3.2 Rolle's Theorum and the Mean Value Theorum

Study 3.2 p. 176 #1-5, 13-19, 23,25,28,29
39-43, 47*, 49

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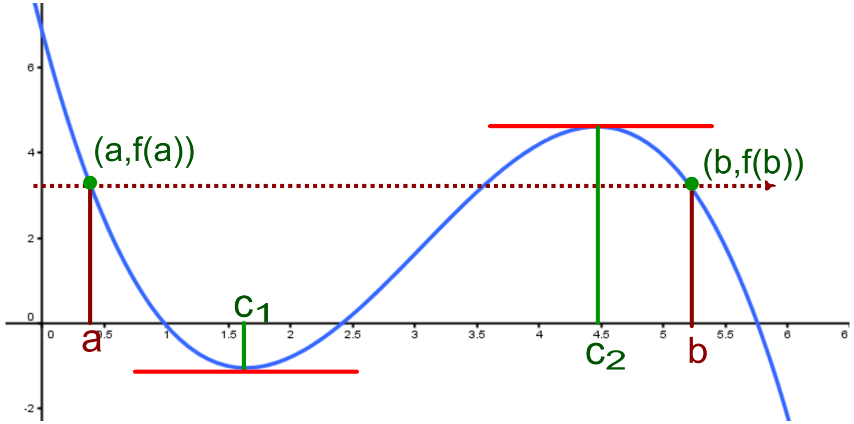
Problems for 3.2

3.2 Rolle's Theorum and the Mean Value Theorum



Rolle's Theorum

Let f be:
1. continuous on closed interval [a,b]
2. differentiable on open interval (a,b)

If

3. f(a) = f(b),

then $\exists$ at least one $c \in (a,b)$

$\ni f'(c) = 0$

(there exists at least one c on the interval from a to b such that f ' (c) =0)

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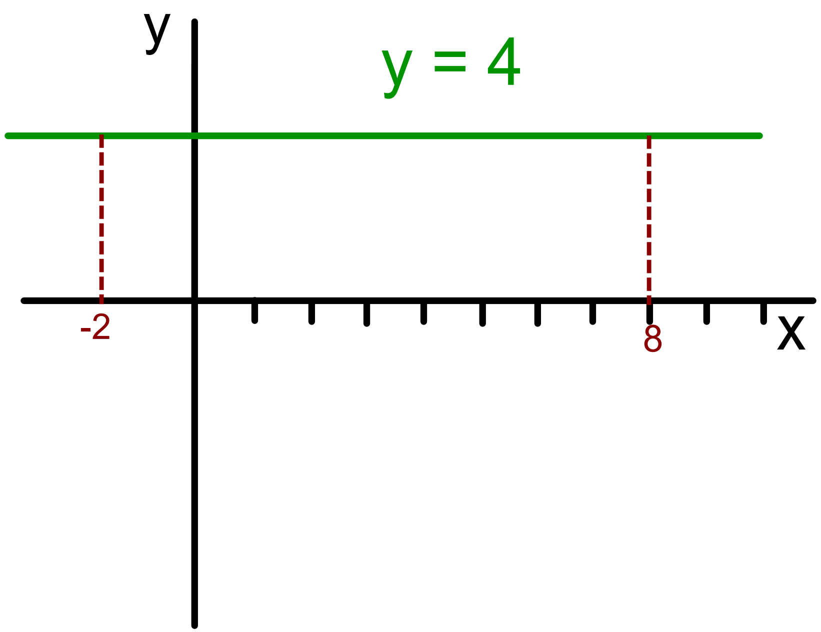
Problems for 3.2

3.2 Rolle's Theorum and the Mean Value Theorum

Does RT apply to y = 4 on the interval [-2, 8] ?



