2.3 Product and Quotient Rules of Differentiation

GOALS: 1. For functions that are not simple, recognize those with products and/or quotients.
2. Learn the Product Rule and apply it properly.
3. Learn the Quotient Rule and apply it properly.

Study 2.3 # 1-7, 11, 15, 17, 21, 22, 23, 24, 29, 31, 37, 39, 43, 53, 57, 59 61, 63, 67, 75, 83, 87, 93, 97, 101

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2.3 Product and Quotient Rules of Differentiation

$y = x^5 + x^3$ F: $\frac{dy}{dx}$

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2.3 Product and Quotient Rules of Differentiation

$y = x^5 + x^3$

$y = x^3(x^2+1)$ F: $\frac{dy}{dx}$

How do find derivative?

2.3 Product and Quotient Rules of Differentiation

$y = x^5 + x^3$

$y = x^3(x^2+1)$ F: $\frac{dy}{dx}$

How do find derivative?

What is the **wrong** approach?

$(3x^2)(2x)$

C

2.3 Product and Quotient Rules of Differentiation

$y = x^3(x^2+1)$ F: $\frac{dy}{dx}$

How do find derivative?

$x^m x^n = x^{m+n}$

$y = x^5 + x^3$

$\frac{dy}{dx}$ $5x^4 + 3x^2$

$\neq 6x^3$

2.3 Product and Quotient Rules of Differentiation

$y = (x-1)(x+1)$ F: $\frac{dy}{dx}$

How do find derivative?

2.3 Product and Quotient Rules of Differentiation

$y = (x-1)(x+1)$ F: $\frac{dy}{dx}$

How do find derivative?

$y = x^2 - 1$

$\frac{dy}{dx} = 2x$

2.3 Product and Quotient Rules of Differentiation

$y = \sqrt{x}(x+1)$ F: $\frac{dy}{dx}$

How do find derivative?

2.3 Product and Quotient Rules of Differentiation

$y = \sqrt{x}(x+1)$ F: $\frac{dy}{dx}$

How do find derivative?

$y = \sqrt{x}\,x + \sqrt{x}$

$= x^{1/2}x^1 + x^{1/2}$

$= x^{3/2} + x^{1/2}$

$x^m x^n = x^{m+n}$

$\frac{dy}{dx} = \frac{3}{2}x^{1/2} + \frac{1}{2}x^{-1/2} = \frac{3}{2}x^{1/2} + \frac{1}{2x^{1/2}}$

$= \frac{3x^{1/2}}{2}\cdot\frac{x^{1/2}}{x^{1/2}} + \frac{1}{2x^{1/2}}$

$= \frac{3x+1}{2x^{1/2}} = \frac{3x+1}{2\sqrt{x}}$

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2.3 Product and Quotient Rules of Differentiation

$y = (x+1)(\sqrt{x+y})$ | $y = x\sin x$

What about a product we cannot expand?

We need a rule to handle these.

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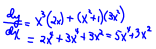
2.3 Product and Quotient Rules of Differentiation

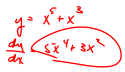
Product Rule

PRODUCT RULE:

If f and g are both differentiable functions,

then $\frac{d}{dx}[f(x)g(x)] = f(x)\,g'(x) + g(x)\,f'(x)$





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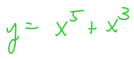
2.3 Product and Quotient Rules of Differentiation

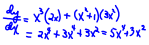
Product Rule

$y = x^3(x^2+1)$ F: $\frac{dy}{dx}$

How do find derivative?



$\frac{d}{dx}[f(x)g(x)] = f(x)\,g'(x) + g(x)\,f'(x)$



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2.3 Product and Quotient Rules of Differentiation

Product Rule

$y = x^3(x^2+1)$ F: $\frac{dy}{dx}$

How do find derivative?

