2.6 Related Rates

Study 2.6, p. 154 # 1-19, 21-25

Goal:
1. Understand how variables change with respect to time.
2. Understand "with respect to".

water drop ripples

pour cement high rise @ 1:30

water drop ripples

pour cement high rise @ 1:30

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Homework Part 1

2.6 Related Rates

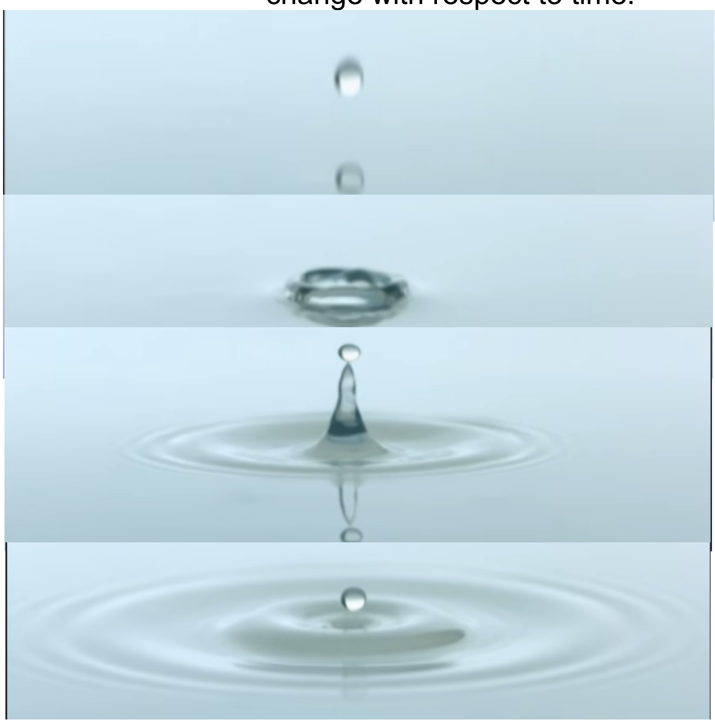
Goal: Understand how variables change with respect to time.

water drop ripples

What variables are changing?

with respect to what?

r increasing w respect to time
A increasing w respect to time and r



water drop ripples

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Homework Part 1

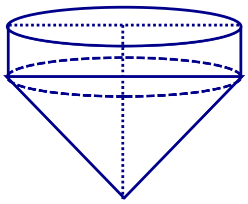
2.6 Related Rates

Goal: Understand how variables change with respect to time.

pour cement
high rise @ 1:30

What variables are changing?

with respect to what?



pour cement high rise

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Homework Part 1

2.6 Related Rates

Refer to problems # 24 - 27

geogebra: Related Rates

What is common to all these problems?

$$\frac{d\,\square}{dt}$$

rate of change of distance with respect to time

$$\frac{dx}{dt} \qquad \frac{dy}{dt}$$

rate of change of volume with respect to time $\frac{dV}{dt}$

rate of change of area with respect to time $\frac{dA}{dt}$

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Homework Part 1

2.6 Related Rates

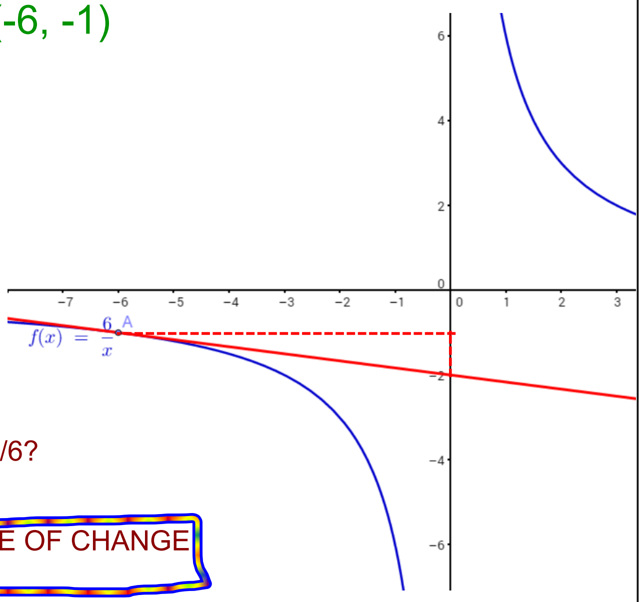
From the previous section: implicit diff.