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Problems for 3.2

3.2 Rolle's Theorum and the Mean Value Theorum

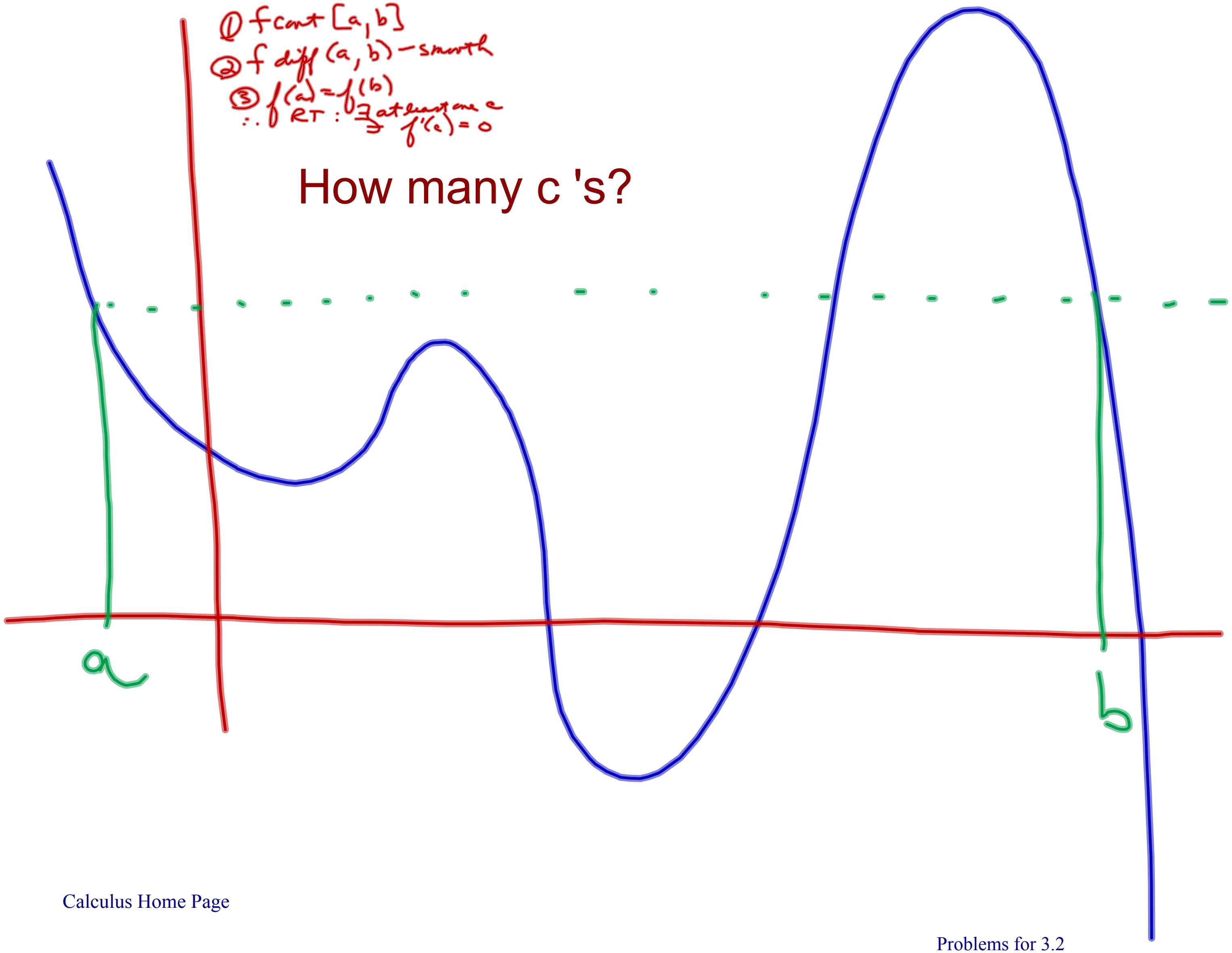
① f cont [a, b]

② f diff (a, b) – smooth

③ f(a) = f(b)

∴ RT: ∃ at least one c ∋ f'(c) = 0

How many c 's?



