3.2 Rolle's Theorum and the Mean Value Theorum

10e: Study 3.2 #1-5,11-17,21,23,26,27,37-45,47*,57

Goals:
1. Understand Rolle's Theorum and when to use it.
2. Understand The Mean Value Theorum and how to use it.

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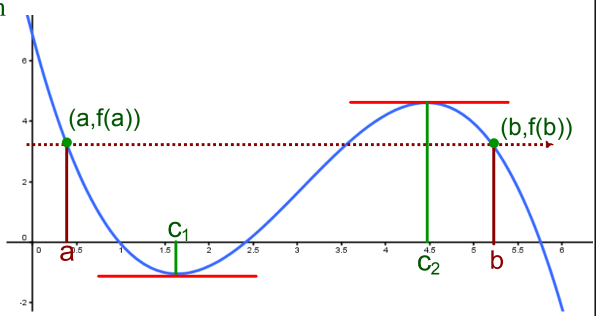
Problems for 3.2

3.2 Rolle's Theorum and the Mean Value Theorum



Rolle's Theorum

Let f be:
1. continuous on closed interval [a,b]
2. differentiable on open interval (a,b)
3. If f(a) = f(b),

then $\exists$ at least one $c \in (a,b)$

$\ni f'(c) = 0$

(there exists at least one c on the interval from a to b such that f ' (c) =0)

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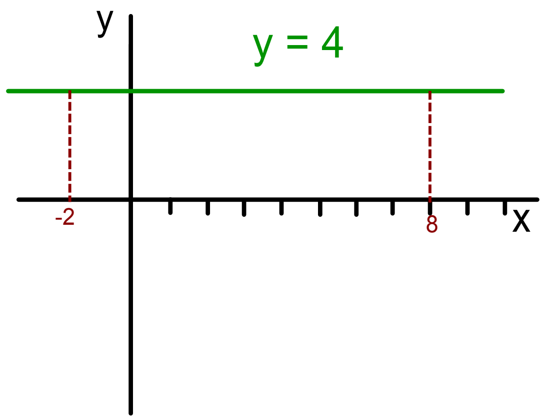
3.2 Rolle's Theorum and the Mean Value Theorum

Does RT apply to y = 4 on the interval [-2, 8] ?



How many c 's?

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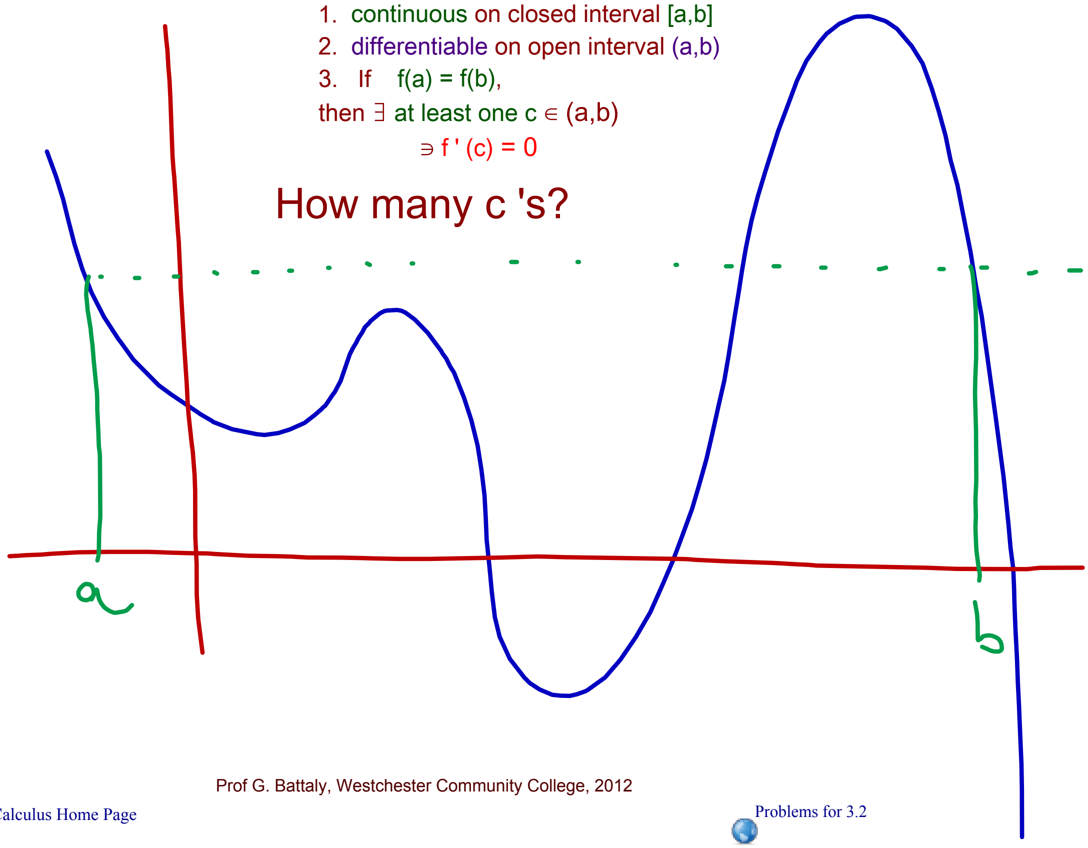
Problems for 3.2

