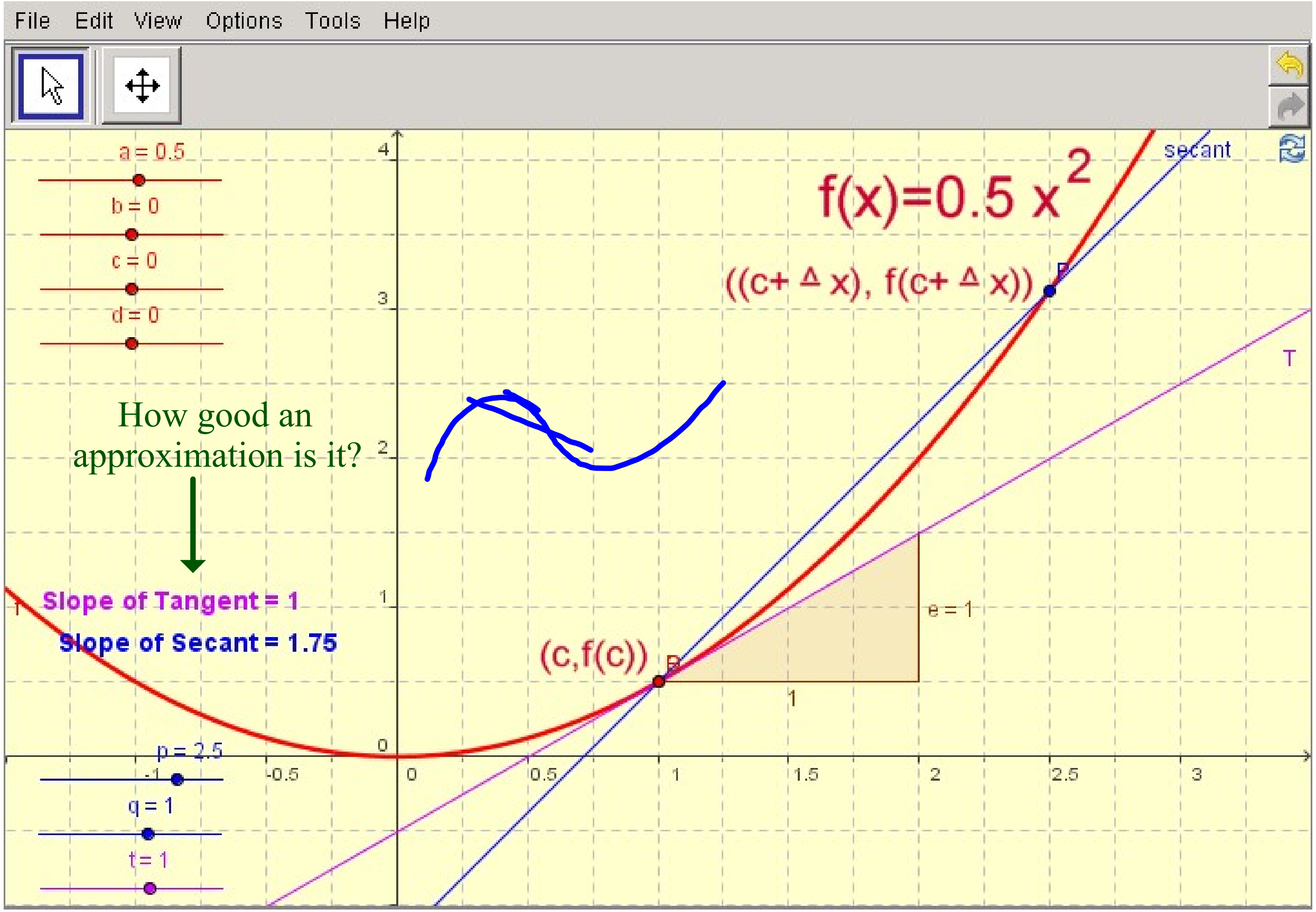
2.1 Slope of the Tangent Line and the Derivative of a Function

Study definitions: Slope of the tangent line, p. 97

Derivative, p. 99**

p. 103-104: 5, 17

Consider the slope of the secant line as an approximation to the slope of the tangent line.



