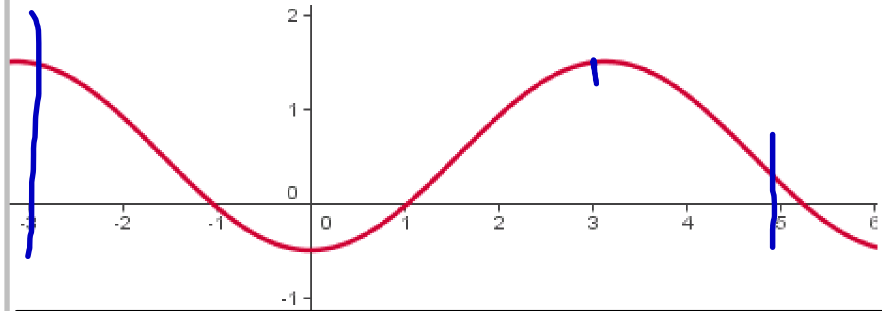
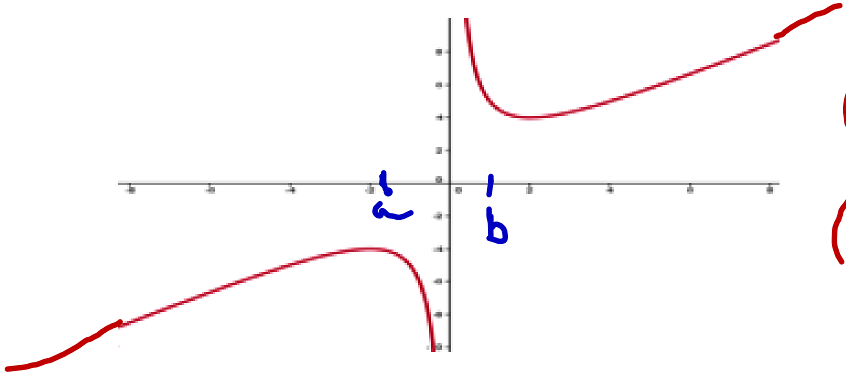
3.3 Increasing & Decreasing Functions and the 1st Derivative Test

p. 186 # 1, 5, 9, 13, ... 37, 39

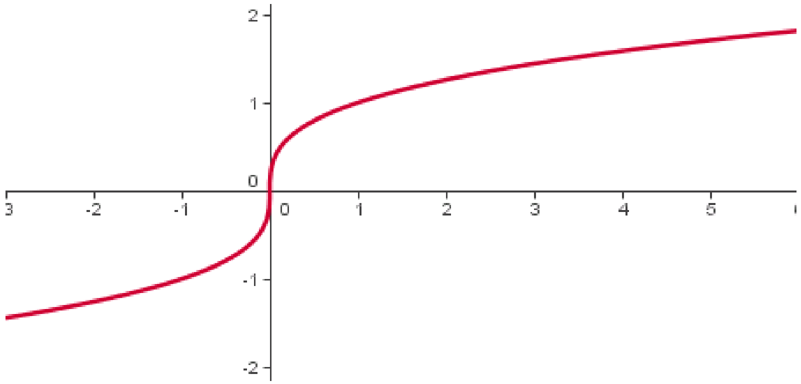
Find: Intervals where function is increasing or decreasing.



Increasing	Decreasing
$(0, 3)$	$(-3, 0)$
	$(3, 5)$



Increasing	Decreasing
$(-\infty, a)$	$(a, 0)$
(b, ∞)	$(0, b)$



Increasing	Decreasing
$(0, \infty)$	
$(-\infty, 0)$	

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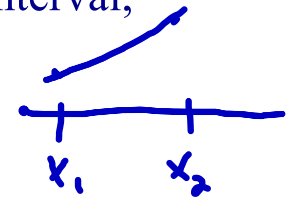
Definitions:

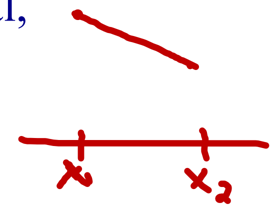
A function **f is increasing** on an interval if,
for any two numbers x_1 and x_2 in the interval,

$x_1 < x_2$ implies that $f(x_1) < f(x_2)$.



A function **f is decreasing** on an interval if,
for any two numbers x_1 and x_2 in the interval,

$x_1 < x_2$ implies that $f(x_1) > f(x_2)$.



How can we tell if a function is increasing or decreasing, if we do not see it's graph?
Is there a way to test for increasing or decreasing?

Hint: Consider the slope of the tangent line. Return to graphs.

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Problems for 3.3

Conclude:

Yes. We can tell if a function is increasing or decreasing, if we consider the slope of the tangent line. In particular we need to look at the sign of the slope. Is it positive or negative?

How can we examine the sign of slope of the tangent line?

Find derivative.

Determine the intervals where the derivative is positive.

is negative.

