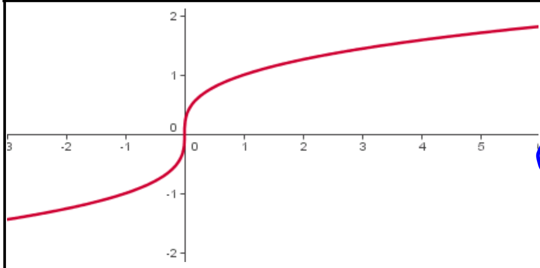
Consider [-3,5]



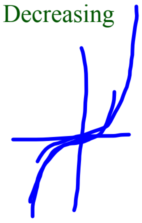

| Increasing | Decreasing |
|---|---|
| $(0,3)$ | $(-3,0)$ |
| | $(3,5)$ |




| Increasing | Decreasing |
|---|---|
| $(-\infty,a)$ | $(a,0)$ |
| $(b,\infty)$ | $(0,b)$ |




| Increasing | Decreasing |
|---|---|
| $(-\infty,\infty)$ | |



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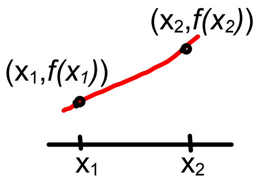
**3.3 Increasing & Decreasing Functions and the 1st Derivative Test**

Definitions:

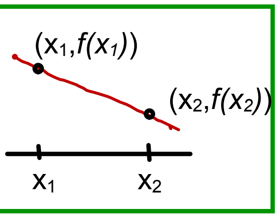
A function **f is increasing** on an interval if,
for any two numbers $x_1$ and $x_2$ in the interval,

**$x_1 < x_2$ implies that $f(x_1) < f(x_2)$.**




A function **f is decreasing** on an interval if,
for any two numbers $x_1$ and $x_2$ in the interval,

**$x_1 < x_2$ implies that $f(x_1) > f(x_2)$.**




Can we tell if a function is increasing or decreasing, if we do not see it's graph? How?
Is there a way to test for increasing or decreasing?

Hint: Consider the slope of the tangent line. Return to graphs.

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**3.3 Increasing & Decreasing Functions and the 1st Derivative Test**

Conclude:

Yes. We can tell if a function is increasing or decreasing, if we consider the slope of the tangent line. In particular we need to look at the sign of the slope. Is it positive or negative?

How can we examine the sign of slope of the tangent line?

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**3.3 Increasing & Decreasing Functions and the 1st Derivative Test**

Conclude:

Yes. We can tell if a function is increasing or decreasing, if we consider the slope of the tangent line. In particular we need to look at the sign of the slope. Is it positive or negative?

How can we examine the sign of slope of the tangent line?

1. Find the derivative.
2. Determine the intervals where the derivative is positive. and where it is negative.

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3.3 Increasing & Decreasing Functions and the 1st Derivative Test

## Test for Increasing & Decreasing Functions

Let f be a function that is continuous on [a,b], and differentiable on (a,b), then:

1. If **$f'(x) > 0$** for all x on (a,b),

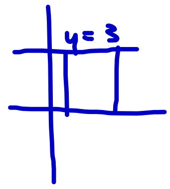
   then f is **increasing** on [a,b]

2. If **$f'(x) < 0$** for all x on (a,b),

   then f is **decreasing** on [a,b]

3. If **$f'(x) = 0$** for all x on (a,b),

   then f is **constant** on [a,b]

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3.3 Increasing & Decreasing Functions and the 1st Derivative Test

Find the intervals where y is increasing and intervals where y is decreasing. $y = -(x-1)^2$

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**3.3 Increasing & Decreasing Functions and the 1st Derivative Test**

Find the intervals where y is increasing and intervals where y is decreasing. $y = -(x - 1)^2$

CN:
1. y defined?
2. x when y' =0?
3. x when y' dne?

$$\frac{dy}{dx} = -2(x-1)(1)$$

CNs: ① y def. ② $\frac{dy}{dx}=0$: $x-1=0$, $x=1$ CN. ③ $\frac{dy}{dx}$ exists all x

| | $(-\infty,1)$ | $(1,\infty)$ |
|---|---|---|
| x | 0 | 2 |
| x − 1 | − | + |
| $\frac{dy}{dx} = -2(x-1)$ | + | − |
| y | incr | decr |

Tabular representation of the number line, the sign of the derivative, and the incr or decr status of the function

y incr on $(-\infty, 1)$
y decr. on $(1, \infty)$

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**3.3 Increasing & Decreasing Functions and the 1st Derivative Test**

Find the intervals where y is increasing and intervals where y is decreasing. $y = x^4 - 2x^2$

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3.3 Increasing & Decreasing Functions and the 1st Derivative Test

Find the intervals where y is increasing and intervals where y is decreasing. $y = x^4 - 2x^2$