$y = x^5 + x^3$

$$\frac{d}{dx}[f(x)g(x)] = f(x)\,g'(x) + g(x)\,f'(x)$$

$$\frac{dy}{dx} = x^3(2x) + (x^2+1)(3x^2)$$

$$= 2x^4 + 3x^4 + 3x^2$$

$$= 5x^4 + 3x^2$$

2.3 Product and Quotient Rules of Differentiation

Product Rule

$$\frac{d}{dx}[f(x)g(x)] = f(x)\,g'(x) + g(x)\,f'(x)$$

$f(x) = (6x+5)(x^3-2)$ F: $f'(x)$

2.3 Product and Quotient Rules of Differentiation

Product Rule

$$\frac{d}{dx}[f(x)g(x)] = f(x)\,g'(x) + g(x)\,f'(x)$$

$f(x) = (6x+5)(x^3-2)$ F: $f'(x)$

$$f'(x) = (6x+5)\frac{d(x^3-2)}{dx} + (x^3-2)\frac{d(6x+5)}{dx}$$

$$= (6x+5)(3x^2) + (x^3-2)(6)$$

$$= 18x^3 + 15x^2 + 6x^3 - 12$$

$$= 24x^3 + 15x^2 - 12$$

2.3 Product and Quotient Rules of Differentiation

Product Rule

$$\frac{d}{dx}[f(x)g(x)] = f(x)\,g'(x) + g(x)\,f'(x)$$

$g(s) = \sqrt{s}\,(4-s^2)$ F: $g'(s)$

2.3 Product and Quotient Rules of Differentiation

Product Rule

$$\frac{d}{dx}[f(x)g(x)] = f(x)\,g'(x) + g(x)\,f'(x)$$

$g(s) = \sqrt{s}\,(4-s^2)$ F: $g'(s)$

$= s^{1/2}(4-s^2)$

$g'(s) = s^{1/2}\frac{d(4-s^2)}{ds} + (4-s^2)\frac{d(s^{1/2})}{ds}$

$= s^{1/2}(-2s) + (4-s^2)\left(\frac{1}{2}s^{-1/2}\right)$

$= -2s^{3/2} + (4-s^2)\left(\frac{1}{2s^{1/2}}\right)$

$= -2s^{3/2} + \frac{4-s^2}{2s^{1/2}}$

$= \frac{-4s^2}{2s^{(1/2)}} + \frac{4-s^2}{2s^{(1/2)}} = \frac{4-5s^2}{2s^{(1/2)}}$

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2.3 Product and Quotient Rules of Differentiation

Product Rule

$$\frac{d}{dx}[f(x)g(x)] = f(x)\,g'(x) + g(x)\,f'(x)$$

G: $y = 2x(4-x^3)$ F: $\frac{dy}{dx}$

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2.3 Product and Quotient Rules of Differentiation

Product Rule

$$\frac{d}{dx}[f(x)g(x)] = f(x)\,g'(x) + g(x)\,f'(x)$$

G: $y = 2x(4-x^3)$ F: $\frac{dy}{dx}$

$$\frac{dy}{dx} = 2x(-3x^2) + (4-x^3)(2)$$

$$= -6x^3 + 8 - 2x^3$$

$$= -8x^3 + 8$$

2.3 Product and Quotient Rules of Differentiation

Product Rule

$$\frac{d}{dx}[f(x)g(x)] = f(x)\,g'(x) + g(x)\,f'(x)$$

6. $g(x) = \sqrt{x}\sin x$ F: $g'(x)$

2.3 Product and Quotient Rules of Differentiation

Product Rule

$$\frac{d}{dx}[f(x)g(x)] = f(x)\,g'(x) + g(x)\,f'(x)$$

6. $g(x) = \sqrt{x}\sin x$ F: $g'(x)$

$g(x) = x^{1/2}\sin x$

$g'(x) = x^{1/2}\frac{d(\sin x)}{dx} + \sin x\frac{d(x^{1/2})}{dx}$

$= x^{1/2}\cos x + \frac{1}{2}x^{-1/2}\sin x$

$= x^{1/2}\cos x + \frac{\sin x}{2x^{1/2}}$

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2.3 Product and Quotient Rules of Differentiation

