

7.7 Approximate Integration

Goals:

1. Acknowledge that some integrands are not integrable.
2. For **definite** integrals, can **approximate the value** by adding up areas of regions under the curve.
3. Several approaches can be used:
 - a) Midpoint Rule
 - b) Trapezoidal Rule
 - c) Simpson's Rule

Study 7.7 # 7, 16

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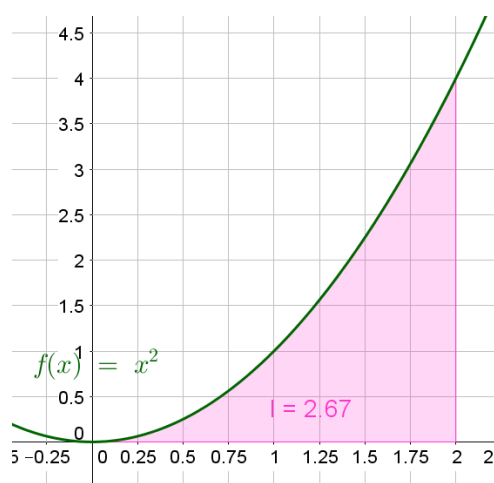
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7.4 Integration using Partial Fractions

$$\int_0^2 x^2 dx = \left[\frac{x^3}{3} \right]_0^2 = 8/3$$

If we could not integrate this function, how could we estimate or approximate its value?



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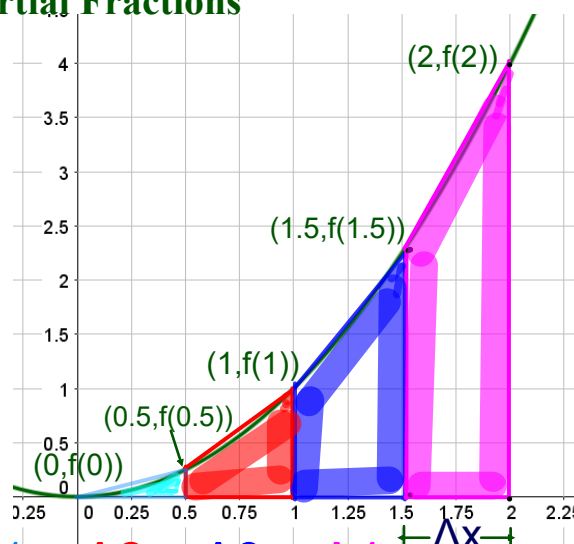
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7.4 Integration using Partial Fractions

$$\int_0^2 x^2 dx$$

Area of trapezoid:
 $\frac{1}{2}(b_1+b_2)h$

for convenience,
 use 4 trapezoids



$$\int_0^2 x^2 dx \approx A1 + A2 + A3 + A4$$

$$A \approx \frac{1}{2}[f(0)+f(\frac{1}{2})](\Delta x) + \frac{1}{2}[f(\frac{1}{2})+f(1)](\Delta x) + \frac{1}{2}[f(1)+f(1\frac{1}{2})](\Delta x) + \frac{1}{2}[f(1\frac{1}{2})+f(2)](\Delta x)$$

where $\Delta x = (b-a)/n$, and $n = \#$ trapezoids

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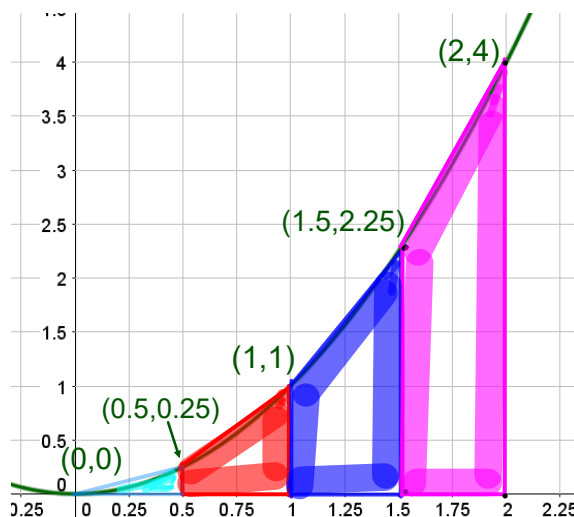
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7.4 Integration using Partial Fractions

$$\int_0^2 x^2 dx$$

Area of trapezoid:
 $\frac{1}{2}(b_1+b_2)h$



$$\int_0^2 x^2 dx \approx A1 + A2 + A3 + A4$$

$$A \approx \frac{1}{2}(0+\frac{1}{4})\frac{1}{2} + \frac{1}{2}(\frac{1}{4}+1)\frac{1}{2} + \frac{1}{2}(1+2\frac{1}{4})\frac{1}{2} + \frac{1}{2}(2\frac{1}{4}+4)\frac{1}{2}$$

factor

$$A \approx \frac{1}{4}(0+\frac{1}{4}+\frac{1}{4}+1+1+2\frac{1}{4}+2\frac{1}{4}+4) = 11/4 = 2.75$$