Test for Increasing & Decreasing Functions

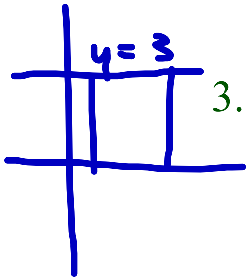
Let f be a function that is continuous on [a,b],
add differentiable on (a,b), then:

1. If **$f'(x) > 0$** for all x on (a,b),

   then f is **increasing** on [a,b]

2. If **$f'(x) < 0$** for all x on (a,b),

   then f is **decreasing** on [a,b]



3. If **$f'(x) = 0$** for all x on (a,b),

   then f is **constant** on [a,b]

p. 186 # 4 Find the intervals where y is increasing and intervals where y is decreasing. $y = -(x - 1)^2$

$$\frac{dy}{dx} = -2(x-1)'(1) = -2(x-1)$$

$$-2(x-1) = 0 \qquad x = 1$$

$x < 1$: $x = 0$, $(-2)(-1) > 0$

$x < 1$, $\frac{dy}{dx} > 0 \Rightarrow$ $(-\infty, 1)$ y is incr.

$x > 1$: $x = 2$, $(-2)(1) < 0$ $\therefore \frac{dy}{dx} < 0$ $(1, \infty)$ y decr.

$(1, \infty)$ y decr.

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Problems for 3.3

p. 186 # 6 Find the intervals where y is increasing and intervals where y is decreasing. $y = x^4 - 2x^2$

$1 - 2 = -1$

$$\frac{dy}{dx} = 4x^3 - 4x = 4x(x^2 - 1) = 4x(x+1)(x-1)$$

CNs: $x = -1, 0, 1$

	$(-\infty, -1)$	$(-1, 0)$	$(0, 1)$	$(1, \infty)$
x	-2	$-\frac{1}{2}$	$\frac{1}{2}$	2
$4x$	$-$	$-$	$+$	$+$
$x+1$	$-$	$+$	$+$	$+$
$x-1$	$-$	$-$	$-$	$+$
$\frac{dy}{dx}$	$-$	$+$	$-$	$+$

y decr. $(-\infty, -1)$, $(0, 1)$
incr $(-1, 0)$, $(1, \infty)$

$(-1, -1)$ rel. min

$(0, 0)$ rel. max

$(1, -1)$ rel. min.

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Problems for 3.3

Title: example 2 (7 of 10)

Wow! We can use this approach to determine max and mins!

The First Derivative Test for Relative Extrema

Let **c be a Critical Number** of the function f that is continuous on the open interval I containing c. If f is differentiable on the interval, except possibly at c, then f(c) can be classified as:

1. **a relative min, if f ' (x) changes from negative to positive at c \ /**

2. **a relative maximum, if f ' (x) changes from positive to negative at c / **

3. **neither a max nor a min if f ' (x) is positive on both sides of c / / or negative on both sides of c \ **

3.3, # 18. G: $f(x) = x2 + 8x + 10$ F: a) CNs, b) inter inc, decr, c) rel.extrema

$f(x) = x^2 + 8x + 10$

$f'(x) = 2x + 8 = 2(x+4)$

CN: $x = -4$

F: a) CNs ✓
b) inter. ↑, ↓ ✓
c) rel. extr.

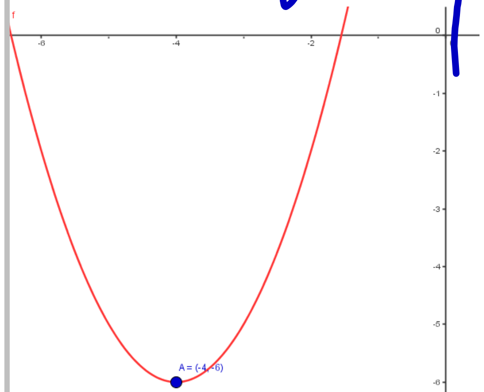
	$(-\infty, -4)$	$(-4, \infty)$
x	-5	0
$f'(x) = 2(x+4)$	$-$	$+$
$f(x)$	decr.	incr.

$f(-4) = (-4)^2 + 8(-4) + 10$

$= 16 - 32 + 10$

-6

$(-4, -6)$ rel. min

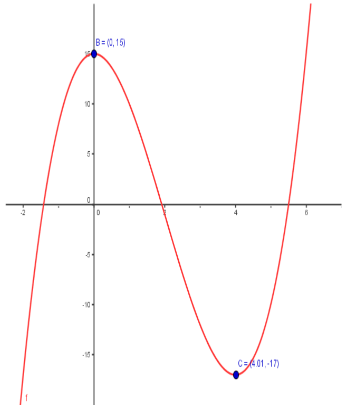


$$f(x) = x^3 - 6x^2 + 15$$

$$f'(x) = 3x^2 - 12x$$

$$3x(x-4) = 0$$

$x = 0$ | $x = 4$



	$(-\infty, 0)$	$(0, 4)$	$(4, \infty)$
x	-1	2	5
$3x$	$-$	$+$	$+$
$x-4$	$-$	$-$	$+$
$f'(x)$	$+$	$-$	$+$
$f(x)$			

(0,15) rel. Max

(4,[illegible]) rel. Min