G: xy = 6 F: dy/dx at (-6, -1)

$$x \cdot \frac{dy}{dx} + y \cdot 1 = 0$$

$$\frac{dy}{dx} = -\frac{y}{x}$$

$$\left.\frac{dy}{dx}\right|_{(-6,-1)} = -\frac{1}{6}$$



How can we interpret dy/dx = -1/6?

slope of tangent line at (-6, -1)

ALSO, INSTANTANEOUS RATE OF CHANGE of y with respect to x!

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Homework Part 1

2.6 Related Rates

When rate of change is with respect to time instead, how do you decide what variables to use?

How do you proceed? Example:

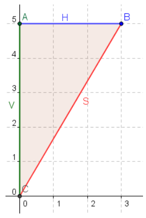
An airplane is flying at an altitude of 5 miles and passes directly over a radar antenna. When the plane is 10 miles away, the radar detects that the distance S is changing at a rate of 240 miles per hour. What is the speed of the plane?

G: altitude 5 mi;
when plane 10 mi away, S changing at 240 mph

F: speed of plane

rate of change with respect to time

5



geogebra: Related Rates

6. Check:
Did you include units? Yes
Does the answer make sense? Yes. dH/dt is directly proportional to S and inversely prop. to H, so it should be larger than dS/dt.
If there were angles involved, did you use radians? No angles

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Homework Part 1

2.6 Related Rates

step-by-step

RELATED RATES:

1. **Identify: a "given" rate, a "to find" rate, other conditions**
2. **Determine the relationship (equation) between the given and the to find. Use a diagram, if possible, or known formula.**
3. **If possible, use the given information to reduce the number of variables.**
4. **Differentiate implicitly with respect to time. This always involves the chain rule. For example,**

$$\frac{dV}{dt} = \frac{dV}{dr} \bullet \frac{dr}{dt} \quad or \quad \frac{dy}{dt} = \frac{dy}{dx} \bullet \frac{dx}{dt}$$

5. **Substitute the given values and solve for the unknown rate.**
6. **Check:**

- **Did you include units?**
- **Does the answer make sense?**
- **If there were angles involved, did you use radians?**

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Homework Part 1

2.6 Related Rates

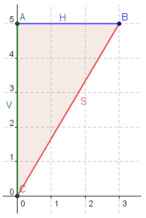
geogebra: Related Rates

An airplane is flying at an altitude of 5 miles and passes directly over a radar antenna. When the plane is 10 miles away, the radar detects that the distance S is changing at a rate of 240 miles per hour. What is the speed of the plane?

G: altitude 5 mi;
when plane 10 mi away, S changing at 240 mph

F: speed of plane

5



Variables **H** and **S**
Then rates of change: $\frac{dH}{dt}$ $\frac{dS}{dt}$

1. Identify: a "given" rate, a "to find" rate, other variables.

2. Determine the relationship (equation) between the 'given' and the 'to find'. Use a diagram, if possible, or known formula.

3. If possible, use the given information to reduce the number of variables. (Not needed here.)

4. Differentiate implicitly with respect to time. This always involves the **chain rule**.

5. Substitute the given values and solve for the unknown rate.

6. Check:
Did you include units? Yes
Does the answer make sense? Yes. dH/dt is directly proportional to S and inversely prop. to H, so it should be larger than dS/dt.
If there were angles involved, did you use radians? No angles

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Homework Part 1

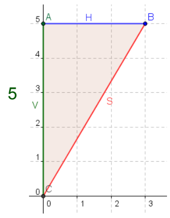
2.6 Related Rates

geogebra: Related Rates

An airplane is flying at an altitude of 5 miles and passes directly over a radar antenna. When the plane is 10 miles away, the radar detects that the distance S is changing at a rate of 240 miles per hour. What is the speed of the plane?