3.2 Rolle's Theorum and the Mean Value Theorum

1. continuous on closed interval [a,b]
2. differentiable on open interval (a,b)
3. If f(a) = f(b),
then ∃ at least one c ∈ (a,b)
∋ f ' (c) = 0

How many c 's?



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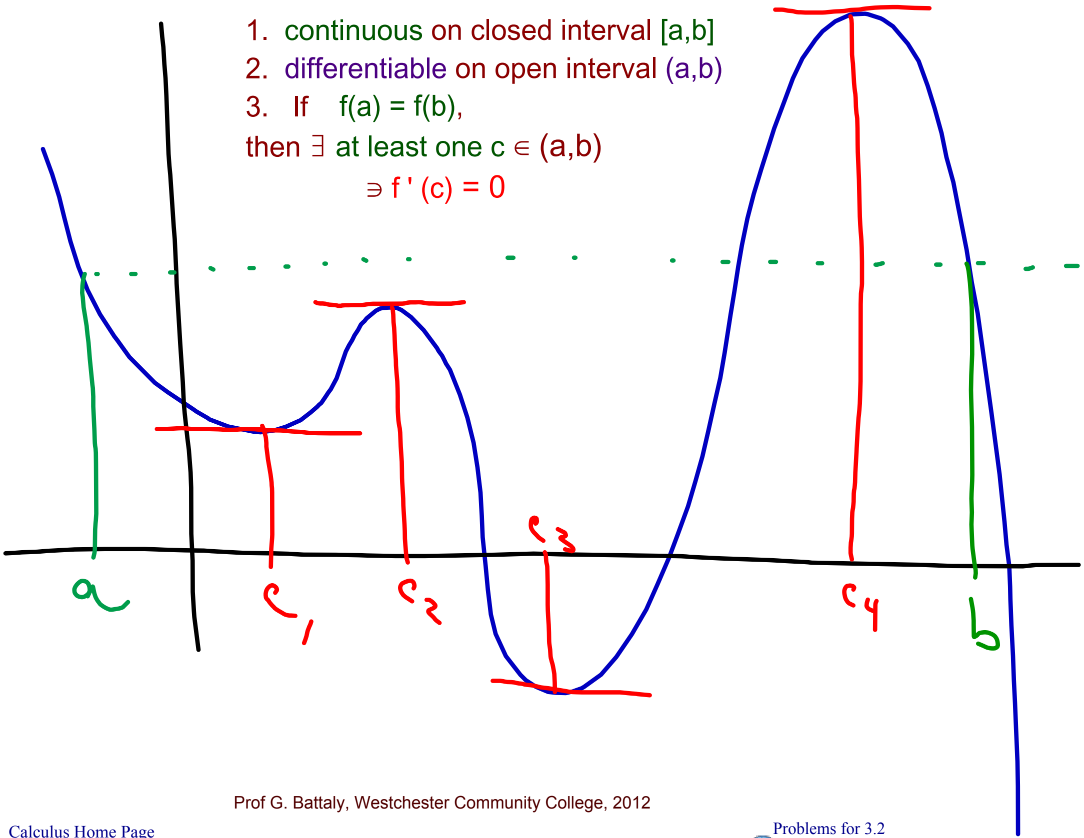
Problems for 3.2

3.2 Rolle's Theorum and the Mean Value Theorum

1. continuous on closed interval [a,b]
2. differentiable on open interval (a,b)
3. If $f(a) = f(b)$,

then $\exists$ at least one $c \in (a,b)$

$\ni f'(c) = 0$



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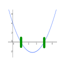
Problems for 3.2

3.2 Rolle's Theorum and the Mean Value Theorum

12. G: $f(x) = x^2 - 5x + 4$

F: Does RT apply on [1, 4] ?

