3.1 Extrema on an Interval

Study 3.1 # 1-5, 11-25, 29, 33, 41, 63-66 all

Goals:
1. Understand critical numbers and how to find them.
2. Understand the difference between relative extrema and absolute extrema.
3. Understand how critical numbers relate to the relative extrema of the function.
4. Find absolute extrema on a closed interval.

First Direct Observation of Carbon Dioxide’s Increasing Greenhouse Effect at the Earth’s Surface

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Describe what happens to the graph on \([-1, 4]\)

Looking at \(y\) values as \(x\) changes,
- Is there a maximum \(y\) value on \([-1,4]\)?
- Is there a minimum \(y\) value on \([-1,4]\)?

Would the minimum \(y\) value be different on \([0,4]\)?
3.1 Extrema on an Interval

Definition of Extrema: Let \( f \) be defined on an interval \( I \) containing \( c \). (Note that \( I \) can be open or closed.)

1. \( f(c) \) is a **minimum of \( f \)** on \( I \) if \( f(c) \leq f(x) \) for all \( x \) in \( I \).
2. \( f(c) \) is a **maximum of \( f \)** on \( I \) if \( f(c) \geq f(x) \) for all \( x \) in \( I \).

The maximum and minimum values are **extreme values**. Note that these are values of \( f \) (or \( y \)), not values of \( x \).

The minimum and maximum on the interval are called the **absolute minimum** and **absolute maximum**.

Extreme Value Theorem

If \( f \) is continuous on a **closed** interval \([a, b]\), then \( f \) has both a minimum and a maximum on the interval.

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Extrema on an Interval

Describe what happens to the graph on \([-1, 4]\)

We need a way to describe the points where the graph of \( f \) turns:
- from up to down
- from down to up

Restrict the interval to an **open interval** containing the point. Then,

**Definition of Relative Extrema:**

1. If there is an **open interval** containing \( c \) on which \( f(c) \) is a maximum, then \( f(c) \) is called a **relative maximum** of \( f \). Or \( f \) has a relative max at \((c, f(c))\).
2. If there is an **open interval** containing \( c \) on which \( f(c) \) is a minimum, then \( f(c) \) is called a **relative minimum** of \( f \). Or \( f \) has a relative min at \((c, f(c))\).
Problem 1. At each labeled point, determine if there is a relative min or max, or an absolute min or max.

A none
B Absolute Max
C none
D Absolute Min
E relative max
F relative min
G none
What is the slope of the tangent line at these extrema?
Can the slope help us to find the extrema?
If so, for which extrema is it helpful?

Definition of **Critical Number**: CN

Let $f$ be defined at $c$ if $f'(c) = 0$, or if $f$ is not differentiable at $c$,

then $c$ is a critical number of $f$. 

CN : $\begin{cases} f'(c) = 0 \\ f'(c) \text{ DNE} \end{cases}$
3.1 Extrema on an Interval

\[ g(x) = y^2(x^2 - y) = x^4 - 4x^2 \]

CN:
1. y defined?
2. x when \( y' = 0 \)?
3. x when \( y' \) dne?

\[ g'(x) = 4x^3 - 8x \]

There are 3 Critical Numbers

\[ x = 0, x = \pm \sqrt{2} \]
3.1 Extrema on an Interval

\[ y = |x| \quad F: \text{CN} \]

**CN:**
1. \( y \) defined?
2. \( x \) when \( y' = 0? \)
3. \( x \) when \( y' \) dne?

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**Images:**
- Images of handwritten notes showing calculations and explanations related to finding extrema on an interval for the function \( y = |x| \). The notes include points about the definition of \( y \), the point where \( y' = 0 \), and a discussion on differentiability at \( x = 0 \).
3.1 Extrema on an Interval

Relationship between extrema and CN?

Do all extrema occur at CNs?

Do all CNs occur at extrema?

Extrema \( \Rightarrow \) CN

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At extrema, \( f'(x) = 0 \) or \( f'(x) \text{ dne} \)

Extrema \( \Rightarrow \) CN

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Relationship between extrema and CN?

Do all extrema occur at CNs? Yes.

Do all CNs occur at extrema? Not always, but sometimes.

Find Absolute Extrema, Closed Interval

1. Find CV's for the interval and corresponding $y$ values

2. Evaluate to find $y$ at the endpoints

3. Compare all $y$ values for both the CV's and the endpts.

    Largest - ABSOLUTE MAX

    Smallest - ABSOLUTE MIN

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3.1 Extrema on an Interval

Given: \( f(x) = x^2 + 2x - 4 \) on \([-1, 1]\)

Find the absolute extrema at critical numbers (CN) or endpoints (endpt).

Evaluate:

1. Is definition valid for all \( x \)?

   \( f'(x) = 0 \) or undefined

   \( f'(x) = 2x + 2 \)

   \( 2x + 2 = 0 \)

   \( x + 1 = 0 \)

   \( x = -1 \)

2. Evaluate at critical numbers and endpoints:

   - CN: \( x = -1 \)
   - Endp: \( x = -1, 1 \)

   \[ f(-1) = -5 \]
   \[ f(1) = -1 \]

3. Determine absolute max and min:

   - Absolute max: \( -5 \) at \( x = -1 \)
   - Absolute min: \( -1 \) at \( x = 1 \)

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3.1 Extrema on an Interval

\[ y = 1 \cdot x \cdot 1 \]

\[ F: \text{abs.extr.in}[2,3] \]

at CN or endpt

\[ x \quad y \]

CNs

Endpts

CN:
1. y defined?
2. x when y' =0?
3. x when y' dne?

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Homework for 3.1
3.1 Extrema on an Interval

\[ f(x) = \frac{2x}{x^2 + 1} \]

**CN:**
1. \( y \) defined?
2. \( x \) when \( y' = 0? \)
3. \( x \) when \( y' \) dne?

**at CN or endpt**

\[ x \quad \gamma \]

\[ \text{CNs} \]

\[ \text{Endpts} \]

\[ -2 \quad -1 \quad 1 \quad 2 \quad 0 \quad \frac{1}{2} \quad 0 \quad \frac{1}{2} \quad \text{DNE} \]
3.1 Extrema on an Interval

Given: \( h(t) = \frac{t}{t-2} \)

Finding: \( \text{Abs. Extremes on } [3, 5] \)

Conditions:
1. \( y \) defined?
2. \( x \) when \( y' = 0 \)?
3. \( x \) when \( y' \) dne?

CN:
- at CN or endpts
- \( x \)
- \( y \)
- CNs
- Endpts

Steps:

1. **h defined for all x**
   
   \[ h'(t) = \frac{(t-2)(1) - t(1)}{(t-2)^2} = \frac{t-2-t}{(t-2)^2} = \frac{-2}{(t-2)^2} \]

2. **No CNs**
   
   \[ h'(t) \text{ dne } \]
   
   \[ 2 = 2 \]
3.1 Extrema on an Interval

\( f(x) = x^3 - 12x \) \( [0, 4] \)

CN:
1. \( y \) defined?
2. \( x \) when \( y' = 0 \)?
3. \( x \) when \( y' \) dne?

at CN or endpt+
\( x \) \( y \)

CNs
Endpts

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\( f'(x) = 3x^2 - 12 = 3(x^2 - 4) \)

CN:
2. \( 3(x^2 - 4) = 0 \)
3. \( f'(x) \) \( x \leq \pm 2 \) \( \forall x \)

\( x \) \( y \)
2 \( 8 - 24 = -16 \)

CN:
1. \( y \) defined?
2. \( x \) when \( y' = 0 \)?
3. \( x \) when \( y' \) dne?

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