3.1 Extrema on an Interval

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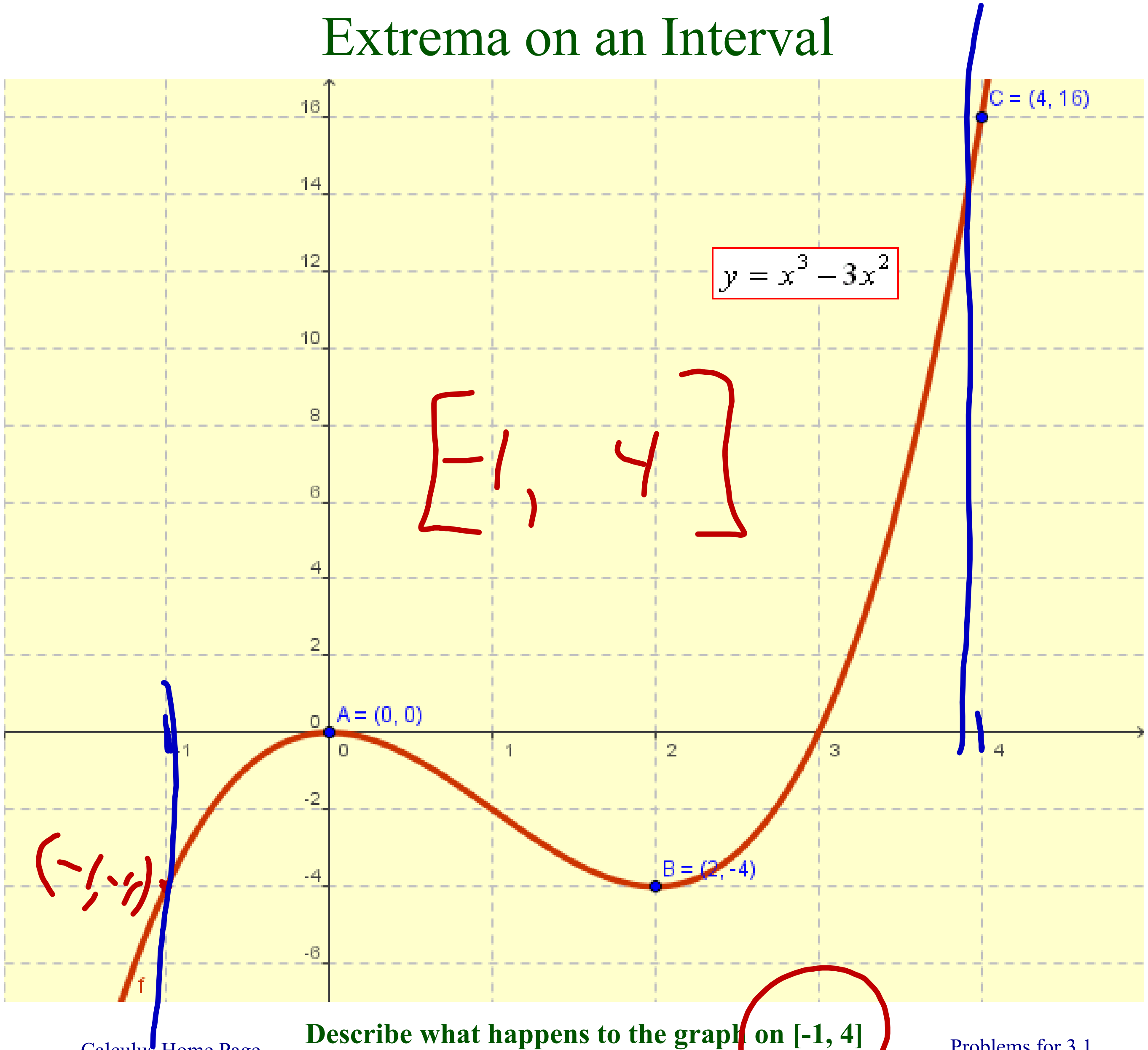
Problems for 3.1