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7.4 Integration using Partial Fractions

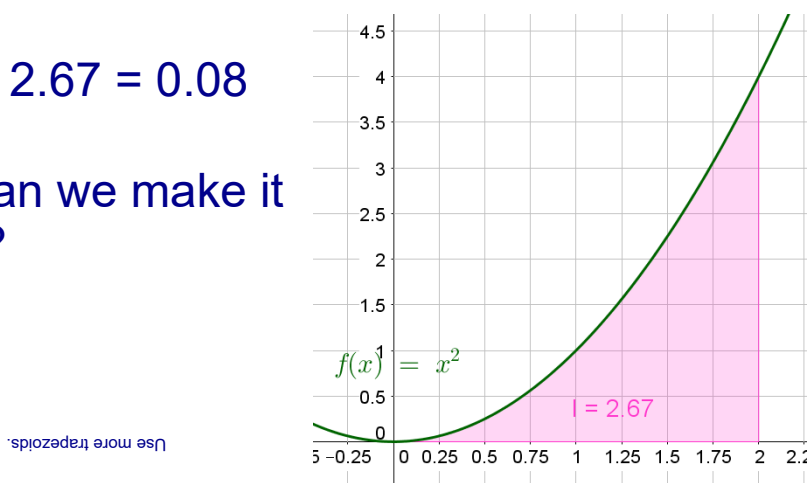
$$\int_0^2 x^2 dx = \left[\frac{x^3}{3} \right]_0^2 = 8/3$$

How good an estimate did we get?

$$A \approx \frac{1}{4}(0 + \frac{1}{4} + \frac{1}{4} + 1 + 1 + 2\frac{1}{4} + 2\frac{1}{4} + 4) = 11/4 = 2.75$$

$$2.75 - 2.67 = 0.08$$

How can we make it better?



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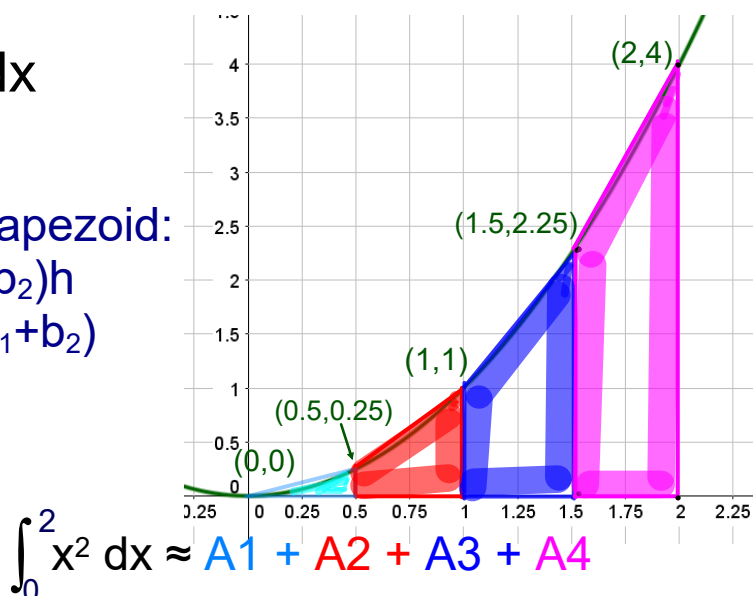
7.4 Integration using Partial Fractions

$$\int_0^2 x^2 dx$$

Area of trapezoid:

$$\frac{1}{2}(b_1 + b_2)h$$

$$= \frac{1}{2}h(b_1 + b_2)$$



$$A \approx \frac{1}{2}\Delta x [f(0) + f(\frac{1}{2}) + f(\frac{1}{2}) + f(1) + f(1) + f(1\frac{1}{2}) + f(1\frac{1}{2}) + f(2)]$$

$$A \approx \frac{1}{2}\Delta x [f(0) + 2f(\frac{1}{2}) + 2f(1) + 2f(1\frac{1}{2}) + f(2)]$$