If yes, F: $c \ni f'(c) = 0$



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Problems for 3.2

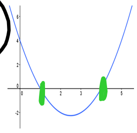
3.2 Rolle's Theorum and the Mean Value Theorum

12. G: $f(x) = x^2 - 5x + 4$

F: Does RT apply on ~~[1, 4]~~ ? Yes!

If yes, F: $c \ni f'(c) = 0$

$c = \frac{5}{2}$



① f cont on $[1,4]$? Yes } polyn.

② f " $(1,4)$? Yes.

③ $f(1) = f(4)$? Yes. } RT applies

$f(1) = 1 - 5 + 4 = 0$

$f(4) = 16 - 20 + 4 = 0$

$f'(x) = 2x - 5$

$2x - 5 = 0$

$c = x = 5/2 \overset{?}{\in} (1,4)$

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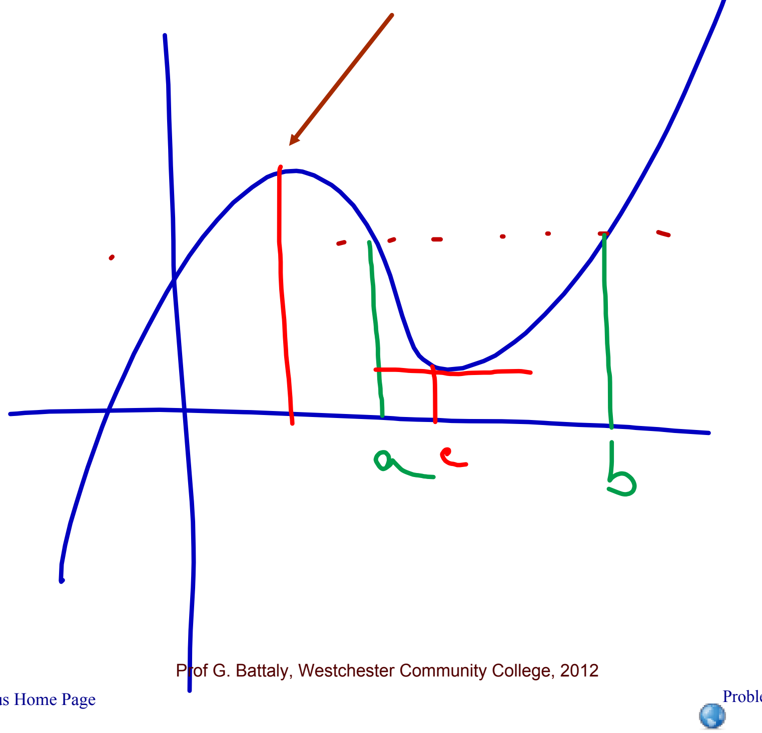
Problems for 3.2

3.2 Rolle's Theorum and the Mean Value Theorum

When Rolle's Theorum applies and you have found possible values for c by finding the first derivative and setting it equal to 0, be sure to check that the possible c values are on the interval (a, b). It is possible that you have found a CN that is not on the interval.



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Problems for 3.2

3.2 Rolle's Theorum and the Mean Value Theorum

$y = x - x^{1/3}$

F: Does RT apply on [0,1]?
If yes, find c on (0,1)
where f '(c) =0

x=0 not on (0,1)

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Problems for 3.2

3.2 Rolle's Theorum and the Mean Value Theorum

$y = x - x^{1/3}$

F: Does RT apply on [0,1]?
If yes, find c on (0,1)
where f '(c) =0

$c = \frac{1}{3\sqrt{3}} = \frac{\sqrt{3}}{9}$

3 (0, 0) (1, 0) Yes.

1 cont. [0,1]? yes

2 diff (0,1)? yes.

∴ RT applies.