$1-2=-1$

$\frac{dy}{dx} = 4x^3 - 4x = 4x(x^2-1) = 4x(x+1)(x-1)$

CNs: $x = -1, 0, 1$

other CNs? No. dy/dx exists for all x

| | $(-\infty,-1)$ | $(-1,0)$ | $(0,1)$ | $(1,\infty)$ |
|---|---|---|---|---|
| x | $-2$ | $-\frac{1}{2}$ | $\frac{1}{2}$ | $2$ |
| $4x$ | $-$ | $-$ | $+$ | $+$ |
| $x+1$ | $-$ | $+$ | $+$ | $+$ |
| $x-1$ | $-$ | $-$ | $-$ | $+$ |
| $\frac{dy}{dx}$ | $-$ | $+$ | $-$ | $+$ |
| y | decr | incr | decr | incr |

$(-1, -1)$ rel. min

$(0, 0)$ rel. max

$(1, -1)$ rel. min.

y decr. $(-\infty,-1)$, $(0,1)$
incr $(-1,0)$, $(1,\infty)$

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3.3 Increasing & Decreasing Functions and the 1st Derivative Test

**Wow!** We can use this approach to determine max and mins!

## The First Derivative Test for Relative Extrema

Let **c be a Critical Number** of the function f that is continuous on the open interval I containing c. If f is differentiable on the interval, except possibly at c, then f(c) can be classified as:

1. **a relative min, if f ' (x) changes from negative to positive at c.** \ /
2. **a relative maximum, if f ' (x) changes from positive to negative at c** / \
3. **neither a max nor a min if f ' (x) is positive on both sides of c** / / **or negative on both sides of c** \ \

step-by-step: 1st Deriv Test

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3.3 Increasing & Decreasing Functions and the 1st Derivative Test

**Wow!** We can use this approach to determine max and mins!

**To Find Relative Extrema of a continuous function using intervals and the First Derivative Test:**

1. **Find critical numbers [f '(c) = 0 or f '(c) undefined]**
2. **Determine intervals for evaluation of f ' and begin the interval table:**
   a) **Locate the critical numbers along a number line containing the domain of the function.**
   b) **Determine the intervals, using the critical numbers as endpoints.**
3. **Continue the interval table by:**
   a) **Selecting a test value for each interval.**
   b) **Express f '(x) in factored form, and write each factor in the first column.**
   c) **Find the sign of each factor in each interval and indicate the sign on the table.**
   d) **For each interval, find the sign of f '(x) by determining the number of negative factors.**
4. **Determine whether f(x), the original function, is increasing (when f '(x) >0) or decreasing (when f '(x) <0) on each interval.**
5. **The critical value for which f(x) is increasing to the left and decreasing to the right is a relative max. / **
6. **The critical value for which f(x) is decreasing to the left and increasing to the right is a relative min. \ /**
7. **Find the corresponding f or y value for each critical value determined to be a relative max or min, and write the ordered pair (c,f(c)).**

step-by-step: 1st Deriv Test

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3.3 Increasing & Decreasing Functions and the 1st Derivative Test

G: $f(x) = x^2 + 8x + 10$  F: a) CNs, b) inter inc, decr, c) rel.extrema

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step-by-step: 1st Deriv Test

Problems for 3.3

### 3.3 Increasing & Decreasing Functions and the 1st Derivative Test

G: $f(x) = x^2 + 8x + 10$ F: a) CNs, b) inter inc, decr, c) rel.extrema

$f(x) = x^2 + 8x + 10$

F: a) CNs, b) inter. ↑ ↓ c) rel. extr.

$f'(x) = 2x + 8 = 2(x+4)$

CN: $x = -4$

other CNs? No.
dy/dx exists for all x

| | $(-\infty,-4)$ | $-4$ | $(-4,\infty)$ |
|---|---|---|---|
| $x$ | $-5$ | | $0$ |
| $f'(x)=2(x+4)$ | $-$ | | $+$ |
| $f(x)$ | decr. | | incr. |

$f(-4) = (-4)^2 + 8(-4) + 10$
$= 16 - 32 + 10$
$-6$

$(-4, -6)$ rel. min

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### 3.3 Increasing & Decreasing Functions and the 1st Derivative Test

$f(x) = x^3 - 6x^2 + 15$

F: intervals where f is incr, decr
relative extrema

step-by-step: 1st Deriv Test

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### 3.3 Increasing & Decreasing Functions and the 1st Derivative Test

F: intervals where f is incr, decr
relative extrema

step-by-step: 1st Deriv Test

$f(x) = x^3 - 6x^2 + 15$

$64 - 96 + 15 = 79 - 96$

$f'(x) = 3x^2 - 12x$

CNs $f'(x) = 0$ or ~~DNE~~

$3x(x-4) = 0$

$3x = 0 \quad | \quad x - 4 = 0$

$x = 0 \quad | \quad x = 4$ CN.

| | $(-\infty, 0)$ | $(0, 4)$ | $(4, \infty)$ |
|---|---|---|---|
| $x$ | $-1$ | $1$ | $5$ |
| $3x$ | $-$ | $+$ | $+$ |
| $x-4$ | $-$ | $-$ | $+$ |
| $f'(x)$ | $+$ | $-$ | $+$ |
| $f(x)$ | incr | decr | incr. |

$(0, 15)$ rel. MAX

$(4, -17)$ rel MIN

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