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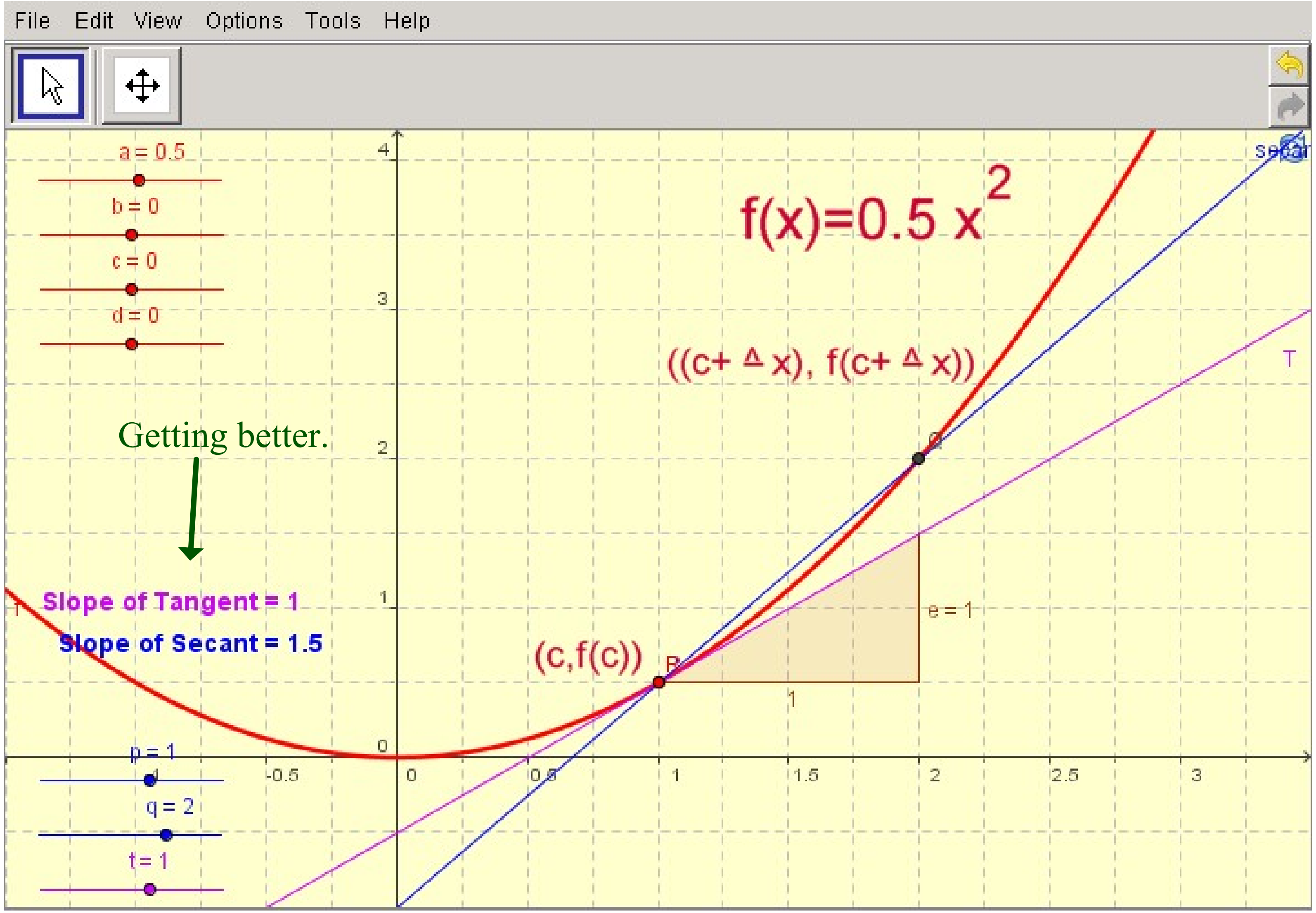
Consider the slope of the secant line as an approximation to the slope of the tangent line.