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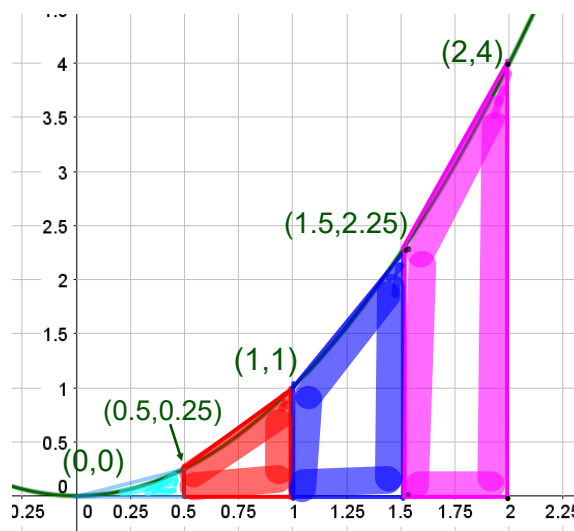
7.4 Integration using Partial Fractions

$$\int_0^2 x^2 dx$$

Area of trapezoid:

$$\frac{1}{2}(b_1+b_2)h$$

$$= \frac{1}{2}h(b_1+b_2)$$



$$\int_a^b x^2 dx \approx A_0 + A_1 + A_2 + A_3 + \dots + A_{n-1} + A_n$$

$$A \approx \frac{1}{2}\Delta x [f(x_0) + 2f(x_1) + 2f(x_2) + \dots + 2f(x_{n-1}) + f(x_n)]$$

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7.4 Integration using Partial Fractions

Trapezoidal Rule

$$\int_a^b f(x) dx \approx \frac{1}{2}\Delta x [f(x_0) + 2f(x_1) + 2f(x_2) + \dots + 2f(x_{n-1}) + f(x_n)]$$

$$\text{where } \Delta x = (b-a)/n \quad \text{and } x_i = a + i\Delta x$$

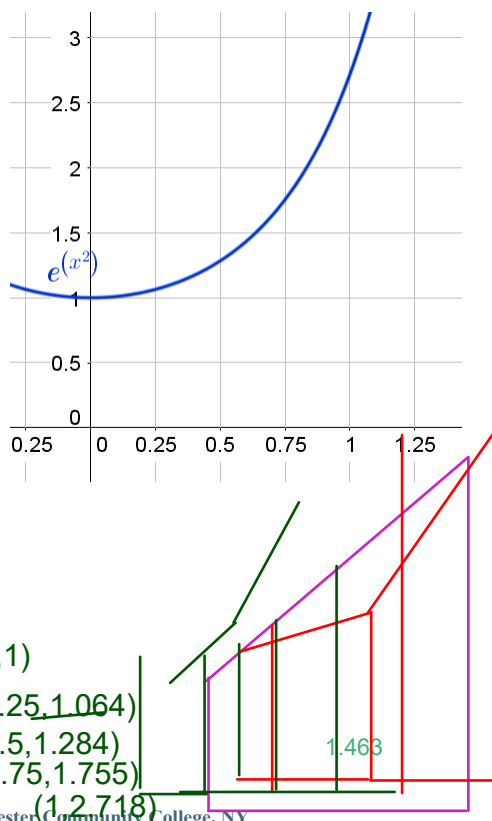
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7.4 Integration using Partial Fractions

$$\int_0^1 e^{x^2} dx = ??$$



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7.4 Integration using Partial Fractions

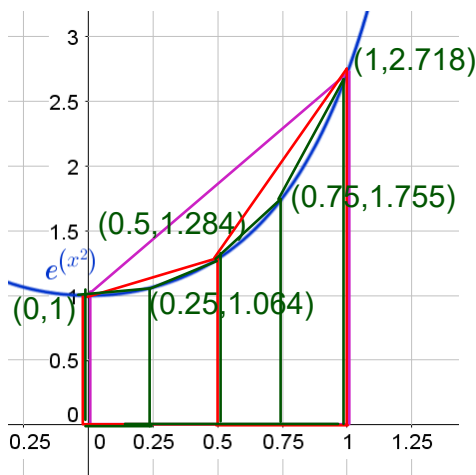
$$\int_0^1 e^{x^2} dx = ??$$

$$n=1 \\ \approx (1/2)(1+2.718) = 1.859$$

$$n=2 \\ \approx (1/4)(1+2(1.284)+2.718) = 1.572$$

$$n=4 \\ \approx (1/8)(1+2(1.064)+2(1.284)+2(1.755)+2.718) = 1.4905$$

$$\approx 1.491 - 1.463 = 0.023$$



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