8.1 Arc Length

Goals:

1. Review formula for length of arc of a circle:
   s = r θ
2. Consider finding an arc length for curves that are not circles. How to approach?
3. Review the distance formula
4. Use the distance formula to generate an integral for finding the length of any arc.
5. Apply the formula for arc length:

$$L = \int_a^b \sqrt{1 + (\frac{dy}{dx})^2}\, dx$$

Study 8.1 # 1, 3, 5, 9, 15, 17

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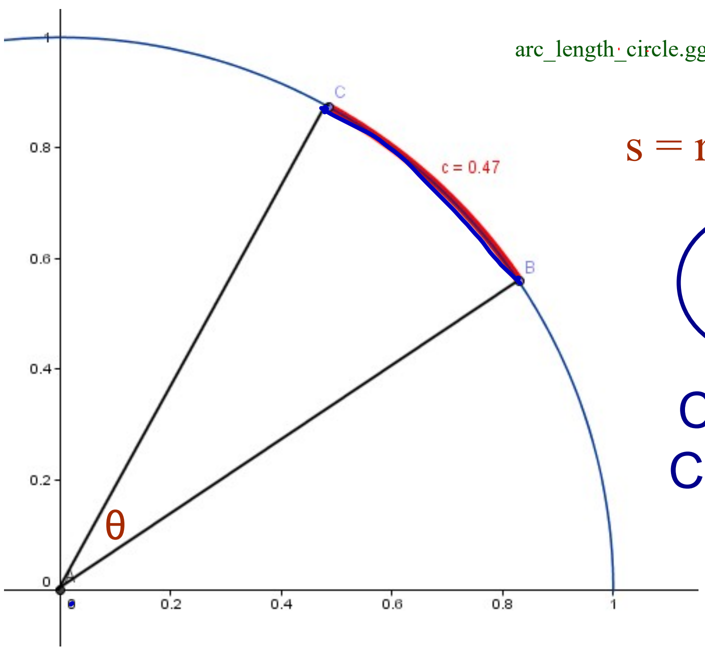
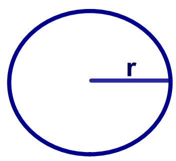
8.1 Arc Length

arc_length_circle.ggb



s = r θ



C = 2 π r

C = (angle) r

π = C/d

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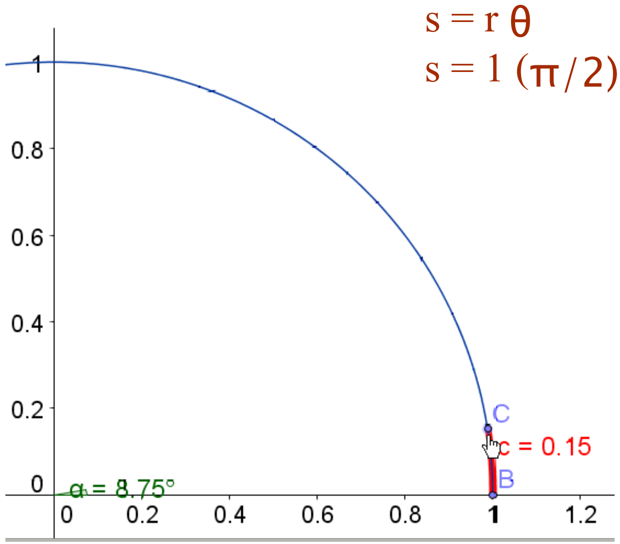
8.1 Arc Length

$s = r\,\theta$

$s = 1\,(\pi/2) = 1.57$



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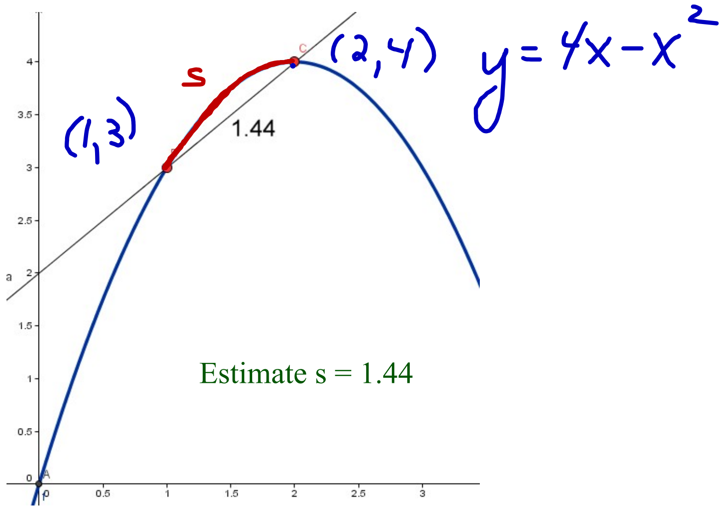
8.1 Arc Length