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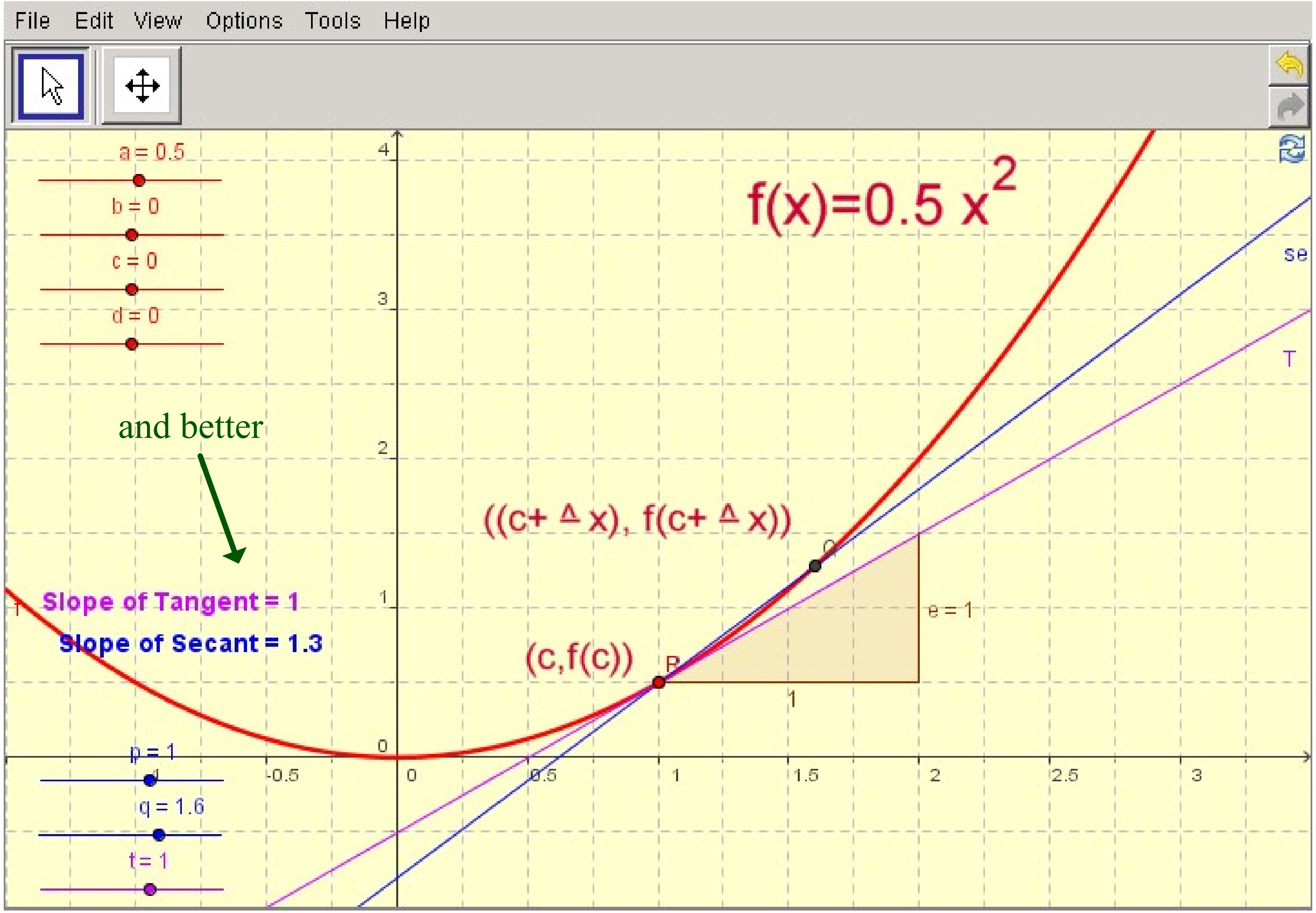