Quotient Rule

$g(t) = \frac{t^2+2}{2t-7} = (t^2+2)(2t-7)^{-1}$

QUOTIENT RULE:

If f and g are both differentiable functions,

then the quotient $\frac{f}{g}$ is differentiable at all x

such that $g(x) \neq 0$ and

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$

$\frac{DdN - NdD}{D^2}$

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2.3 Product and Quotient Rules of Differentiation

Quotient Rule

$$\frac{d\left[\frac{f(x)}{g(x)}\right]}{dx} = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$

$g(t) = \dfrac{t^2+2}{2t-7}$

2.3 Product and Quotient Rules of Differentiation

Quotient Rule

$$\frac{d\left[\frac{f(x)}{g(x)}\right]}{dx} = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$

QR: $\frac{D\cdot N - N\cdot D}{D^2}$

$h(t) = \dfrac{t^2+2}{2t-7}$

$$h'(t) = \frac{(2t-7)\frac{d}{dt}(t^2+2) - (t^2+2)\frac{d}{dt}(2t-7)}{(2t-7)^2}$$

$$= \frac{(2t-7)(2t) - (t^2+2)(2)}{(2t-7)^2}$$

$$= \frac{4t^2-14t-2t^2-4}{(2t-7)^2}$$

$$= \frac{2t^2-14t-4}{(2t-7)^2}$$

2.3 Product and Quotient Rules of Differentiation

Quotient Rule

$\frac{DdN-NdD}{D^2}$

12. $f(t) = \frac{\cos t}{t^3} = t^{-3}\cos t$

P.R.

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2.3 Product and Quotient Rules of Differentiation

Quotient Rule

12. $f(t) = \frac{\cos t}{t^3} = t^{-3}\cos t$

Quotient Rule

$\frac{DdN-NdD}{D^2}$

P.R.

$f'(t) = t^{-3}(-\sin t) + (\cos t)(-3t^{-4})$

$= -\frac{(\sin t)t}{t^3 \cdot t} - \frac{3\cos t}{t^4}$

$= \frac{-t\sin t - 3\cos t}{t^4}$

$= -\frac{(t\sin t + 3\cos t)}{t^4}$

$f'(t) = \frac{t^3(-\sin t) - (\cos t)(3t^2)}{(t^3)^2}$

$\frac{t^2[-t\sin t - 3\cos t]}{t^6}$

$= \frac{-t\sin t - 3\cos t}{t^4}$

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2.3 Product and Quotient Rules of Differentiation

Quotient Rule

29. $G: f(x) = \frac{2x+5}{\sqrt{x}}$ $x > 0$ $F: f'(x)$

Can use:

1. Simplification
2. Product Rule
3. Quotient Rule

29. $G: f(x) = \frac{2x+5}{\sqrt{x}}$ $F: f'(x)$ Quotient Rule Quotient Rule

$f(x) = x^{-1/2}(2x+5) = 2x^{1/2} + 5x^{-1/2}$

$f'(x) = 2 \cdot \frac{1}{2} x^{-1/2} + 5\left(-\frac{1}{2} x^{-3/2}\right) = x^{-1/2} - \frac{5}{2} x^{-3/2}$

$= \frac{1}{\sqrt{x}} - \frac{5}{2x\sqrt{x}} = \frac{2x-5}{2x\sqrt{x}}$

$f(x) = (2x+5)\, x^{-1/2}$

$f'(x) = (2x+5)\left(-\frac{1}{2} x^{-3/2}\right) + \left(x^{-1/2}\right)(2)$

$= \frac{-(2x+5)}{2x^{3/2}} + \frac{2}{x^{1/2}} \cdot \frac{2x}{2x} = \frac{-2x-5+4x}{2x\sqrt{x}} = \frac{2x-5}{2x\sqrt{x}}$

$f(x) = \frac{2x+5}{\sqrt{x}} = \frac{2x+5}{x^{1/2}}$ $\frac{D\,dN - N\,dD}{D^2}$

$f'(x) = \frac{x^{1/2}(2) - (2x+5)\left(\frac{1}{2} x^{-1/2}\right)}{\left(x^{1/2}\right)^2}$