Extrema on an Interval



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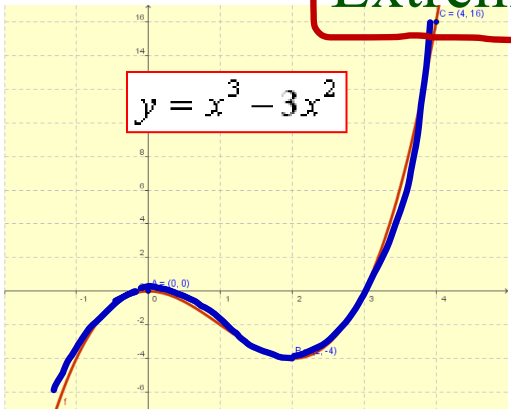
Describe what happens to the graph on [-1, 4]

Problems for 3.1

Extrema on an Interval



Describe what happens to the graph on [-1, 4]

Fr. -1 to 0 graph incr.
0 to 2 decr.
2 to 4 incr.

Beyond [-1, 4], for all x, tendency is: fr. neg to pos.

We need a way to describe the points where the graph of f turns:
from up to down, from down to up.

3.1 Extrema on an Interval

Definition of Extrema: Let f be defined on and interval I containing c. *(Note that I can be open or closed.)*

1. f(c) is a **minimum of f** on I if $f(c) \leq f(x)$ for all x in I.

2. f(c) is a **maximum of f** on I if $f(c) \geq f(x)$ for all x in I.

The maximum and minimum values are **extreme values**.
Note that these are values of f (or y), not values of x.

The minimum and maximum on the interval are called the absolute minimum and absolute maximum.

Extreme Value Theorem

If f is continuous on a **closed** interval [a, b], then f has both a minimum and a maximum on the interval.

Definition of Relative Extrema:

1. If there is an **open interval** contain c on which $f(c)$ is a maximum, then $f(c)$ is called a **relative maximum** of f. Or f **has a relative max at** $(c, f(c))$.

2. If there is an **open interval** contain c on which $f(c)$ is a minimum, then $f(c)$ is called a **relative minimum** of f. Or f **has a relative min at** $(c, f(c))$.

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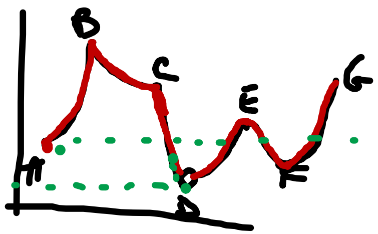
Problem 1. At each labeled point, determine if there is a relative min or max, or an absolute min or max.



A neither

E rel. max

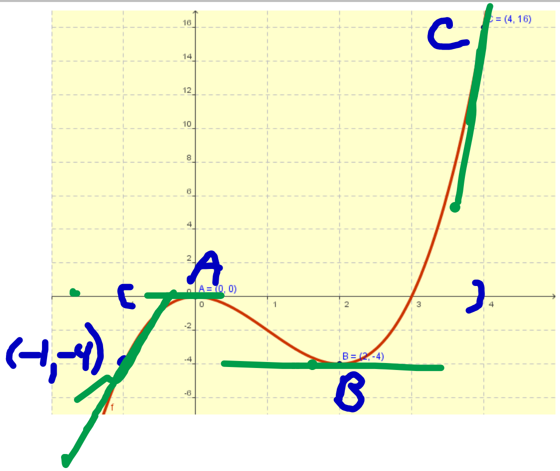
F. rel. min.

B absolute max, rel. max.

G neither

C As drawn in book, neither, as drawn here it appears to be a rel. max.

D Neith



ABS. MIN = -4
" MAX = 16

rel min = -4
rel max = 0

what m_T at ↑

Def. of Critical # CN Or CV, critical value

Let f be defined at c. If $f'(c) = 0$ or if f is not differentiable at c, then c is a critical # of f.