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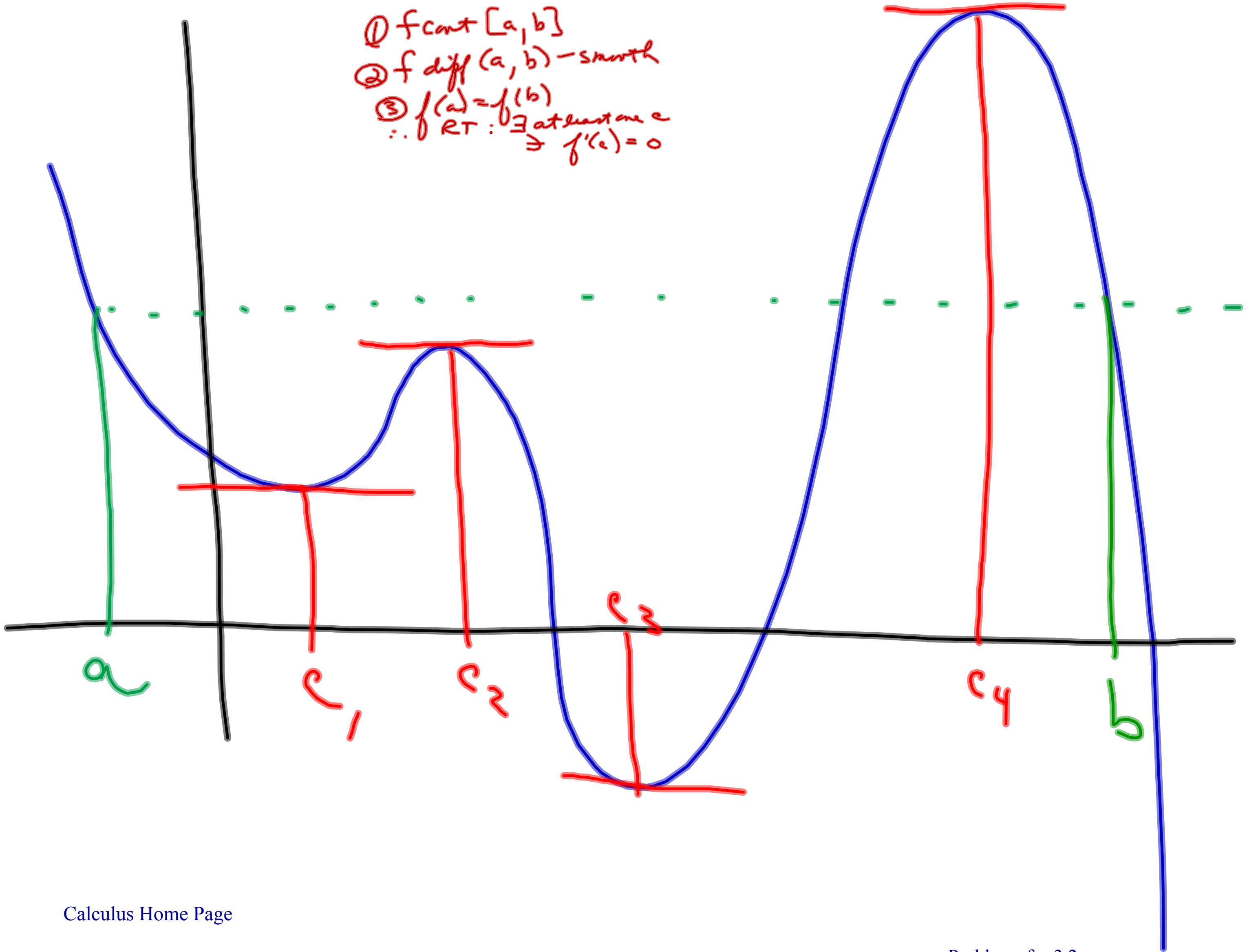
Problems for 3.2

3.2 Rolle's Theorum and the Mean Value Theorum

① f cont [a,b]
② f diff (a,b) – smooth
③ f(a) = f(b)
∴ RT: ∃ at least one c ∋ $f'(c) = 0$



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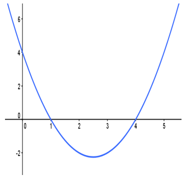
Problems for 3.2

3.2 Rolle's Theorum and the Mean Value Theorum

12. G: $f(x) = x^2 - 5x + 4$

F: Does RT apply on [1, 4] ?
If yes, F: $c \ni f'(c) = 0$



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Problems for 3.2

3.2 Rolle's Theorum and the Mean Value Theorum

12. G: $f(x) = x^2 - 5x + 4$

F: RT? $[1,4]$

If yes, $c \ni f'(c) = 0$

RT:

① f cont on $[1,4]$? Yes. Polyn.

② f diff. on $(1,4)$? Yes "

③ $f(1) \stackrel{?}{=} f(4)$ Yes.

$f(1) = 1^2 - 5 + 4 = 0$

$f(4) = 4^2 - 5(4) + 4 = 16 - 20 + 4 = 0$

$\therefore f(1) = f(4) = 0$

Yes. RT. applies.