$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$d = \sqrt{(2-1)^2 + (4-3)^2} = \sqrt{2} = 1.41$$

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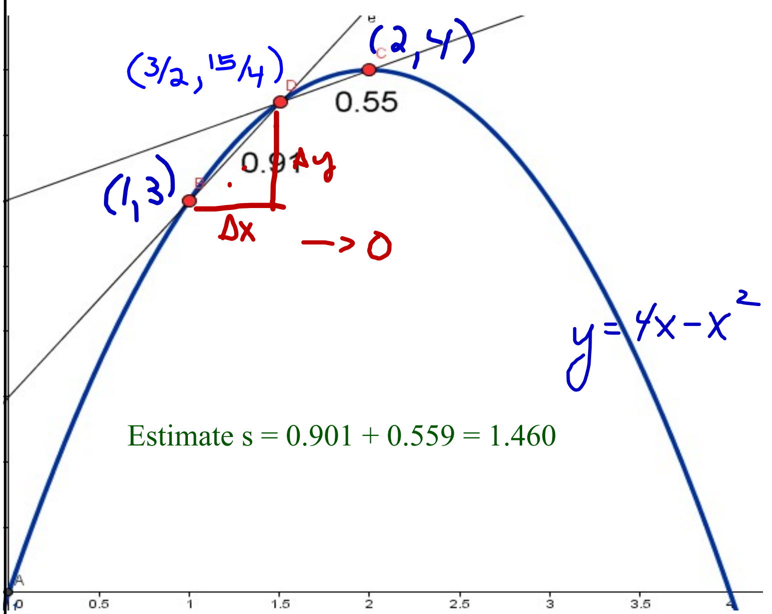
8.1 Arc Length



Sum of two line segments gets a better estimate.
Can increase the number of line segments and
let delta x approach 0.

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8.1 Arc Length

$$d = \sqrt{(x_i - x_{i-1})^2 + (y_i - y_{i-1})^2}$$

$$d = \sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{(\Delta x)^2 + (\Delta y)^2 \frac{(\Delta x)^2}{(\Delta x)^2}}$$

$$= \sqrt{(\Delta x)^2 + \frac{(\Delta y)^2}{(\Delta x)^2}(\Delta x)^2} = \sqrt{(\Delta x)^2 + \left[\frac{\Delta y}{\Delta x}\right]^2 (\Delta x)^2}$$

$$\frac{\Delta y}{\Delta x}$$

(x_i, y_i)

(x_{i-1}, y_{i-1})

$$d = \sqrt{\left[1 + \left(\frac{\Delta y}{\Delta x}\right)^2\right](\Delta x)^2} = \sqrt{\left[1 + \left(\frac{\Delta y}{\Delta x}\right)^2\right]}(\Delta x)$$

$$S = \lim_{\|\Delta\| \to 0} \sum_{i=1}^{n} \sqrt{1 + \left[\frac{\Delta y_i}{\Delta x_i}\right]^2}\, \Delta x_i$$

$$s = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

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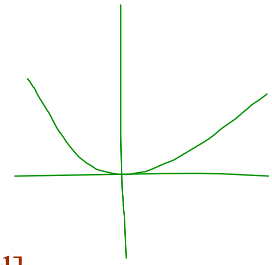
8.1 Arc Length

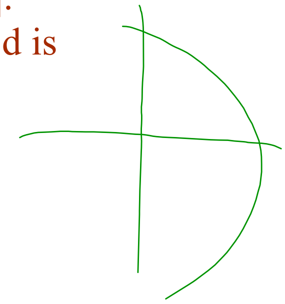
Let y = f(x) be a smooth curve on [a,b].
Then the arc length of f between a and b is

$$s = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$



Let x = g(y) be a smooth curve on [c,d].
Then the arc length of g between c and d is

$$s = \int_c^d \sqrt{1 + (g'(y))^2}\, dy$$



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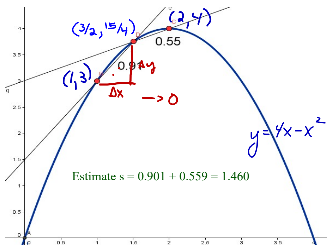
8.1 Arc Length

G: $y = 4x - x^2$ F: L, [1,2]



