2.5 Implicit Differentiation

p.146 # 1-17, 21-31, 51

G: $x^2+y^2=100$

F: $\frac{dy}{dx}$

If we solve explicitly for x,
we have 2 functions,
the upper semicircle, + radical,
and the lower semicircle, - radical.

$y^2 = 100 - x^2$

$y = \pm\sqrt{100-x^2} = \pm(100-x^2)^{1/2}$

for $y>0$: $\frac{dy}{dx} = \frac{1}{2}(100-x^2)^{-1/2}(-2x)$

$\frac{dy}{dx} = \frac{-x}{\sqrt{100-x^2}}$

for $y<0$, $\frac{dy}{dx} = \frac{x}{-\sqrt{100-x^2}}$

$\frac{dy}{dx} = \frac{-x}{y}$

$x^2 + y^2 = 100$ $\qquad$ $F : \frac{dy}{dx}$

Instead of solving explicitly, use the **Chain Rule** to solve implicitly.

$$2x + 2y\frac{dy}{dx} = 0$$

$$\frac{2y}{2y}\frac{dy}{dx} = \frac{-2x}{2y}$$

$$\frac{dy}{dx} = \frac{-2x}{2y} = \boxed{\frac{-x}{y}}$$

$$\frac{2x}{2} = \frac{10}{2}$$

$$x = 5$$

2. $x^2 - y^2 = 16$

$2x - 2y\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{x}{y}$

$-2y\frac{dy}{dx} = -2x$

$\frac{dy}{dx} = \frac{-2x}{-2y} = \frac{x}{y}$

4. $x^3+y^3=8$

$3x^2+3y^2\frac{dy}{dx}=0$

$3y^2\frac{dy}{dx}=-3x^2$

$\frac{dy}{dx}=-\frac{x^2}{y^2}$

$\frac{dy}{dx}=\frac{-x^2}{y^2}$

10. $(2\sin x)\cos y = 1$ $\quad$ F: $\frac{dy}{dx}$

$$(2\sin x)\left(-\sin y \frac{dy}{dx}\right) + \cos y\,(2\cos x) = 0$$

$$-2\sin x \sin y \frac{dy}{dx} = -2\cos x \cos y$$

$$\frac{dy}{dx} = \frac{\cos x \cos y}{\sin x \sin y}$$

18. $x^2+y^2-4x+6y+9=0$ $\quad$ F: $\frac{dy}{dx}$

$$2x+2y\frac{dy}{dx}-4+6\frac{dy}{dx}+0=0$$

$$2y\frac{dy}{dx}+6\frac{dy}{dx}=4-2x$$

$$(2y+6)\frac{dy}{dx}=4-2x$$

$$\frac{dy}{dx}=\frac{4-2x}{2y+6}=\frac{2-x}{y+3}$$

22. $x^2 - y^3 = 0$ F: $\left.\frac{dy}{dx}\right|_{(1,1)}$

$2x - 3y^2 \frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{-2x}{-3y^2} = \frac{2x}{3y^2}$

at $(1,1)$ $\frac{dy}{dx} = \frac{2}{3}$

18. $x^2 + y^2 - 4x + 6y + 9 = 0$

$y^2 + 6y + \underline{9} = -x^2 + 4x$

$(y+3)^2 = -x^2 + 4x$

$y + 3 = \pm\sqrt{-x^2 + 4x}$

$y = -3 \pm \sqrt{-x^2 + 4x}$

$(x+b)^2 =$

$a^2 + 2ab + b^2$

$x^2 + 2bx + b^2$