$\frac{dy}{dx} = 1 - \frac{1}{3}x^{-2/3}$

$= 1 - \frac{1}{3x^{2/3}}$

$= \frac{3x^{2/3} - 1}{3x^{2/3}}$

DNE at x=0

x=0 not on (0,1)

for f '(c) =0: $3x^{2/3} - 1 = 0$

$3x^{2/3} = 1$

$\left(x^{2/3}\right)^{3/2} = \left(\frac{1}{3}\right)^{3/2}$

$x = \left(\frac{1}{3}\right)^{3/2}$

$(a^m)^n = a^{mn}$

$\boxed{x = \frac{1}{3\sqrt{3}} \in (0,1)}$

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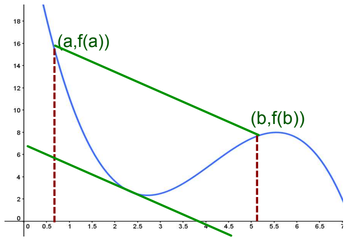
Problems for 3.2

3.2 Rolle's Theorum and the Mean Value Theorum

Suppose
① f(x) cont [a,b]
② f(x) diff. (a,b)
f(a) ≠ f(b)



What happens to f ' (x) on the interval?
Compare f ' (x) to m_{sec}.
Does f ' (x) = m_{sec} anywhere on interval (a,b)?

Mean Value Theorem

$$m_{sec} = \frac{f(b)-f(a)}{b-a}$$

geogebra demo

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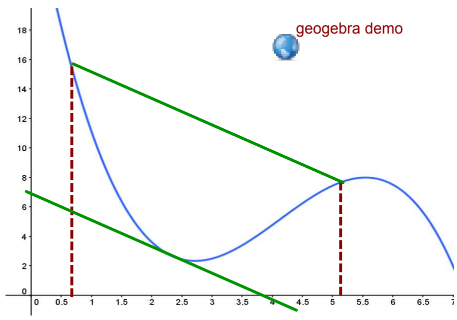
Problems for 3.2

3.2 Rolle's Theorum and the Mean Value Theorum

Mean Value Theorum



geogebra demo

Let f be:
1. continuous on closed interval [a,b] and
2. differentiable on open interval (a,b)

then ∃ at least one c ∈ (a,b) ∋

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

Interpretation:

There exists at least one *c* on the interval from *a* to *b* such that the derivative at *c* equals the slope of the secant line joining the endpoints.

ALSO: There exists at least one c on the interval where the instantaneous rate of change equals the average value.

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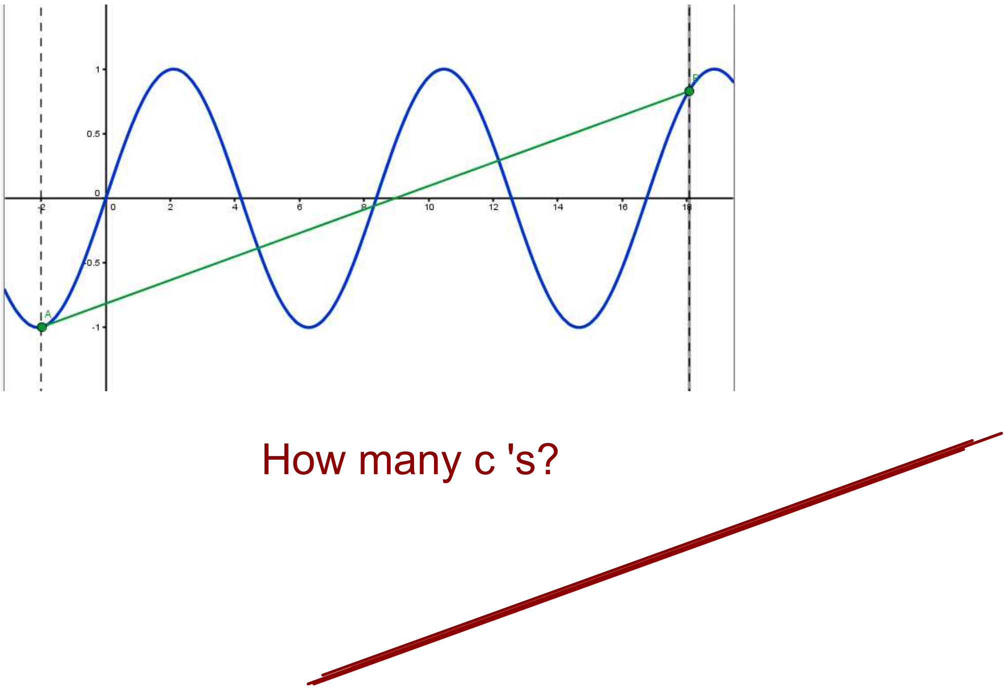
Problems for 3.2

3.2 Rolle's Theorum and the Mean Value Theorum



How many c 's?