G: altitude 5 mi;
when plane 10 mi away, S changing at 240 mph

F: speed of plane



Variables **H** and **S**
Then rates of change: $\frac{dH}{dt}$ $\frac{dS}{dt}$

1. Identify: a "given" rate, a "to find" rate, other variables.

$$H^2 + 5^2 = S^2$$

2. Determine the relationship (equation) between the 'given' and the 'to find'. Use a diagram, if possible, or known formula.

3. If possible, use the given information to reduce the number of variables. (Not needed here.)

$$2H \cdot \frac{dH}{dt} + 0 = 2S \cdot \frac{dS}{dt}$$

4. Differentiate implicitly with respect to time. This always involves the **chain rule**.

remember **F: speed of plane** $\frac{dH}{dt}$

at $s = 10$, $\frac{ds}{dt} = 240\, m/h$.

$$2H \cdot \frac{dH}{dt} = 2(10mi) \cdot 240\ mi/h$$

5. Substitute the given values and solve for the unknown rate.

$$\frac{dH}{dt} = \frac{\left[(10mi) \cdot 240\frac{mi}{h}\right]}{H}$$

need H:
(from above)

$H^2 + 5^2 = S^2$

$H^2 = 10^2 - 5^2 = 75$

$$\frac{dH}{dt} = \frac{\left[(10mi) \cdot 240\frac{mi}{h}\right]}{5\sqrt{3}\ mi}$$

$$\frac{dH}{dt} = \frac{\left[(10mi) \cdot 240\frac{mi}{h}\right]}{5\sqrt{3}\ mi} = \frac{480}{\sqrt{3}}\frac{mi}{h} = 277.1\ \frac{mi}{h}$$

6. Check:
Did you include units? Yes
Does the answer make sense? Yes. dH/dt is directly proportional to S and inversely prop. to H, so it should be larger than dS/dt.
If there were angles involved, did you use radians? No angles $\frac{dH}{dt} = \frac{S}{H}\frac{dS}{dt} > \frac{dS}{dt}$ since $S > H$ and $\frac{S}{H} > 1$

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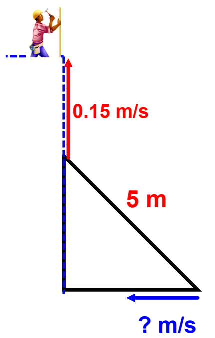
Homework Part 1

2.6 Related Rates

G: Using a rope, a worker pulls a 5 m plank up the side of a building at a rate of 0.15 m/s.

F: How fast sliding on ground when 2.5 m from building?



1. Identify: a "given" rate, a "to find" rate, other variables.

2. Determine the relationship (equation) between the 'given' and the 'to find'. Use a diagram, if possible, or known formula.

3. If possible, use the given information to reduce the number of variables. (Not needed here.)

4. Differentiate implicitly with respect to time. This always involves the **chain rule**.

5. Substitute the given values and solve for the unknown rate.

6. Check:
Did you include units? Yes
Does the answer make sense? Yes. x is decreasing
If there were angles involved, did you use radians? No angles

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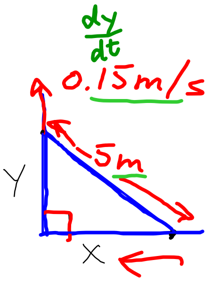
Homework Part 1

2.6 Related Rates

G: Using a rope, a worker pulls a 5 m plank up the side of a building at a rate of 0.15 m/s.