$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

ON THE CALCULATOR:
Y1 = *f(x)*
Y2 = nDeriv(Y1,X,X)
Y3 = √(1 + (Y2)²)
2nd CALC / ∫ f(x) dx

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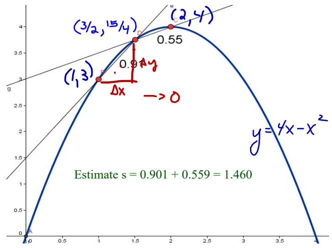
8.1 Arc Length

G: $y = 4x - x^2$ F: L, [1,2]

$dy/dx = 4-2x$

$(dy/dx)^2 = (4-2x)^2$



$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$L = \int_1^2 \sqrt{1+(4-2x)^2}\, dx$$

$L = 1.479$

ON THE CALCULATOR:
Y1 = *f(x)*
Y2 = nDeriv(Y1,X,X)
Y3 = $\sqrt{(1 + (Y2)^2)}$
2nd CALC / ∫ f(x) dx

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8.1 Arc Length

G: $y = 2x^{3/2} + 3$ F: L, [0,9]

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Homework

8.1 Arc Length

G: $y = 2x^{3/2} + 3$ F: L, [0,9]

$dy/dx = 3x^{1/2}$

$(dy/dx)^2 = (3x^{1/2})^2 = 9x$

$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$L = \int_0^9 \sqrt{1+9x}\; dx$$

$u = 1+9x$

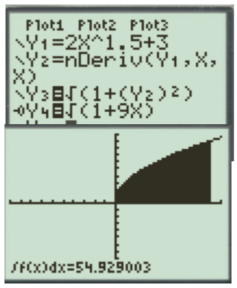
$du = 9dx$

x	u
0	1
9	82

$$L = \frac{1}{9}\int_0^9 \sqrt{1+9x}\; 9dx$$

$$L = \frac{1}{9}\int_1^{82} \sqrt{u}\; du$$

$$L = \frac{1}{9}\,\frac{2}{3}\, u^{3/2}\Big]_1^{82} = \frac{2}{27}\left[82^{3/2} - 1\right] = 54.93$$



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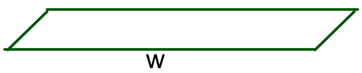
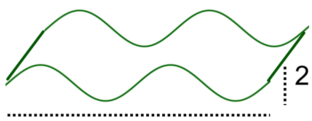
8.1 Arc Length

A corrugated metal panel is 28 inches wide and 2 in in depth. The edge has the shape of a sine wave described by the equation: $y = \sin(\pi x/7)$

The panel is made from a single flat sheet of metal. How wide is the sheet?





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8.1 Arc Length

A corrugated metal panel is 28 inches wide and 2 in in depth. The edge has the shape of a sine wave described by the equation: $y = \sin(\pi x/7)$
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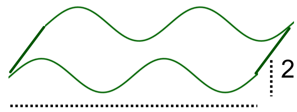
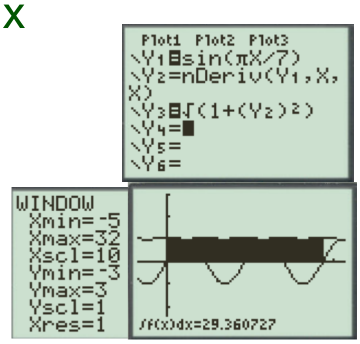


$dy/dx = (\pi/7)\cos(\pi x/7)$

$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$L = \int_0^{28} \sqrt{1+[(\pi/7)\cos(\pi x/7)]^2}\, dx$$

$= 29.36$ in.



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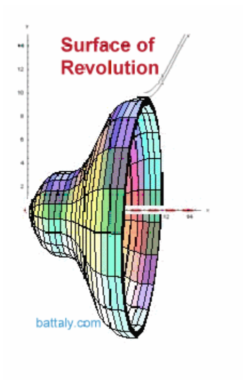
8.2 Area, Surface of Revolution



If we start with an arc length, and rotate it around an axis of revolution, we have a surface of revolution.

Surface measurement: Surface Area

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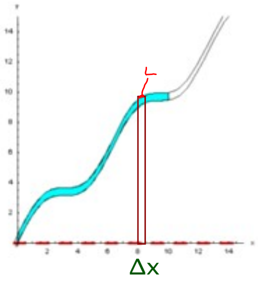
8.2 Area, Surface of Revolution



Let Δx be small enough so that the surface being rotated is like the surface of a cylinder. Then the surface area is

$SA = 2\pi\, r\, L$

where L is the arc length.

$$S = 2\pi \int_a^b r(x)\sqrt{1+[f\,'(x)]^2}\,dx$$

8.2 Area, Surface of Revolution

Let y = *f(x)* has continuous derivative on [a,b]. The Area S of the surface of revolution formed by revolving the graph of *f* about a horizontal axis is:

$$S = 2\pi \int_a^b r(x)\sqrt{1+[f\,'(x)]^2}\,dx$$

where r(x) = distance between *f* and the axis of revolution.