$= \frac{\left(2x^{1/2} - \frac{2x+5}{2x^{1/2}}\right)}{x} \cdot \frac{2x^{1/2}}{2x^{1/2}} = \frac{4x - (2x+5)}{2x^{3/2}}$

$= \frac{2x-5}{2x^{3/2}}$

2.3 Product and Quotient Rules of Differentiation

Quotient Rule

$$y = \frac{1 - \sin x}{\cos x} \qquad F: \frac{dy}{dx}$$

$$\frac{DdN - NdD}{D^2}$$

$$\frac{d(\sin x)}{dx} = \cos x$$

$$\frac{d(\cos x)}{dx} = -\sin x$$

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2.3 Product and Quotient Rules of Differentiation

Quotient Rule

$$y = \frac{1-\sin x}{\cos x} \qquad F: \frac{dy}{dx}$$

$$\frac{DdN - NdD}{D^2}$$

one way to find the derivative, though not the easiest

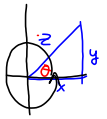
$$\frac{dy}{dx} = \frac{\cos x \frac{d(1-\sin x)}{dx} - (1-\sin x)\frac{d(\cos x)}{dx}}{\cos^2 x}$$

$$= \frac{\cos x(-\cos x) - (1-\sin x)(-\sin x)}{\cos^2 x}$$

$$= \frac{-\cos^2 x + \sin x(1-\sin x)}{\cos^2 x} = \frac{-\cos^2 x + \sin x - \sin^2 x}{\cos^2 x}$$

$$= \frac{-(\sin^2 x + \cos^2 x) + \sin x}{\cos^2 x} = \frac{\sin x - 1}{\cos^2 x}$$

$$= \frac{1}{\cos x}\cdot\frac{\sin x}{\cos x} - \frac{1}{\cos^2 x} \rightarrow \sec x \tan x - \sec^2 x$$



$$\sin^2\theta + \cos^2\theta$$

$$\left(\frac{y}{z}\right)^2 + \left(\frac{x}{z}\right)^2 = \frac{y^2}{z^2} + \frac{x^2}{z^2} = \frac{x^2+y^2}{z^2} = \frac{z^2}{z^2} = 1$$

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2.3 Product and Quotient Rules of Differentiation

Quotient Rule

$\frac{D\,dN - N\,dD}{D^2}$

$y = \frac{1-\sin x}{\cos x}$ F: $\frac{dy}{dx}$

easier to use trig identities

$y = \frac{1}{\cos x} - \frac{\sin x}{\cos x}$

G: $y = \sec x - \tan x$ F: dy/dx

$\frac{dy}{dx} = \sec x \tan x - \sec^2 x$

2.3 Product and Quotient Rules of Differentiation

co-functions

$\frac{d(\sin x)}{dx} = \cos x$ $\frac{d(\cos x)}{dx} = -\sin x$

$\frac{d(\tan x)}{dx} = \sec^2 x$ $\frac{d(\cot x)}{dx} = -\csc^2 x$

$\frac{d(\sec x)}{dx} = \sec x \tan x$ $\frac{d(\csc x)}{dx} = -\csc x \cot x$

2.3 Product and Quotient Rules of Differentiation

derivative practice

$y = x^4 - 3x^2 + 1$ F: y'' $\frac{d^2y}{dx^2} = f''(x)$

$\frac{dy}{dx} = 4x^3 - 6x$

$\frac{d^2y}{dx^2} = 12x^2 - 6$

$\frac{d^3y}{dx^3} = 24x$

$\frac{d^4y}{dx^4} = 24$ $\frac{d^5y}{dx^5} = 0$

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2.3 Product and Quotient Rules of Differentiation

derivative practice

$h(t) = \sqrt{t}(1 - t^2)$

$= t^{\frac{1}{2}}(1 - t^2) = t^{\frac{1}{2}} - t^{\frac{5}{2}}$

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2.3 Product and Quotient Rules of Differentiation