F: How fast sliding on ground when 2.5 m from building? F: $\left.\frac{dx}{dt}\right|_{x=2.5}$

1. Identify: a "given" rate, a "to find" rate, other variables.



$x,\ y,\ \underset{F:}{\frac{dx}{dt}},\ \underset{G:}{\frac{dy}{dt}}$

2. Determine the relationship (equation) between the 'given' and the 'to find'. Use a diagram, if possible, or known formula.

3. If possible, use the given information to reduce the number of variables. (Not needed here.)

4. Differentiate implicitly with respect to time. This always involves the **chain rule**.

5. Substitute the given values and solve for the unknown rate.

6. Check:
Did you include units? Yes
Does the answer make sense? Yes. x is decreasing
If there were angles involved, did you use radians? No angles

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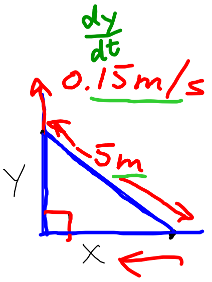
Homework Part 1

2.6 Related Rates

F: How fast sliding on ground when 2.5 m from building? F: $\left.\frac{dx}{dt}\right|_{x=2.5}$



$x,\ y,\ \underset{F:}{\frac{dx}{dt}},\ \underset{G:}{\frac{dy}{dt}}$

1. Identify: a "given" rate, a "to find" rate, other variables.

$$x^2 + y^2 = 5^2$$

2. Determine the relationship (equation) between the 'given' and the 'to find'. Use a diagram, if possible, or known formula.

3. If possible, use the given information to reduce the number of variables. (Not needed here.)

$$2x\cdot\frac{dx}{dt} + 2y\cdot\frac{dy}{dt} = 0$$

4. Differentiate implicitly with respect to time. This always involves the **chain rule**.

$$x\cdot\frac{dx}{dt} + y\cdot\frac{dy}{dt} = 0$$

$$2.5m\cdot\frac{dx}{dt} + y\cdot(0.15\ m/s) = 0$$

5. Substitute the given values and solve for the unknown rate.

$$\frac{dx}{dt} = -y\cdot\frac{\left(0.15\frac{m}{s}\right)}{2.5m}$$

need y:
(from above)
$2.5^2 + y^2 = 5^2$
$y^2 = 5^2 - 2.5^2 = 25 - 6.25$
$y = \sqrt{25-6.25} = 4.33$

$$\frac{dx}{dt} = -(4.33m)\cdot\frac{\left(0.15\frac{m}{s}\right)}{2.5m} = -0.26\ m/s$$

6. Check:
Did you include units? Yes
Does the answer make sense? Yes. x is decreasing
If there were angles involved, did you use radians? No angles

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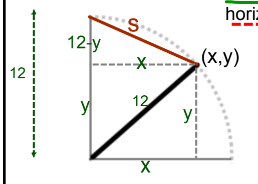
Homework Part 1

2.6 Related Rates



A winch at the top of a 12-m building pulls a 12 m pipe to a vertical position. The winch pulls in the rope at a rate of -0.2 m/sec. Find the rate of vertical change and the rate of horizontal change at the end of the pipe when y = 6.

1. Identify: a "given" rate, a "to find" rate, other variables.

2. Determine the relationship (equation) between the 'given' and the 'to find'. Use a diagram, if possible, or known formula.

3. If possible, use the given information to **reduce the number of variables**.

4. Differentiate implicitly with respect to time. This always involves the **chain rule**.

5. Substitute the given values and solve for the unknown rate.

6. Check:
Did you include units? Yes
Does the answer make sense? Yes.
x decreasing, y increasing; so dx/dt <0 and dy/dt >0
If there were angles involved, did you use radians? N
angles

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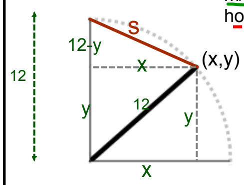
Homework Part 1

2.6 Related Rates



A winch at the top of a 12-m building pulls a 12 m pipe to a vertical position. The winch pulls in the rope at a rate of -0.2 m/sec. Find the rate of vertical change and the rate of horizontal change at the end of the pipe when y = 6.