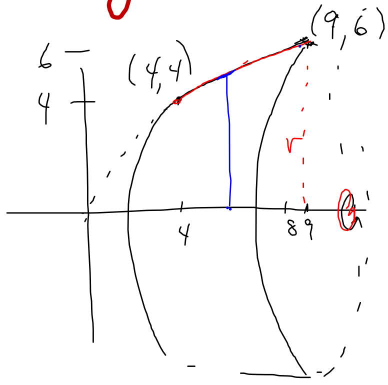
If *x=g(y)* on [c,d]. The Area S of the surface of revolution formed by revolving the graph of *g* about a verticle axis is:

$$S = 2\pi \int_c^d r(y)\sqrt{1+[g\,'(y)]^2}\,dy$$

where r(y) = distance between *g* and the axis of revolution.

8.2 Area, Surface of Revolution

· 6: $y = 2\sqrt{x} = 2x^{1/2}$ F: SA ⟲ x-axis $[4, 9]$



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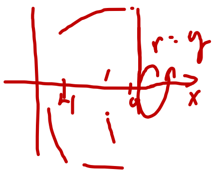
8.2 Area, Surface of Revolution

40. 6: $y = 2\sqrt{x}$ F: SA ⟲ x-axis $[4, 9]$ $\frac{dy}{dx} = x^{-1/2}$

$$SA = 2\pi \int_a^b r(x) \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$= 2\pi \int_4^9 y \sqrt{1 + \left(\frac{1}{x^{1/2}}\right)^2}\, dx$$



$$= 2\pi \int_4^9 2x^{1/2} \sqrt{1 + \frac{1}{x}}\, dx$$

$$= 2\pi \int^9 2x^{1/2} \sqrt{\frac{x+1}{x}}\, dx = 2\pi \int_4^9 2x^{1/2} \frac{\sqrt{x+1}}{x^{1/2}}\, dx$$

$$= 4\pi \int_4 (x+1)^{1/2}\, dx \qquad du = dx$$

$$= 4\pi \cdot \frac{2}{3} (x+1)^{3/2} \Big]_4^9 = \frac{8}{3}\pi \left[10^{3/2} - 5^{3/2}\right]$$

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8.2 Area, Surface of Revolution

ON THE CALCULATOR:

Y1 = *f(x)*

Y2 = nDeriv(Y1,X,X)

Y3 = $\sqrt{(1 + (Y2)^2)}$ for arc length (def integral)

Y4 = 2 π Y1 Y3 if r(x)=f(x) for Surface Area (def Int)

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8.1 Arc Length

$y = \ln(\cos x)$ F: L, $[0, \pi/3]$

defined all values on ↑

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8.1 Arc Length

$y = \ln(\cos x)$ F: L, $[0, \pi/3]$

$$s = \int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

$$\frac{dy}{dx} = \frac{1}{\cos x}\cdot(-\sin x)$$

$$\frac{dy}{dx} = -\frac{\sin x}{\cos x} = -\tan x$$

$$= \int_0^{\pi/3} \sqrt{1+(-\tan x)^2}\,dx$$

$$\frac{\sin^2}{\cos^2} + \frac{\cos^2}{\cos^2} = \frac{1}{\cos^2}$$

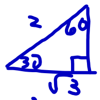
$$\tan^2 + 1 = \sec^2$$

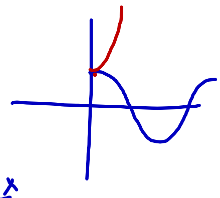
$$= \int_0^{\pi/3} \sqrt{1+\tan^2 x}\,dx$$

$$= \int_0^{\pi/3} \sqrt{\sec^2 x}\,dx$$

$$= \int_0^{\pi/3} \sec x\,dx = \ln|\sec x + \tan x|\Big]_0^{\pi/3}$$

$$= \ln\left|\tfrac{1}{2}+\sqrt{3}\right| - \ln|1+0|$$





$$= \ln\left|\tfrac{1}{2}+\sqrt{3}\right|$$

$$\sec x \cdot \frac{\sec x + \tan x}{\sec x + \tan x} = \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x}$$

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

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