derivative practice

$h(t) = \sqrt{t}(1-t^2)$

$h(t) = t^{1/2}(1-t^2)$

$h'(t) = t^{1/2}\frac{d(1-t^2)}{dt} + (1-t^2)\frac{d(t^{1/2})}{dt}$

$= t^{1/2}(-2t) + (1-t^2)\left(\frac{1}{2}t^{-1/2}\right)$

$\frac{2t^{1/2}(-2t^{3/2})}{2t^{1/2}} + \frac{1-t^2}{2t^{1/2}} = \frac{-4t^2+1-t^2}{2t^{1/2}} = \frac{-5t^2+1}{2t^{1/2}}$

$= t^{1/2}(1-t^2) = t^{1/2} - t^{5/2}$

$h'(t) = \frac{1}{2}t^{-1/2} - \frac{5}{2}t^{3/2}$

$= \frac{1}{2t^{1/2}} - \frac{5}{2}t^{3/2}\cdot\frac{t^{1/2}}{t^{1/2}}$

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2.3 Product and Quotient Rules of Differentiation

derivative practice

$f(x) = x\cos x$

F: $f'(x)$, $f'\left(\frac{\pi}{4}\right)$

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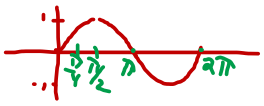
G: $f(x) = x\cos x$ F: $f'\left(\frac{\pi}{4}\right)$

$f'(x) = x\frac{d(\cos x)}{dx} + (\cos x)\frac{d(x)}{dx}$

$= x(-\sin x) + \cos x$

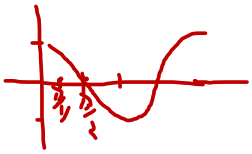
$= -x\sin x + \cos x$



$f'\left(\frac{\pi}{4}\right) = -\frac{\pi}{4}\sin\frac{\pi}{4} + \cos\frac{\pi}{4}$

$= -\frac{\pi}{4}\cdot\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}$

$= \frac{1}{\sqrt{2}}\left(1 - \frac{\pi}{4}\right)$



derivative practice

G: $f(x) = 8x^6 - 10x^5 + 5x^3$ F: $f''(x)$

G: $f(x) = 8x^6 - 10x^5 + 5x^3$ F: $f''(x)$

$f'(x) = 48x^5 - 50x^4 + 15x^2$

$f''(x) = 240x^4 - 200x^3 + 30x$

derivative practice

Quotient Rule

27. G: $f(x) = x\left(1 - \frac{4}{x+3}\right)$ F: $f'(x)$

27. G: $f(x)=x\left(1-\frac{4}{x+3}\right)$ F: $f'(x)$ Quotient Rule

$f(x)=x-\frac{4x}{x+3}=\frac{x(x+3)-4x}{x+3}$

$f'(x)=1-\left[\frac{(x+3)(4)-4x(1)}{(x+3)^2}\right]=1-\left[\frac{4x+12-4x}{(x+3)^2}\right]$

$=1-\frac{12}{(x+3)^2}=\frac{(x+3)^2-12}{(x+3)^2}$

$\frac{x^2+6x+9-12}{(\quad)^2}=\frac{x^2+6x-3}{(\quad)^2}$ $(x)=3$ $(\quad)+(\quad)=6$

$-3\ \ 1\ \ 6\ \ -3$

$1\ \ 3\ \ -12$

G: $f(x)=x+32x^{-2}$ F: $f''(x)$ derivative practice

G: $f(x) = x + 32x^{-2}$ F: $f''(x)$

$f'(x) = 1 + 32(-2x^{-3})$

$= 1 - 64x^{-3}$

$f''(x) = (-64)(-3x^{-4})$

$= 192x^{-4}$

$f'(x)$

$f''(x)$

$f'''(x)$

$y = x^4 + 3x^2 - 1$ F: $\frac{d^2y}{dx^2}$

$\frac{dy}{dx} = 4x^3 + 6x$

$\frac{d^2y}{dx^2} = 12x^2 + 6$

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