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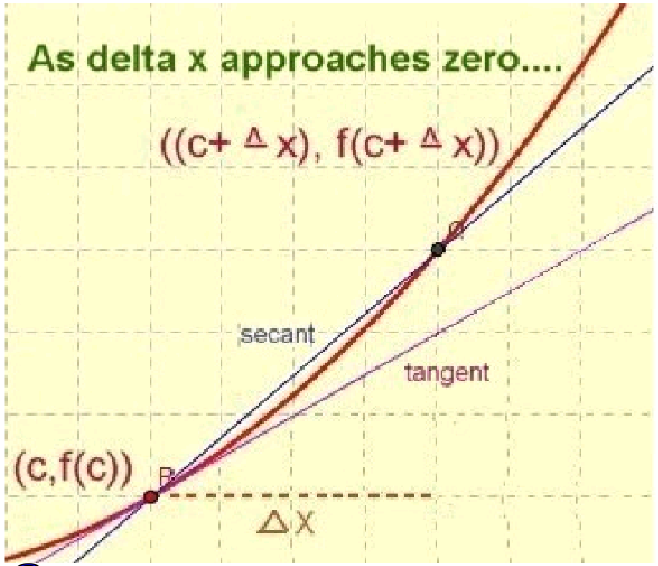

secant_tangent_slopes_dx.swf

Slope of the secant line:

$$m_s = \frac{\Delta y}{\Delta x}$$

$$= \frac{y_2 - y_1}{x_2 - x_1}$$

$$m_s = \frac{f(c+\Delta x) - f(c)}{c + \Delta x - c} =$$

$$m_s = \frac{f(c+\Delta x) - f(c)}{\Delta x}$$

Slope of the tangent line at a point (c, f(c)):

$$m_T = \lim_{\Delta x \to 0} \frac{f(c+\Delta x) - f(c)}{\Delta x}$$ at a point $(c, f(c))$

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8. $f(x) = 5 - x^2$ F: m_T at $(2,1)$

$$m_T = \lim_{\Delta x \to 0} \frac{f(c+\Delta x) - f(c)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{f(2+\Delta x) - f(2)}{\Delta x}$$

Ex: Finding the slope of a tangent line at a particular point. Use equation derived above.

$f(\ ) = 5 - (\ )^2$

$f(2) = 5 - 2^2 = 1$

$f(2+\Delta x) = 5 - (2+\Delta x)^2 = 5 - (4 + 4\Delta x + (\Delta x)^2)$

$= 5 - 4 - 4\Delta x - (\Delta x)^2 = 1 - 4\Delta x - (\Delta x)^2$

$$\lim_{\Delta x \to 0} \frac{1 - 4\Delta x - (\Delta x)^2 - 1}{\Delta x} = \lim_{\Delta x \to 0} \frac{-4\Delta x - (\Delta x)^2}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\cancel{\Delta x}(-4 - \Delta x)}{\cancel{\Delta x}} = \lim_{\Delta x \to 0} -4 - \Delta x$$

$$= -4$$

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~~Derivatie~~ Derivative of a Function

$(x, f(x))$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

provided the limit exists

This generalizes from a specific point (c,f(c)) to any point on the curve (x,f(x))

It can be interpreted as the slope of the tangent line to the curve at any point on the curve.

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G: $f(x) = x^2 + 2x + 1$ F: eq of tan line at $(-3,4)$

$y = mx + b$

$$\boxed{f'(x)} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$\frac{dy}{dx}$

Ex: Finding the equation of a tangent line at a particular point requires first finding the slope of the tangent at that point. We use the definition of the derivative as an example here.

$f(\ ) = (\ \ )^2 + 2(\ \ ) + 1$

$y - y_1 = m(x - x_1)$

$f(x+\Delta x) = (x+\Delta x)^2 + 2(x+\Delta x) + 1$

$= x^2 + 2x\Delta x + (\Delta x)^2 + 2x + 2\Delta x + 1$

$f(x+\Delta x) - f(x) = \cancel{x^2} + 2x\Delta x + (\Delta x)^2 + \cancel{2x} + 2\Delta x + \cancel{1}$

