# 2.4 Chain Rule: Derivative of Composite Functions

Study 2.4
p. 137 # 1 - 27;
[37,41,45,.....69 (every 4th)]
71, 73, 77, 79, 81,95, 108

$y = \left[\sin(2x^3+1)\right]^2$

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# 2.4 Chain Rule: Derivative of Composite Functions

$y = (2x^3 + 1)^2$

F: dy/dx

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# 2.4 Chain Rule: Derivative of Composite Functions

$y = (2x^3+1)^2$ F: dy/dx $f(g(x))$

$= (2x^3+1)(2x^3+1)$

$\frac{dy}{dx} = (2x^3+1)(6x^2) + (2x^3+1)(6x^2)$

$= 12x^2(2x^3+1)$

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# 2.4 Chain Rule: Derivative of Composite Functions

$y = (2x^3+1)^2$

F: dy/dx

$f(x) = 2x^3+1$

$f(\ ) = 2(\ )^3+1$

$g(\ ) = (\ \ \ \ )^2$

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## 2.4 Chain Rule: Derivative of Composite Functions

$y = (2x^3+1)^2$ F: $\frac{dy}{dx}$

Can we use the Power Rule directly and get the correct answer? NO!

$\frac{dy}{dx} \neq 2(2x^3+1)^1$

NEED FACTOR: $6x^2$

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# 2.4 Chain Rule: Derivative of Composite Functions

$$y = (2x^3 + 1)^2$$

Without the Chain Rule, we are missing a factor of $6x^2$.

$$\frac{dy}{dx} = 2(2x^3+1)^1 \frac{d(2x^3+1)}{dx}$$

$$= 2(2x^3+1)(6x^2)$$

$$= 12x^2(2x^3+1)$$

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# 2.4 Chain Rule: Derivative of Composite Functions

$y = (2x^3 + 1)^{10}$ F: $dy/dx$

## 2.4 Chain Rule: Derivative of Composite Functions

$y = (2x^3+1)^{10}$ F: $dy/dx$

$u = 2x^3+1$

$y = u^{10}$

$\frac{du}{dx} = 6x^2$

$\frac{dy}{du} = 10u^9$

$\frac{dy}{dx} = 10(2x^3+1)^9 \cdot 6x^2$

$= 60x^2(2x^3+1)^9$

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# 2.4 Chain Rule: Derivative of Composite Functions

$y = (2x^3+1)^{10}$ F: dy/dx

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**Chain Rule**

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

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# 2.4 Chain Rule: Derivative of Composite Functions

$$y = (\sin x^2)^3 \qquad \frac{dy}{dx} = ?$$

Hint: $y = (\sin x^2)^3$

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# 2.4 Chain Rule: Derivative of Composite Functions

$$y = (\sin x^2)^3$$

$$\frac{dy}{dx} = 3(\sin x^2)^2 \cdot (\cos x^2)(2x)$$

$$= 6x(\cos x^2)(\sin x^2)^2$$

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## 2.4 Chain Rule: Derivative of Composite Functions

13. G: $y = \sqrt[3]{6x^2+1}$ F: $\frac{dy}{dx}$

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# 2.4 Chain Rule: Derivative of Composite Functions

13. G: $y = \sqrt[3]{6x^2+1}$

$y = u^{1/3}$

$\frac{dy}{du} = \frac{1}{3}u^{-2/3}$

$\frac{dy}{dx} = \frac{4x}{(6x^2+1)^{2/3}}$

F: $\frac{dy}{dx}$

$u = 6x^2+1$

$\frac{du}{dx} = 12x$

$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

$= \frac{1}{3}(6x^2+1)^{-2/3}(12x)$

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# 2.4 Chain Rule: Derivative of Composite Functions

30. $h(t) = \left(\frac{t^2}{t^3+2}\right)^2$

F: $h'(t)$

## 2.4 Chain Rule: Derivative of Composite Functions

30. $h(t) = \left(\frac{t^2}{t^3+2}\right)^2$ F: $h'(t)$

$$= \frac{t^4}{(t^3+2)^2} = \frac{t^4}{t^6+4t^3+4}$$

$$\left(\frac{a}{b}\right)^2 = \frac{a^2}{b^2}$$

30. $h(t) = \left(\dfrac{t^2}{t^3+2}\right)^2$

F: $h'(t)$

$\dfrac{dh}{dt} = \dfrac{dh}{du} \cdot \dfrac{du}{dt}$

$\dfrac{DdN - NdD}{D^2}$

$u = \dfrac{t^2}{t^3+2}$

$h(u) = u^2$

$\dfrac{dh}{du} = 2u$

$\dfrac{du}{dt} = \dfrac{(t^3+2)(2t) - t^2(3t^2)}{(t^3+2)^2}$

$= \dfrac{2t^4 + 4t - 3t^4}{(t^3+2)^2} = \dfrac{-t^4 + 4t}{(t^3+2)^2}$

$\dfrac{dh}{dt} = \dfrac{2t^2}{(t^3+2)} \cdot \dfrac{t(4-t^3)}{(t^3+2)^2}$

$= \dfrac{2t^3(4-t^3)}{(t^3+2)^3}$

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p.137 #12. $g(x) = \sqrt{9-4x}$ F: $g'(x)$

