2.5 Implicit Differentiation

Study 2.5, # 1-17, 21-31, 51, 53

Goal: to find the derivative of a variable that is not expressed explicitly in terms of other variables.

Examples: Find *dy/dx*

$xy = 1$ $\sin y = x$

$x^2 + y^2 = 25$

2.5 Implicit Differentiation

$G: x^2+y^2=100$ $F: \frac{dy}{dx}$

If we solve explicitly for x,
we have 2 functions,
the upper semicircle, a pos. radical,
and the lower semicircle, a neg. radical.

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2.5 Implicit Differentiation

G: $x^2+y^2=100$ F: $\frac{dy}{dx}$

If we solve explicitly for x, we have 2 functions, the upper semicircle, + radical, and the lower semicircle, - radical.

$y^2 = 100 - x^2$

$y = \pm\sqrt{100-x^2} = \pm(100-x^2)^{1/2}$

for $y>0$: $\frac{dy}{dx} = \frac{1}{2}(100-x^2)^{-1/2}(-2x)$

$\frac{dy}{dx} = \frac{-x}{\sqrt{100-x^2}} = \frac{-x}{y}$

for $y<0$: $\frac{dy}{dx} = \frac{x}{\sqrt{100-x^2}} = \frac{x}{-y}$

$\boxed{\frac{dy}{dx} = \frac{-x}{y}}$

2.5 Implicit Differentiation

$x^2+y^2=100$ F: $\frac{dy}{dx}$

derivative with **respect to x**

Since $y = \pm\sqrt{100-x^2}$

treat y^2 as a composite function

Instead of solving explicitly, use the **Chain Rule** on y^2 to solve implicitly.

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2.5 Implicit Differentiation

$x^2 + y^2 = 100$ $\quad F: \frac{dy}{dx}$

derivative with **respect to x**

$2x + 2y\frac{dy}{dx} = 0$

Since $y = \pm\sqrt{100 - x^2}$

treat y^2 as a composite function

$2y\frac{dy}{dx} = -2x$

Instead of solving explicitly, use the **Chain Rule** on y^2 to solve implicitly.

$\frac{dy}{dx} = \frac{-2x}{2y} = \frac{-x}{y}$

2.5 Implicit Differentiation

$x^2 - y^2 = 16$ $\quad \frac{dy}{dx} = ?$

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2.5 Implicit Differentiation

$x^2 - y^2 = 16$

$\frac{dy}{dx} = \frac{x}{y}$

$2x - 2y\frac{dy}{dx} = 0$

$-2y\frac{dy}{dx} = -2x$

$\frac{dy}{dx} = \frac{-2x}{-2y} = \frac{x}{y}$

Class Notes: Prof. G. Battaly, Westchester Community College, NY

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2.5 Implicit Differentiation

$x^3 + y^3 = 8$

$\frac{dy}{dx} = ?$

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2.5 Implicit Differentiation

$$x^3+y^3=8$$

$$3x^2+3y^2\frac{dy}{dx}=0$$

$$3y^2\frac{dy}{dx}=-3x^2$$

$$\frac{dy}{dx}=-\frac{x^2}{y^2}$$

$$\frac{dy}{dx}=\frac{-x^2}{y^2}$$

2.5 Implicit Differentiation

$$(2\sin x)\cos y=1$$

Find $\frac{dy}{dx}$

2.5 Implicit Differentiation

$(2\sin x)\cos y = 1$ $\quad F: \frac{dy}{dx}$

$(2\sin x)(-\sin y \frac{dy}{dx}) + \cos y (2\cos x) = 0$

$-2\sin x \sin y \frac{dy}{dx} = -2\cos x \cos y$

$\frac{dy}{dx} = \frac{\cos x \cos y}{\sin x \sin y}$

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2.5 Implicit Differentiation

$x^2 + y^2 - 4x + 6y + 9 = 0$ $\quad F: \frac{dy}{dx}$

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2.5 Implicit Differentiation

$x^2+y^2-4x+6y+9=0$ $\quad F: \frac{dy}{dx}$

$2x+2y\frac{dy}{dx}-4+6\frac{dy}{dx}+0=0$

$2y\frac{dy}{dx}+6\frac{dy}{dx}=4-2x$

$(2y+6)\frac{dy}{dx}=4-2x$

$\frac{dy}{dx}=\frac{4-2x}{2y+6}=\frac{2-x}{y+3}$

2.5 Implicit Differentiation

$x^2+y^2-4x+6y+9=0$

$y^2+6y+9=-x^2+4x$

$(y+3)^2=-x^2+4x$

$y+3=\pm\sqrt{-x^2+4x}$

$y=-3\pm\sqrt{-x^2+4x}$

$(x+b)^2=$

$a^2+2ab+b^2$

$x^2+2bx+b^2$

2.5 Implicit Differentiation

$x^2 - y^3 = 0$ F: $\left.\frac{dy}{dx}\right|_{(1,1)}$

2.5 Implicit Differentiation

$x^2 - y^3 = 0$ F: $\left.\frac{dy}{dx}\right|_{(1,1)}$

$2x - 3y^2\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{-2x}{-3y^2} = \frac{2x}{3y^2}$

at $(1,1)$, $\frac{dy}{dx} = \frac{2}{3}$

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2.5 Implicit Differentiation

G: $x^2+y^2=36$

F: eq. tang line and normal line at (6,0) and at $(5,\sqrt{11})$

2.5 Implicit Differentiation

G: $x^2+y^2=36$

$2x+2y\frac{dy}{dx}=0$

$\frac{dy}{dx}=\frac{-2x}{2y}=-\frac{x}{y}$

$\left.\frac{dy}{dx}\right|_{(6,0)}=\frac{-6}{0}$ DNE

$\left.\frac{dy}{dx}\right|_{(5,\sqrt{11})}=\frac{-5}{\sqrt{11}}=m_T$

$m_N=\frac{\sqrt{11}}{5}$

F: eq. tang line and normal line at (6,0) and at $(5,\sqrt{11})$

$y-y_1=m_T(x-x_1)$

$y-0=m_T(x-6)$

$y=m_T(x-6)$

$x=6$ tang.

$y=0$ normal

$y-\sqrt{11}=m_T(x-5)$

tangent

$y-\sqrt{11}=\frac{-5}{\sqrt{11}}(x-5)$

$y=\frac{-5}{\sqrt{11}}x+\frac{25}{\sqrt{11}}+\sqrt{11}$

normal

$y-\sqrt{11}=\frac{\sqrt{11}}{5}(x-5)$

$y=\frac{\sqrt{11}}{5}x-\sqrt{11}+\sqrt{11}$

$y=\frac{\sqrt{11}}{5}x$

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