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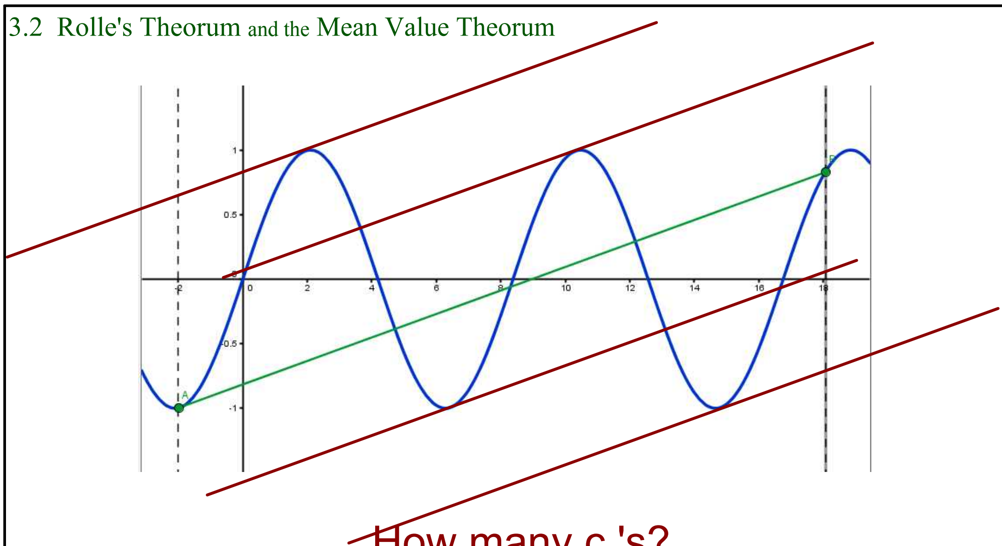
Problems for 3.2

3.2 Rolle's Theorum and the Mean Value Theorum



How many c 's?

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Problems for 3.2

3.2 Rolle's Theorum and the Mean Value Theorum

G: $f(x) = x(x^2 - x - 2)$

F: a) MVT apply on [-1, 1]?

b) If yes, find c (-1, 1)

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

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Problems for 3.2

3.2 Rolle's Theorum and the Mean Value Theorum

p.177 #40.

G: $f(x) = x(x^2 - x - 2)$

F: a) MVT on $[-1,1]$? Yes.

b) If yes, c?

① f cont on $[-1,1]$? Yes. Polyn.

② f diff on $(-1,1)$? Yes Polyn.

$f(x) = x^3 - x^2 - 2x$

$f'(x) = 3x^2 - 2x - 2$

$3x^2 - 2x - 2 = -1$

$3x^2 - 2x - 1 = 0$

$3x^2 - 3x + x - 1 = 0$

$(-3)(1) = -3$

$(-3) + (+1) = -2$

$3x(x-1) + 1(x-1) = 0$

$(3x+1)(x-1) = 0$

$3x+1 = 0$ | $x - 1 = 0$

$3x = -1$ | $\cancel{x = 1} \notin (-1,1)$

$x = -1/3$

$-\frac{1}{3} \in (-1,1)$

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$f'(c) = \frac{f(1) - f(-1)}{1 - (-1)}$$

$f(1) = 1(1 - 1 - 2) = -2$

$f(-1) = (-1)(1 + 1 - 2) = 0$

$$f'(c) = \frac{-2 - 0}{2} = -1$$

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Problems for 3.2

3.2 Rolle's Theorum and the Mean Value Theorum

p.176 #17

G: $f(x) = \frac{x^2-2x-3}{x+2}$ $x \neq -2$

F: a) RT on $[-1,3]$?

b) If yes, $c \ni f'(c)=0$

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Problems for 3.2

3.2 Rolle's Theorum and the Mean Value Theorum

p.176 #17

G: $f(x) = \frac{x^2-2x-3}{x+2}$ $x \neq -2$

F: a) RT on $[-1,3]$? Yes

b) If yes, $c \ni f'(c)=0$

① $f(x)$ cont on $[-1,3]$? Yes. poly. $-2 \notin [-1,3]$

② " diff on $(-1,3)$? Yes. "

③ $f(-1) \stackrel{?}{=} f(3)$

$0 = 0$ ✓ Yes.