p.137 #12. $g(x) = \sqrt{9-4x}$ F: $g'(x)$

$y = (9-4x)^{1/2}$

$u = 9-4x$

$\frac{du}{dx} = -4$

$y = u^{1/2}$

$\frac{dy}{du} = \frac{1}{2} u^{-1/2} = \frac{1}{2\sqrt{u}}$

$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

$= \frac{1}{2\sqrt{9-4x}} \cdot (-4) = \frac{-2}{\sqrt{9-4x}}$

# Chain Rule

If:

1) *y = f(u)* is a differentiable function of *u*, and
2) *u=g(x)* is a differentiable function of *x*,

then

*y = f(g(x))* is a differentiable function of *x*, and

$$\frac{dy}{dx} = \frac{dy}{du} \bullet \frac{du}{dx}$$

or

$$\frac{d}{dx}[f(g(x))] = f'(g(x))g'(x)$$

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36. $g(t) = \sqrt{\sqrt{t+1} + 1}$

$= \dfrac{1}{4\sqrt{t+1}\,\sqrt{\sqrt{t+1}+1}}$

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36. $g(t) = \sqrt{\sqrt{t+1} + 1} = \left((t+1)^{1/2} + 1\right)^{1/2}$

$$g'(t) = \frac{1}{2}\left[(t+1)^{1/2} + 1\right]^{-1/2} \frac{d\left[(t+1)^{1/2} + 1\right]}{dt}$$

$$= \frac{1}{2}\left[(t+1)^{1/2} + 1\right]^{-1/2} \cdot \left[\frac{1}{2}(t+1)^{-1/2}(1)\right]$$

$$= \frac{1}{2\sqrt{\sqrt{t+1} + 1}} \cdot \frac{1}{2\sqrt{t+1}}$$

$$\sqrt{a}\sqrt{b} = \sqrt{ab}$$

$$= \frac{1}{4\sqrt{t+1}\sqrt{\sqrt{t+1} + 1}}$$

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70. G: $f(x) = \dfrac{1}{(x^2-3x)^2}$ F: $f'(x)$ at $(4, \frac{1}{16})$

$f'(4)$

70. G: $f(x) = \frac{1}{(x^2-3x)^2}$ F: $f'(x)$ at $(4, \frac{1}{16})$

$f'(4)$

$f(x) = (x^2-3x)^{-2}$

$f'(x) = -2(x^2-3x)^{-3} \cdot (2x-3)$

$= \frac{-2(2x-3)}{(x^2-3x)^3}$

$f'(4) = \frac{-2(8-3)}{(16-12)^3} = \frac{-10}{64} = -\frac{5}{32}$

38. $y = \sqrt{\frac{2x}{x+1}} = \left(\frac{2x}{x+1}\right)^{1/2}$

38. $y = \sqrt{\dfrac{2x}{x+1}} = \left(\dfrac{2x}{x+1}\right)^{1/2}$

$$\frac{dy}{dx} = \frac{1}{2}\left(\frac{2x}{x+1}\right)^{-1/2} \frac{d\left(\frac{2x}{x+1}\right)}{dx}$$

$$= \frac{1}{2}\left(\frac{2x}{x+1}\right)^{-1/2}\left[\frac{(x+1)2 - 2x(1)}{(x+1)^2}\right] = \frac{1}{2}\left[\frac{x+1}{2x}\right]^{\frac{1}{2}} \cdot \frac{2}{(x+1)^2}$$

$(x+1)2 - 2x(1) = 2x + 2 - 2x$

$$= \frac{(x+1)^{1/2}}{\sqrt{2x}} \cdot \frac{1}{(x+1)^2} = \frac{1}{\sqrt{2x}\,(x+1)^{3/2}}$$

10. $y = (9x+2)^{2/3}$ F: $\frac{dy}{dx}$

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10. $y = (9x+2)^{2/3}$ F: $\frac{dy}{dx}$

$y = u^{2/3}$ $u = 9x+2$

$\frac{dy}{du} = \frac{2}{3}u^{-1/3}$ $\frac{du}{dx} = 9$

$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

$= \frac{2}{3}u^{-1/3} \cdot 9 = \frac{6}{u^{1/3}} = \frac{6}{(9x+2)^{1/3}}$

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12. $y = \sqrt{5-3x}$ F: dy/dx