$\frac{ds}{dt}$

$s, x, y,$ $\boxed{F: \left.\frac{dx}{dt}\right|_{y=6} \quad \left.\frac{dy}{dt}\right|_{y=6}}$

1. Identify: a "given" rate, a "to find" rate, other variables.

bottom right triangle:

$$x^2 + y^2 = 12^2$$

top left triangle:

$$x^2 + (12-y)^2 = s^2$$

2. Determine the relationship (equation) between the 'given' and the 'to find'. Use a diagram, if possible, or known formula.

solve bottom right for x^2 and substitute in top left to get just 2 variables:

$$12^2 - y^2 + (12-y)^2 = s^2$$

simplify: $12^2 - y^2 + 12^2 - 24y + y^2 = s^2$

$$2(12^2) - 24y = s^2$$

3. If possible, use the given information to **reduce the number of variables**.

$$-24\frac{dy}{dt} = 2s\frac{ds}{dt}$$

$$-24\frac{dy}{dt} = 2s(-0.2)\frac{m}{s} = -0.4s\ mps$$

4. Differentiate implicitly with respect to time. This always involves the **chain rule**.

need s:
(from above)

$s^2 = 2(12^2) - 24(6) = 144$

s=12

$$\frac{dy}{dt} = \frac{-0.4s}{-24}\ mps = \frac{s}{60}\ mps$$

$$\frac{dy}{dt} = \frac{12}{60}\ mps = \frac{1}{5}\ mps$$

5. Substitute the given values and solve for the unknown rate.

Still need *dx/dt* From equation for bottom right triangle:

$$2x\cdot\frac{dx}{dt} + 2y\cdot\frac{dy}{dt} = 0$$

$$\frac{dx}{dt} = -\frac{2y}{2x}\frac{dy}{dt} = -\frac{y}{x}\frac{dy}{dt} = -\frac{6}{x}\frac{1}{5}\ mps$$

$x^2 + 6^2 = 12^2$
$x^2 = 144 - 36$
$x^2 = 108 = 36(3)$
$x = 6\sqrt{3}$

Still need *x* Use bottom right equation above to get x = 6 $\sqrt{3}$

$$\frac{dx}{dt} = -\frac{6}{6\sqrt{3}}\frac{1}{5} = -\frac{1}{5\sqrt{3}}\ mps$$

6. Check:
Did you include units? Yes
Does the answer make sense? Yes.
x decreasing, y increasing; so dx/dt <0 and dy/dt >0
If there were angles involved, did you use radians? No
angles

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Homework Part 1

2.6 Related Rates

$$y = 4(x^2 - 5x) \qquad a) \text{ F: } \left.\frac{dy}{dt}\right|_{x=3} \qquad \text{G: } \frac{dx}{dt} = 2$$

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Homework Part 1

2.6 Related Rates

$$y = 4(x^2 - 5x) \qquad a) \text{ F: } \left.\frac{dy}{dt}\right|_{x=3} \qquad \text{G: } \frac{dx}{dt} = 2$$