$f'(x) = 2x - 5$

$2x - 5 = 0$

$x = 5/2 = c.$ $\{\in (1,4)\}$

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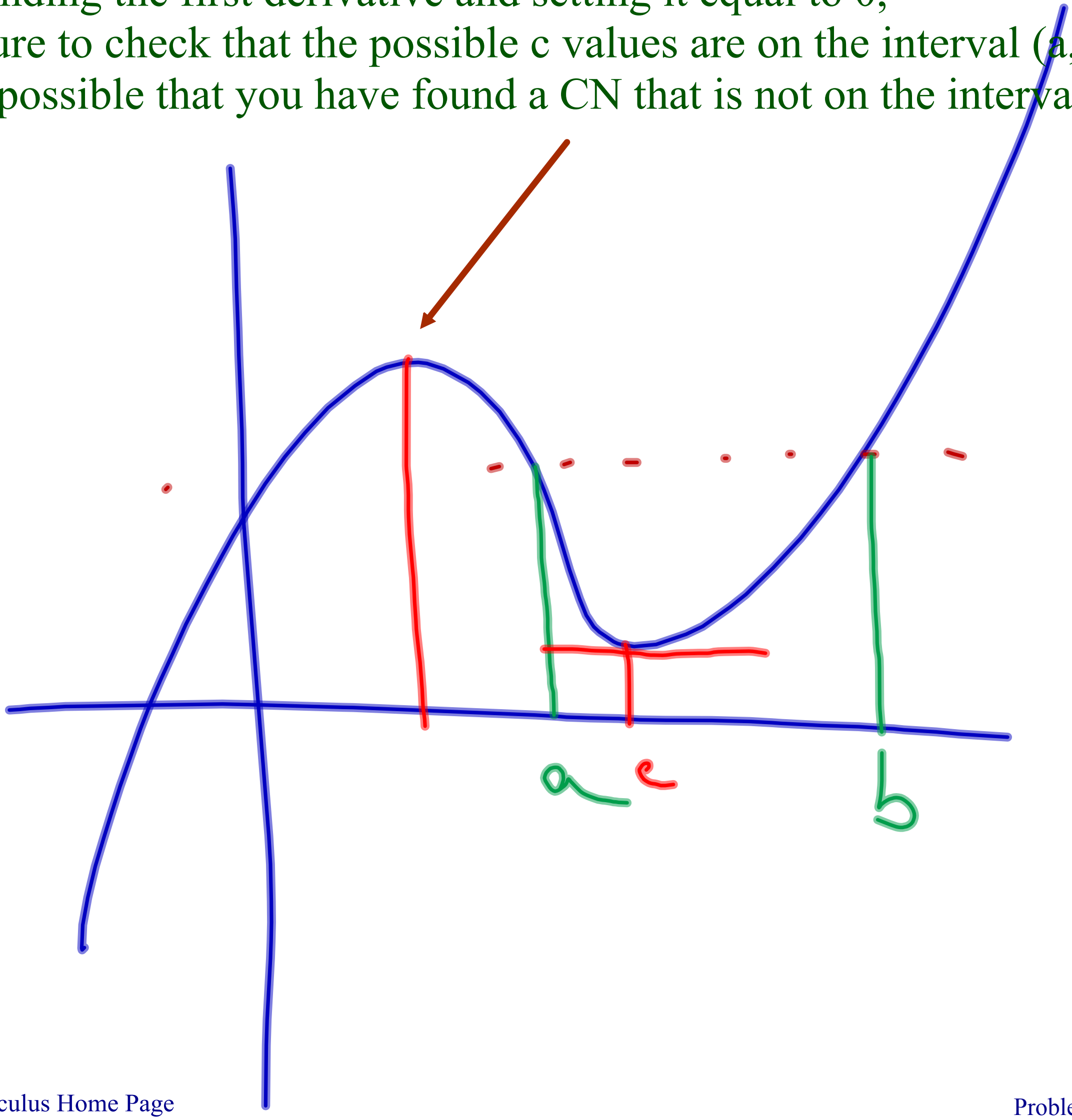
Problems for 3.2

3.2 Rolle's Theorum and the Mean Value Theorum

When Rolle's Theorum applies and you have found possible values for c
by finding the first derivative and setting it equal to 0,
be sure to check that the possible c values are on the interval (a, b).
It is possible that you have found a CN that is not on the interval.



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3.2 Rolle's Theorum and the Mean Value Theorum

$y = x - x^{1/3}$

F: Does RT apply on [0,1]?
If yes, find c on (0,1)
where f '(c) =0

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Problems for 3.2

3.2 Rolle's Theorum and the Mean Value Theorum

$$y = x - x^{1/3}$$

$$\frac{dy}{dx} = 1 - \frac{1}{3}x^{-2/3}$$

$$= 1 - \frac{1}{3x^{2/3}}$$

① y cont [0,1]

② y diff (0,1)

③ $f(0) \stackrel{?}{=} f(1)$

RTapply.