$- (\cancel{x^2} + \cancel{2x} + \cancel{1})$

$= 2x\Delta x + (\Delta x)^2 + 2\Delta x$

$$\lim_{\Delta x \to 0} \frac{2x\Delta x + (\Delta x)^2 + 2\Delta x}{\Delta x}$$

$$\lim_{\Delta x \to 0} 2x + \Delta x + 2 = 2x + 2 = f'(x)$$

The tangent line would be of the form:

y - y 1 = m (x-x1) using (-3,4)

$y = x^2 + 2x + 1$

$f'(x) = 2x + 2$

$f'(-3) = 2(-3) + 2 = -6 + 2 = -4$

y - 4 = (-4)(x - (-3)) = (-4)(x+3) = -4x - 12

Therefore, y = -4x - 8

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Alternative Form of Derivative at $x=c$

$$f'(c) = \lim_{x \to c} \frac{f(x)-f(c)}{x-c}$$

p.104 #10 G: $h(t) = t^2+3$ F: m_T at $(-2, 7)$

$$h'(-2) = \lim_{t \to -2} \frac{h(t)-h(-2)}{t-(-2)}$$

$$= \lim_{t \to -2} \frac{t^2+3-[(-2)^2+3]}{t+2} = \lim_{t \to -2} \frac{t^2+3-(7)}{t+2}$$

$$= \lim_{t \to -2} \frac{t^2-4}{t+2} = \lim_{t \to -2} \frac{(t+2)(t-2)}{t+2} = \lim_{t \to -2} (t-2) = -2-2 = -4$$

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$$f(x) = x^2 + 3 \qquad \text{F}: m_T \ (-2, 7)$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

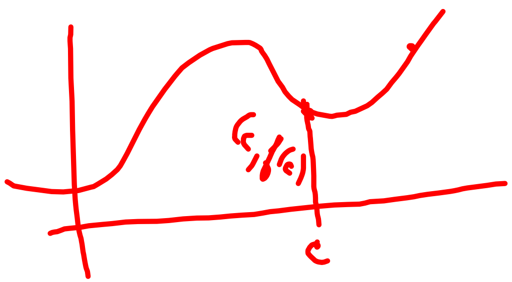
$$f(x+\Delta x) = (x+\Delta x)^2 + 3$$

$$= x^2 + 2x\Delta x + (\Delta x)^2 + 3$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{x^2 + 2x\Delta x + (\Delta x)^2 + 3 - x^2 - 3}{\Delta x} = \lim_{\Delta x \to 0} \frac{2x\Delta x + (\Delta x)^2}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\cancel{\Delta x}^{\,1}(2x+\Delta x)}{\cancel{\Delta x}_{\,1}} = \lim_{\Delta x \to 0} 2x + \Delta x = 2x$$

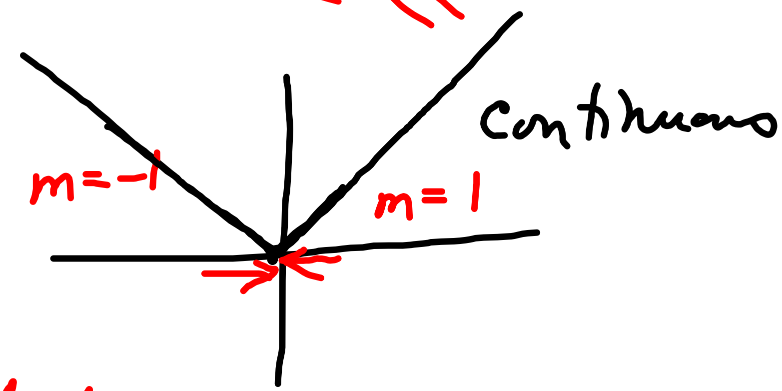
$$\text{at } (-2, 7): \ f'(-2) = -4 = m_T$$



f is differentiable at pt $\Longrightarrow$ continuous at pt.

$\not\Longleftarrow$

$y = |x|$



$\hookrightarrow$ not diff. at $x=0$

$$y' = \frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x} \quad \text{DNE}$$

bec. $\lim_{x\to 0^-} \ldots \neq \lim_{x \to 0^+}$

No sharp turns!