$$\frac{dy}{dt} = 4\left(2x\frac{dx}{dt} - 5\frac{dx}{dt}\right)$$

$$= 4\left[(2x-5)\frac{dx}{dt}\right]$$

$$= 4\left[(6-5)(2)\right] = 8$$

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Homework Part 1

2.6 Related Rates

$y = 4(x^2 - 5x)$

b) F: $\left.\frac{dx}{dt}\right|_{x=1}$ G: $\frac{dy}{dt} = 5$

2.6 Related Rates

$y = 4(x^2 - 5x)$

b) F: $\left.\frac{dx}{dt}\right|_{x=1}$ G: $\frac{dy}{dt} = 5$

$$\frac{dy}{dt} = 4\left(2x\frac{dx}{dt} - 5\frac{dx}{dt}\right)$$

$$5 = 4\left(2(1)\frac{dx}{dt} - 5\frac{dx}{dt}\right)$$

$$5 = -12\frac{dx}{dt}$$

$$\frac{dx}{dt} = -\frac{5}{12}$$

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Homework Part 1

2.6 Related Rates

$G: y = 2(x^2 - 3x)$

$F:$ a) $\left.\frac{dy}{dt}\right|_{x=3}$,

b) $\left.\frac{dx}{dt}\right|_{x=1}$; $\frac{dy}{dt} = 5$

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Homework Part 1

2.6 Related Rates

$G: y = 2(x^2 - 3x)$

$F:$ a) $\left.\frac{dy}{dt}\right|_{x=3}$,

b) $\left.\frac{dx}{dt}\right|_{x=1}$; $\frac{dy}{dt} = 5$

$$\frac{dy}{dt} = 2\left[2x\frac{dx}{dt} - 3\frac{dx}{dt}\right] = 4x\frac{dx}{dt} - 6\frac{dx}{dt}$$

$$\frac{dy}{dt} = (4x - 6)\frac{dx}{dt} = 6\frac{dx}{dt}$$

b) $$\frac{dx}{dt} = \frac{dy/dt}{4x-6} = \frac{5}{4-6} = \frac{5}{-2}$$

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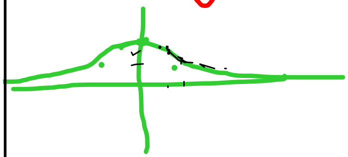
Homework Part 1

2.6 Related Rates

$$G: y = \frac{1}{1+x^2}, \quad \frac{dx}{dt} = 2 \text{ cm/s}. \qquad F: \left.\frac{dy}{dt}\right|_{x=-2}$$



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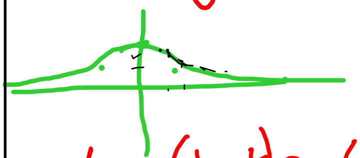
2.6 Related Rates

$$G: y = \frac{1}{1+x^2}, \quad \frac{dx}{dt} = 2 \text{ cm/s}. \qquad F: \boxed{\frac{dy}{dt}}\Bigg|_{\underline{x=-2}}$$



$$\frac{dy}{dt} = \frac{(1+x^2)0 - \left(2x\frac{dx}{dt}\right)}{(1+x^2)^2}$$

$$\frac{DdN - NdD}{D^2}$$

$$= \frac{-2x}{(1+x^2)^2}\frac{dx}{dt}$$

$$= \frac{-2(-2)}{(1+4)^2} \cdot 2 \text{ cm/s} = \frac{8}{25} \text{ cm/s}.$$

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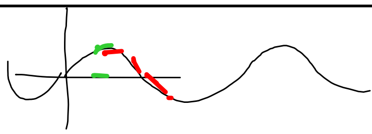
2.6 Related Rates



G: $y = \sin x$ $\quad \frac{dx}{dt} = 2$ cm/sec.

F: $\left.\frac{dy}{dt}\right|_{x=\frac{\pi}{6}}$

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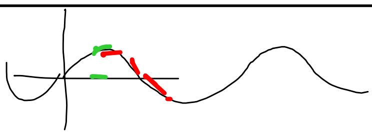
Homework Part 1

2.6 Related Rates



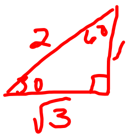
G: $y = \sin x$ $\quad \frac{dx}{dt} = 2$ cm/sec.