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Problems for 3.2

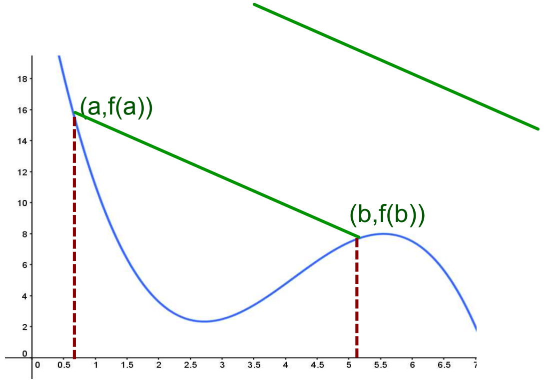
3.2 Rolle's Theorum and the Mean Value Theorum

Suppose

① f(x) cont [a,b]

② f(x) diff. (a,b)

$f(a) \neq f(b)$



What happens to f ' (x) on the interval?

Compare f ' (x) to m_{sec}.

Does f ' (x) = m_{sec} anywhere on interval (a,b)?

Mean Value Theorm

$$m_{sec} = \frac{f(b) - f(a)}{b - a}$$

geogebra demo

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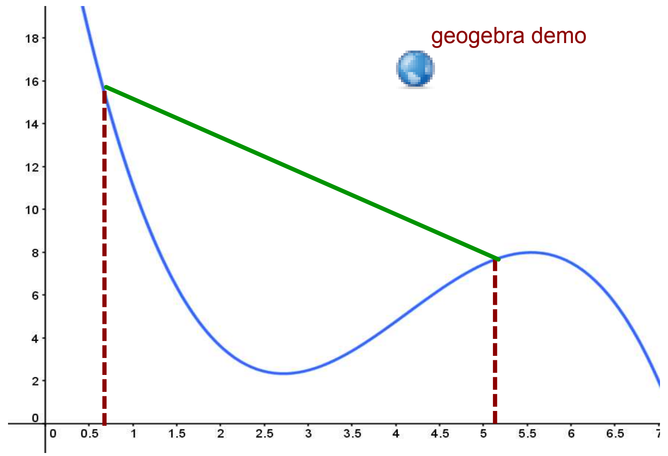
Problems for 3.2

3.2 Rolle's Theorum and the Mean Value Theorum



geogebra demo

Mean Value Theorum

Let f be:

1. continuous on closed interval [a,b] and
2. differentiable on open interval (a,b)

then ∃ at least one c ∈ (a,b) ∋

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

(there exists at least one c on the interval from a to b such that the derivative at c equals the slope of the secant line joining the endpoints)

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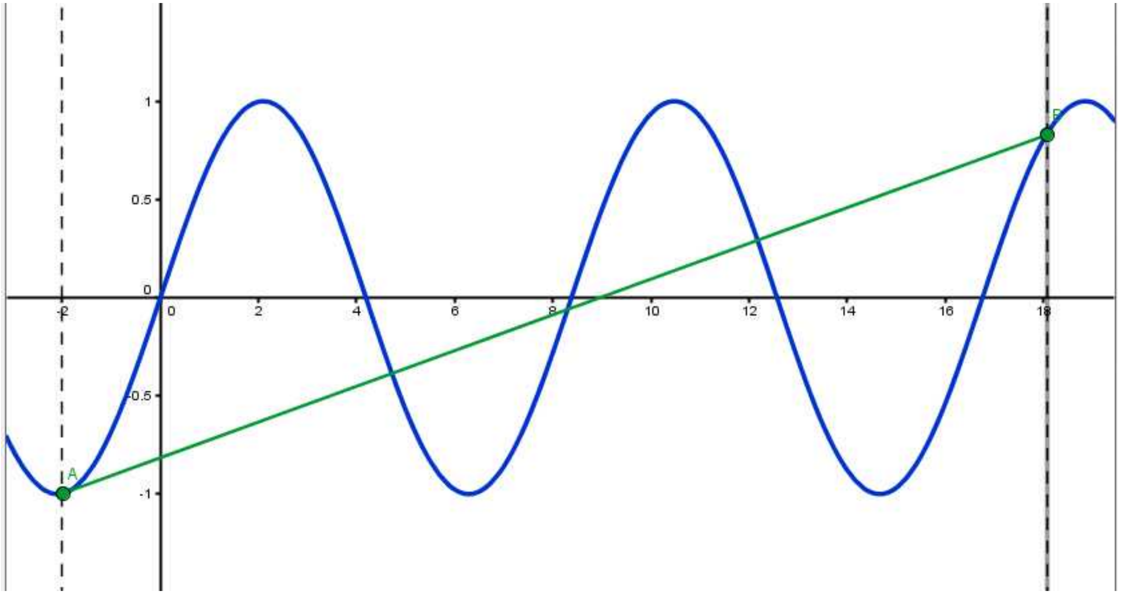
Problems for 3.2