CN: ① $f'(c) = 0$
② $f'(c)$ DNE

c is CN
X value

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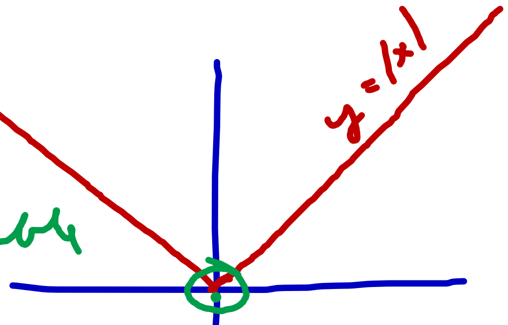
$y = |x|$



not differentiable at $x=0$

is CN bec. $f(0)$ defined.
but $f'(0)$ DNE

$g(x) = x^2(x^2-4) = x^4 - 4x^2$

$g'(x) = 4x^3 - 8x$

CN: $g'(x) = 0$ | $\cancel{g'(x)\text{ undef.}}$

$4x(x^2-2)$

$\boxed{x=0 \quad x = \pm\sqrt{2}}$

There are 3 Critical Numbers

CV: $f'(c) = 0$

or. $f'(c)$ DNE

$\longrightarrow$

$\not\longleftarrow$

Not necessarily extrema

F: CV

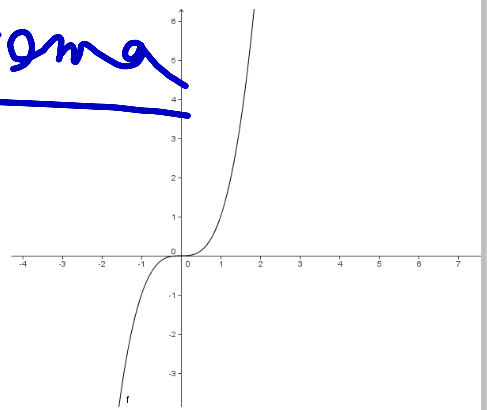
$y = x^3$

$\frac{dy}{dx} = 3x^2$

$3x^2 = 0$

$x = 0$ CV

Does NOT extrema



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Problems for 3.1

Find Absolute Extrema, Closed Int.

1. Find CV's for the interval and corresponding y values
2. Evaluate to find f (or y) at endpts.
3. Compare all y values, both fr. CVs. and for endpts.

largest - ABS MAX

smallest ABS MIN

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Problems for 3.1

G: $f(x) = x^2 + 2x - 4$

F: abs. extr. on $[-1,1]$

$f'(x) = 2x + 2$

$f'(c) = 0$ or ~~DNE~~

$2x + 2 = 0$

$x = -1$ cv.