F: $\left.\frac{dy}{dt}\right|_{x=\frac{\pi}{6}}$

$$y = \sin x$$

$$\frac{dy}{dt} = (\cos x)\frac{dx}{dt} = \left(\cos\frac{\pi}{6}\right) 2 \text{ cm/s.}$$

$$= \frac{\sqrt{3}}{2} \cdot 2 \text{ cm/s.} = \sqrt{3} \text{ cm/s.}$$



$\sin 30 = \frac{1}{2}$

$\frac{dy}{dt} = 2\cos x$ cm/s

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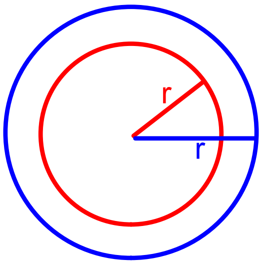
2.6 Related Rates



16. G: radius of sphere increases 3 in/min

dr/dt

F: rate of change of V when r =9 in, r =36 in

$\frac{dV}{dt}$

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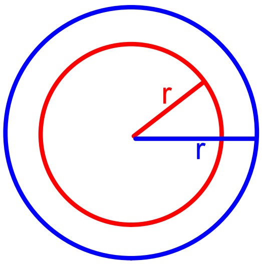
2.6 Related Rates



16. G: radius of sphere increases 3 in/min

dr/dt

F: rate of change of V when r =9 in, r =36 in

$\frac{dV}{dt}$

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = \frac{4}{3}\left(3\pi r^2\right)\frac{dr}{dt} = 4\pi r^2(3) \text{ in}^3/\text{min.}$$

$$= 12\pi r^2 \text{ in}^3/\text{min.}$$

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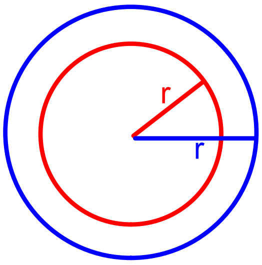
Homework Part 1

2.6 Related Rates



G: *dr/dt* is constant

F: Is dA/dt constant? Why?

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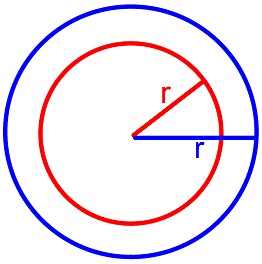
Homework Part 1

2.6 Related Rates



G: *dr/dt* is constant

F: Is dA/dt constant? Why?

$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$\frac{dA}{dt} = 2\pi k r$

Therefore, ***dA/dt*** is NOT constant.
It varies directly as ***r*** varies.

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2.6 Related Rates

$G: xy = 4$

$F: \left.\frac{dy}{dt}\right|_{x=8}$ $\frac{dx}{dt} = 10$

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2.6 Related Rates

$G: xy = 4$

$F: \left.\frac{dy}{dt}\right|_{x=8}$ $\frac{dx}{dt} = 10$

$x\frac{dy}{dt} + y\frac{dx}{dt} = 0$

$8\frac{dy}{dt} + y(10) = 0$

$xy = 4$

$y = \frac{4}{x} = \frac{4}{8}$

$y = \frac{1}{2}$

$8\frac{dy}{dt} + \frac{1}{2} \cdot 10 = 0$

$8\frac{dy}{dt} = -5$

$\frac{dy}{dt} = -\frac{5}{8}$

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2.6 Related Rates

Use Implicit Differentiation

$y = 2x + x^3 - z$	Differentiate with respect to:
	t
	x
	z
	y

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Homework Part 1

2.6 Related Rates

Use Implicit Differentiation

$y = 2x + x^3 - z$	Differentiate with respect to:
$\frac{dy}{dt} = 2\frac{dx}{dt} + 3x^2\frac{dx}{dt} - \frac{dz}{dt}$	t
$\frac{dy}{dx} = 2 + 3x^2 - \frac{dz}{dx}$	x
$\frac{dy}{dz} = 2\frac{dx}{dz} + 3x^2\frac{dx}{dz} - 1$	z
$1 = 2\frac{dx}{dy} + 3x^2\frac{dx}{dy} - \frac{dz}{dy}$	y

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