3.2 Rolle's Theorum and the Mean Value Theorum



How many c 's?

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Problems for 3.2

3.2 Rolle's Theorum and the Mean Value Theorum

p.177 #40.

G: $f(x) = x(x^2 - x - 2)$

F: a) MVT on $[-1,1]$?

b) If yes, $c \ni [a,b]$

3.2 Rolle's Theorum and the Mean Value Theorum

p.177 #40.

G: $f(x) = x(x^2 - x - 2)$

① f cont on $[-1,1]$? yes } Polyn.

② f diff on $(-1,1)$? yes } Polyn.

F: MVT on $[-1,1]$? yes.

b) If yes, $[a,b]$

$f'(c) = \frac{f(b)-f(a)}{b-a}$

$f'(c) = \frac{f(1)-f(-1)}{1-(-1)}$

$f(1) = 1(1-1-2) = -2$

$f(-1) = (-1)(1+1-2) = 0$

$f'(c) = \frac{-2-0}{2} = -1$

$f(x) = x^3 - x^2 - 2x$

$f'(x) = 3x^2 - 2x - 2$

$3x^2 - 2x - 2 = -1$

$3x^2 - 2x - 1 = 0$

$(-3)(1) = -3$

$(-3) + (+1) = -2$

$3x^2 - 3x + x - 1 = 0$

$3x(x-1) + 1(x-1) = 0$

$(3x+1)(x-1) = 0$

$3x+1=0$	$x-1=0$
$3x=-1$	
$x=-\frac{1}{3}$	$\cancel{x=1} \notin (-1,1)$

$-\frac{1}{3} \in (-1,1)$

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3.2 Rolle's Theorum and the Mean Value Theorum

p.176 #17

G: $f(x) = \dfrac{x^2-2x-3}{x+2}$ $\quad x \neq -2$

F: a) RT on $[-1,3]$? yes

b) If yes, $c \ni f'(c)=0$

① $f(x)$ cont on $[-1,3]$? yes. poly. $-2 \notin [-1,3]$

② " diff on $(-1,3)$? yes. "

③ $f(-1) \overset{?}{=} f(3)$

$0 = 0$ ✓ yes.

$f(-1) = \dfrac{1+2-3}{-1+2} = \dfrac{0}{1} = 0$ | $f(3) = \dfrac{9-6-3}{3+2} = \dfrac{0}{5} = 0$

$\therefore$ RT's applies yes. $\therefore \exists\, c \in (-1,3) \ni f'(c)=0$

$f(x) = \dfrac{x^2-2x-3}{x+2}$

$f'(x) = \dfrac{(x+2)(2x-2)-(x^2-2x-3)(1)}{(x+2)^2} = \dfrac{2(x+2)(x-1)}{(x+2)^2}$

$= \dfrac{2(x+2)(x-1)-(x-3)(x+1)}{(x+2)^2}$

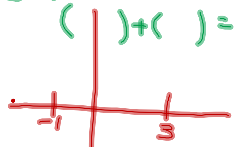
$2(x^2+x-2)-(x^2-2x-3) = 0$

$2x^2+2x-4-x^2+2x+3 = x^2+4x-1=0$

ac =

b = ( )( ) = -1

( )+( ) = 4



$x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \dfrac{-4 \pm \sqrt{16+4}}{2} = \dfrac{-4 \pm \sqrt{20}}{2}$

$= \dfrac{-4 \pm 2\sqrt{5}}{2} = -2 \pm \sqrt{5}$

c = $-2+\sqrt{5} \approx -2+2.... \in (-1,3)$

$\cancel{-2-\sqrt{5} \notin (-1,3)}$

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Problems for 3.2