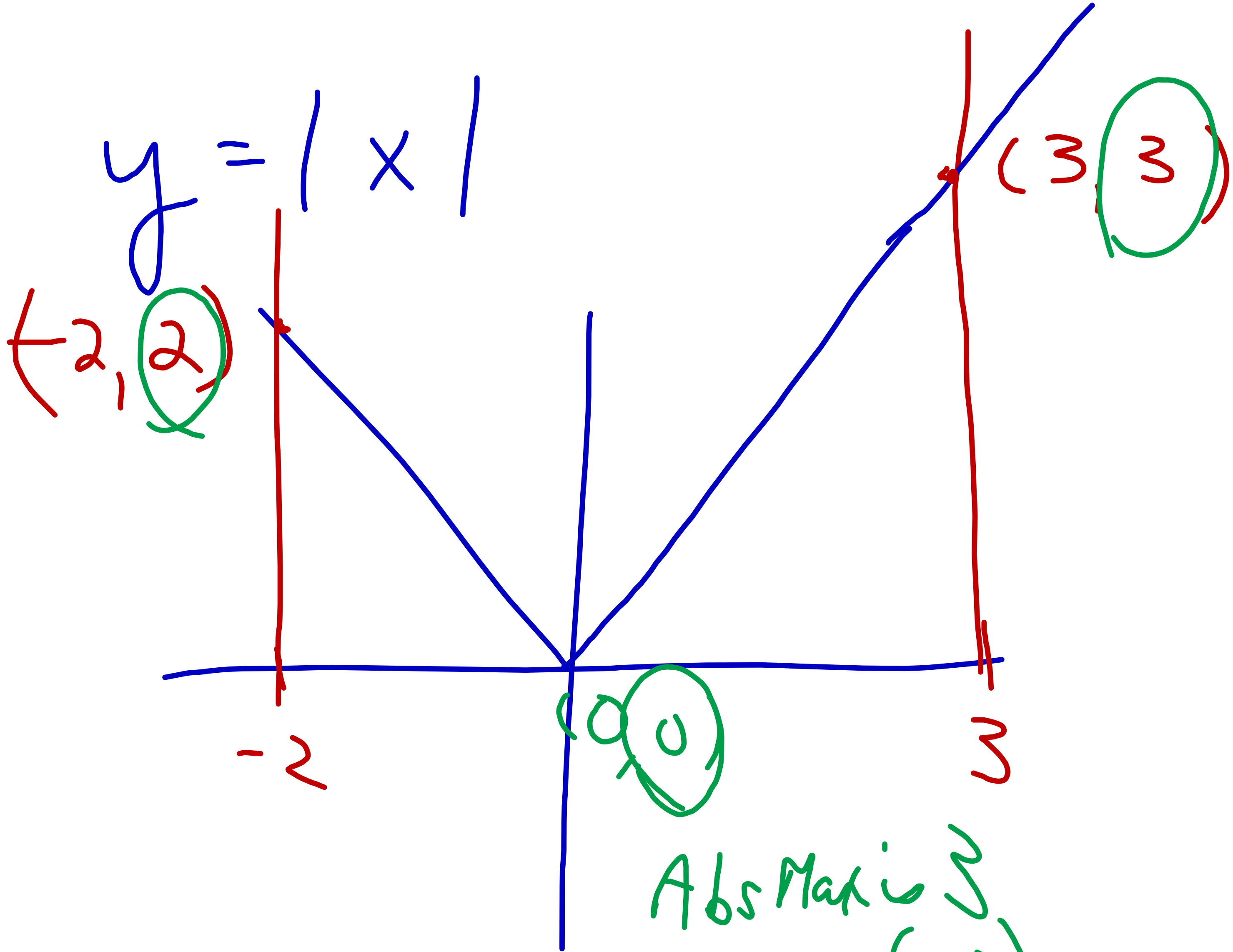
$f(-1) = (-1)^2 + 2(-1) - 4$

$= 1 - 2 - 4 = -5$

$f(1) = 1^2 + 2 - 4 = -1$

	x	y	
cv	-1	-5	ABS MIN
	1	-1	ABS MAX

$y = |x|$



Abs Max is 3

" " (3, 3)

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28G: $f(x) = \dfrac{2x}{x^2+1}$ F: Abs Ext $[2,2]$

CVs:

$$f'(x) = \frac{(x^2+1)(2) - 2x(2x)}{(x^2+1)^2}$$

$$= \frac{2x^2+2-4x^2}{(x^2+1)^2} = \frac{2-2x^2}{(x^2+1)^2}$$

$f'(x) = 0$	$f'(x)$ ~~DNE~~
$2-2x^2=0$	$(x^2+1)^2=0$
$2(1-x^2)=0$	$x^2+1=0$
$2(1+x)(1-x)=0$	no x
$x=+1,-1$ CVs.	

x	y
-1	-1 ABS MIN
1	1 ABS MAX
-2	$-4/5$
2	$4/5$

$\dfrac{2x}{x^2+1} = \dfrac{2(-1)}{2} = -1$

$\dfrac{2}{2} = 1$

$\dfrac{-4}{5}$ $\dfrac{4}{5}$