$f(-1) = \frac{1+2-3}{-1+2} = \frac{0}{1} = 0$ | $f(3) = \frac{9-6-3}{3+2} = \frac{0}{5} = 0$

Yes. ∴ RT's applies ∴ $\exists c \in (-1,3) \ni f'(c) = 0$

$f(x) = \frac{x^2-2x-3}{x+2}$

$$f'(x) = \frac{(x+2)(2x-2)-(x^2-2x-3)(1)}{(x+2)^2} = \frac{2(x+2)(x-1)-(x-3)(x+1)}{(x+2)^2}$$

no common factors

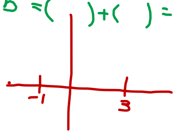
$f'(x) = 0$ when num $= 0$

$2x^2+2x-4-x^2+2x+3 = x^2+4x-1 = 0$

$(\quad x \quad) = 0$

$ac = (\quad)(\quad) = -1$

$b = (\quad)+(\quad) = 4$



$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \frac{-4 \pm \sqrt{16+4}}{2} = \frac{-4 \pm \sqrt{20}}{2}$

$= \frac{-4 \pm 2\sqrt{5}}{2} = -2 \pm \sqrt{5}$

$c = -2+\sqrt{5} \approx -2+2.\ldots \in (-1,3)$

$\cancel{-2-\sqrt{5}}$ $\cancel{\in}$ $(-1,3)$

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Problems for 3.2

G: $f(x) = |2x+1|$

F: MVT apply $[-1,3]$?
If yes, f: c $\ni$
$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

$$|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$$

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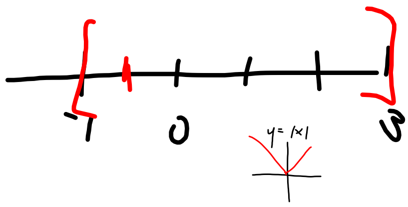
45. G: $f(x) = |2x+1|$ F: MVT apply $[-1,3]$?
If yes, f: c $\ni$
$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

① f cont $[-1,3]$? Yes
② f diff $(-1,3)$? No. Sharp turn at $x = -\frac{1}{2}$

$$|2x+1| = \begin{cases} 2x+1, & 2x+1 \geq 0 \quad x \geq -\frac{1}{2} \quad ? \; (-1,3)? \\ -(2x+1), & 2x+1 < 0 \quad x < -\frac{1}{2} \quad ? \; (-1,3)? \end{cases}$$

$$f'(x) = \begin{cases} 2, & x > -\frac{1}{2} \\ -2 & x < -\frac{1}{2} \end{cases}$$



$\therefore$ MVT does NOT apply

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From 3.1: CNs

6: $f(x) = (\sin x)^2 + \cos x, \quad 0 < x < 2\pi$ F: CN

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Problems for 3.2

From 3.1: CNs

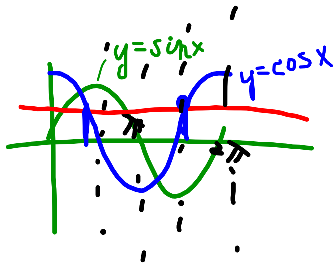
6: $f(x) = (\sin x)^2 + \cos x, \quad 0 < x < 2\pi$ F: CN

$f'(x) = 0$ or ~~DNE~~

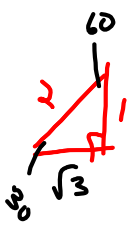
$f'(x) = 2\sin x \cos x + (-\sin x)$ exists all x

$2\sin x \cos x - \sin x = 0$

$\sin x(2\cos x - 1) = 0$

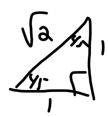


$\sin x = 0$	$2\cos x - 1 = 0$
$\boxed{x = \pi \quad \text{CN}}$	$\cos x = \frac{1}{2}$
	$x = 60^\circ, 300^\circ$
	$\boxed{x = \frac{\pi}{3}, \frac{5\pi}{3} \quad \text{CN}}$



$\tan 45^\circ = 1$

$\sin 30^\circ = \frac{1}{2}$



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