

7.8 Improper Integrals

Goals:

1. Improper Integrals involve Infinity
2. Two types:
 - > Infinite Limits: $\int_a^b$ $\int_a^\infty$ $\int_{-\infty}^b$
 a or b unbounded
 - > Infinite Discontinuities within the limits, a and b
3. Improper Integral
 - > Converges if has limit
 - > Diverges if has no limit

Study 7.8 #1-11, 15, 19, 21, 27-35

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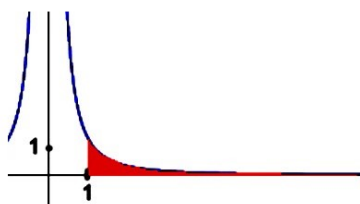
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7.8 Improper Integrals

Consider:

$$\int_1^{\infty} \frac{1}{x^2} dx$$



Does the integral exist?

OR: Is the area finite?

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$$\int_1^2 \frac{1}{x^2} dx = -\frac{1}{x} \Big|_1^2 = -\frac{1}{2} - (-1) = \frac{1}{2}$$

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$$\int_1^2 \frac{1}{x^2} dx = -\frac{1}{x} \Big|_1^2 = -\frac{1}{2} - (-1) = \frac{1}{2}$$

$$\int_1^5 \frac{1}{x^2} dx = -\frac{1}{x} \Big|_1^5 = -\frac{1}{5} - (-1) = \frac{4}{5}$$

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$$\int_1^2 \frac{1}{x^2} dx = -\frac{1}{x} \Big|_1^2 = -\frac{1}{2} - (-1) = \frac{1}{2}$$

$$\int_1^5 \frac{1}{x^2} dx = -\frac{1}{x} \Big|_1^5 = -\frac{1}{5} - (-1) = \frac{4}{5}$$

$$\int_1^{10} \frac{1}{x^2} dx = -\frac{1}{x} \Big|_1^{10} = -\frac{1}{10} - (-1) = \frac{9}{10}$$

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$$\int_1^{10} \frac{1}{x^2} dx = -\frac{1}{x} \Big|_1^{10} = -\frac{1}{10} - (-1) = \frac{9}{10}$$

$$\int_1^{100} \frac{1}{x^2} dx = -\frac{1}{x} \Big|_1^{100} = -\frac{1}{100} - (-1) = \frac{99}{100}$$

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$$\lim_{b \rightarrow \infty} \int_1^b \frac{1}{x^2} dx = \lim_{b \rightarrow \infty} \left[-\frac{1}{b} + 1 \right] = 1$$

$$\int_1^{\infty} \frac{1}{x^2} dx = \lim_{b \rightarrow \infty} \int_1^b \frac{1}{x^2} dx = 1$$

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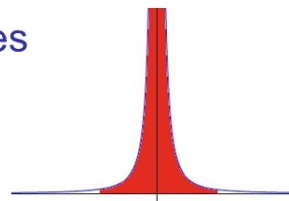
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Improper Integrals –
involve ∞

1. Infinite Limits



2. Infinite Discontinuities



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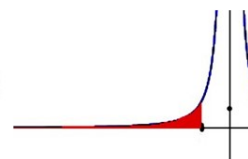
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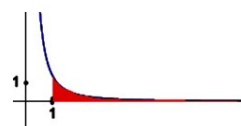
Infinite Limits

$f(x)$ cont:

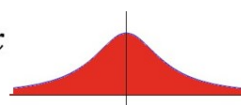
$$(-\infty, b] \quad \int_{-\infty}^b f(x) dx = \lim_{a \rightarrow -\infty} \int_a^b f(x) dx$$



$$[a, \infty) \quad \int_a^{\infty} f(x) dx = \lim_{b \rightarrow \infty} \int_a^b f(x) dx$$



$$(-\infty, \infty) \quad \int_{-\infty}^{\infty} f(x) dx = \int_{-\infty}^c f(x) dx + \int_c^{\infty} f(x) dx$$



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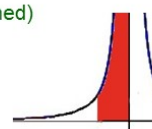
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Infinite Discontinuities

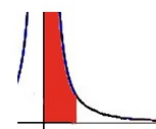
$f(x)$ cont:

(open at end of interval where $f(x)$ is not defined)

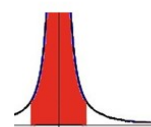
$$[a, b) \quad \int_a^b f(x) dx = \lim_{c \rightarrow b^-} \int_a^c f(x) dx$$



$$(a, b] \quad \int_a^b f(x) dx = \lim_{c \rightarrow a^+} \int_c^b f(x) dx$$



$$\underbrace{[a, b]}_{\substack{\text{disc at} \\ c \in (a, b)}} \quad \int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx$$



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Improper Integral

1. **CONVERGES** if the limit exists (finite value)
2. **DIVERGES** if the limit does NOT exist

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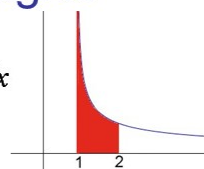
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To Solve an Improper Integral

1. **Sketch integrand.**

- Look for asymptotes
- Compare to the limits of integration

$$\int_1^2 \frac{1}{\sqrt{x-1}} dx$$



2. **If the curve is unbounded at an upper or lower limit**

- replace with a letter
- rewrite the integral as a limit

$$\int_1^2 \frac{1}{\sqrt{x-1}} dx = \lim_{a \rightarrow 1^+} \int_a^2 (x-1)^{-1/2} dx$$

3. **Perform the integration** $\int_a^2 (x-1)^{-1/2} dx = 2\sqrt{x-1} \Big|_a^2 = 2 - 2\sqrt{a-1}$

4. **Apply the limit.**

$$\lim_{a \rightarrow 1^+} \int_a^2 (x-1)^{-1/2} dx = \lim_{a \rightarrow 1^+} [2 - 2\sqrt{a-1}] = 2$$

[examples from homework](#)

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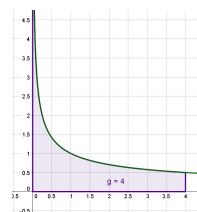
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$$\int_0^4 \frac{1}{\sqrt{x}} dx$$


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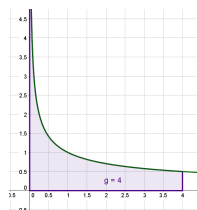
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7.8 Improper Integrals

$$\int_0^4 \frac{1}{\sqrt{x}} dx = \lim_{a \rightarrow 0^+} \int_a^4 x^{-1/2} dx$$



$$\int x^{-1/2} dx = \frac{x^{1/2}}{1/2} + c = 2x^{1/2} + c$$

$$\int_0^4 \frac{1}{\sqrt{x}} dx = \lim_{a \rightarrow 0^+} \int_a^4 x^{-1/2} dx = \lim_{a \rightarrow 0^+} \left[2x^{1/2} \right]_a^4$$

$$= \lim_{a \rightarrow 0^+} [2(4^{1/2}) - 2(a^{1/2})] = 4 - 0 = 4$$

converges


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