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12. $y = \sqrt{5-3x} = (5-3x)^{1/2}$

$u = 5-3x$

$\frac{du}{dx} = -3$

$y = u^{1/2}$

$\frac{dy}{du} = \frac{1}{2}u^{-1/2}$

$\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx} = \frac{1}{2}u^{-1/2}(-3) = \frac{-3}{2u^{1/2}}$

$= \frac{-3}{2\sqrt{5-3x}} = -\frac{3}{2}(5-3x)^{-1/2}$

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| | |
|---|---|
| $\frac{d[\sin u]}{dx} = \cos u \frac{du}{dx}$ | $\frac{d(\cos u)}{dx} = -\sin u \frac{du}{dx}$ |
| $\frac{d(\tan u)}{dx} = \sec^2 u \frac{du}{dx}$ | $\frac{d(\cot u)}{dx} = -\csc^2 u \frac{du}{dx}$ |
| $\frac{d(\sec u)}{dx} = \sec u \tan u \frac{du}{dx}$ | $\frac{d(\csc u)}{dx} = -\csc u \cot u \frac{du}{dx}$ |

$$\sin^2(3x^2-4x+1) + \cos^2(3x^2-4x+1) = 1$$

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42. $y = \sin \pi x$ F: dy/dx

$u = \pi x$

$\frac{du}{dx} = \pi$

$y = \sin u$

$\frac{dy}{dx} = \cos u$

$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

$= \pi \cos \pi x$

$y = \sin \frac{3\pi}{2} x$

$\frac{dy}{dx} = \frac{3\pi}{2} \cos \frac{3\pi}{2} x$

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42. $y = \sin \pi x$

$u = \pi x$

$\frac{du}{dx} = \pi$

$y = \sin u$

$\frac{dy}{du} = \cos u$

$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = (\cos u)\pi = \pi \cos \pi x$

$\frac{d(\sin u)}{dx} = \cos u \frac{du}{dx}$

$\frac{d(\sin \pi x)}{dx} = \pi \cos \pi x$

46. $y = \cos(1-2x)^2$ F: $dy/dx$

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$$y = \cos(1-2x)^2$$

$$F: \frac{dy}{dx}$$

This composite function is composed of 3 nested functions.

$$y = f(g(h(x)))$$

Therefore, the Chain Rule requires 3 factors, where each factor is the derivative of one of the functions in the composite function.

$$y = \cos(1-2x)^2 \qquad F: \frac{dy}{dx}$$

This composite function is composed of 3 nested functions.

$$y = f(g(h(x)))$$

Therefore, the Chain Rule requires 3 factors, where each factor is the derivative of one of the functions in the composite function.

$$\frac{dy}{dx} = -[\sin(1-2x)^2][2(1-2x)][-2]$$

$$= +4(1-2x)\sin(1-2x)^2$$

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44. $h(x) = \sec x^2$  F: $h'(x)$

$h'(x) = (\sec x^2 \tan x^2)(2x)$

$= 2x \sec x^2 \tan x^2$



 

11. $f(t) = \sqrt{5-t}$  F: $f'(t)$

$f(t) = (5-t)^{1/2}$

$f'(t) = \frac{1}{2}(5-t)^{-1/2}(-1)$

$= -\dfrac{1}{2\sqrt{5-t}}$

23. G: $f(x) = x^2(x-2)^4$ PR

$f'(x) = x^2 \frac{d(x-2)^4}{dx} + (x-2)^4 \frac{d(x^2)}{dx}$

$= x^2\left[4(x-2)^3(1)\right] + (x-2)^4(2x)$ $2x(x-2)^3(3x-2)$

$= 4x^2(x-2)^3 + 2x(x-2)^4$

$= 2x(x-2)^3\left[2x + (x-2)\right] = 2x(x-2)^3(3x-2)$

~~$4x(x-2)(x-1)$~~

21. $y = \frac{1}{\sqrt{x+2}} = (x+2)^{-1/2}$

$$\frac{dy}{dx} = -\frac{1}{2}(x+2)^{-3/2}(1)$$

$$= -\frac{1}{2(x+2)^{3/2}}$$

$$= \frac{-1}{2\sqrt{(x+2)^3}} = \frac{-1}{2(x+2)\sqrt{x+2}}$$

61. G: $f(t) = 3\sec^2(\pi t - 1)$ F: $f'(t)$

$$f(t) = 3\left[\sec(\pi t - 1)\right]^2$$

$$f'(t) = 3\left\{2\left[\sec(\pi t - 1)\right]^1 \cdot \sec(\pi t - 1)\tan(\pi t - 1)\,\pi\right\}$$

$$= 6\pi \sec^2(\pi t - 1)\tan(\pi t - 1)$$