7.5 Strategy for Integration: Use as Review

Goals: Integration overview

1. Look for simplest approach:
   > Direct integration
   > U-Substitution
   > Substitute Trig Identity
   > Long division, if appropriate

   Manipulate using algebra or trig
2. Then try more complex approach:
   > If a product, Integration by Parts
   > If sum or diff of squares, Trig Substition
   > If rational expression, Partial Fractions
3. If still no solution:
   > Use Tables of Integrals (appendix)
   > Use Computer Algebra System (CAS)
   eg: geogebra (free app)
   > Approximation (next section)

geogebra webstart

geogebra download

Study 7.5 #1-5,8,9-19,23-31

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7.5 Strategy for Integration: Use as Review

Review Table of Integrals in text (p. 503)

Formulas 1 - 10 are previous, standard.

Can learn others, but all are derivable with techniques learned.

11 can be useful since requires a manipulation not always easy to think of

17, 18 helpful

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7.5 Strategy for Integration: Use as Review

What approach would you take to integrate?

$$\int \frac{\sin^3 x}{\cos x}\, dx$$

$$\int_0^1 (3x+1)^{\sqrt{2}}\, dx$$

$$\int_0^1 \frac{x}{(2x+1)^3}\, dx$$

$$\int t \sin t \cos t \, dt$$

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7.5 Strategy for Integration: Use as Review

What approach would you take to integrate?

$$\int \frac{\sin^3 x}{\cos x}\, dx$$

Trig Identity

$$\int_0^1 (3x+1)^{\sqrt{2}}\, dx$$

U-substitution

$$\int_0^1 \frac{x}{(2x+1)^3}\, dx$$

Partial Fractions

$$\int t \sin t \cos t \, dt$$

Parts

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7.5 Strategy for Integration: Use as Review

What approach would you take to integrate?

$$\int \frac{2x-3}{x^3+3x}\,dx$$

$$\int_0^1 e^2\,dx$$

$$\int_0^{\sqrt{2}/2} \frac{x^2}{\sqrt{1-x^2}}\,dx$$

$$\int ln(1+x^2)\,dx$$

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7.5 Strategy for Integration: Use as Review

What approach would you take to integrate?

$$\int \frac{2x-3}{x^3+3x}\,dx$$

Partial Fractions

$$\int_0^1 e^2\,dx$$

Direct integral

$$\int_0^{\sqrt{2}/2} \frac{x^2}{\sqrt{1-x^2}}\,dx$$

Trig Substitution

$$\int ln(1+x^2)\,dx$$

Parts

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7.5 Strategy for Integration: Use as Review

What approach would you take to integrate?

$$\int_0^1 \frac{1+12x}{1+3x}\,dx$$

$$\int_0^4 \frac{e^{\sqrt{t}}}{\sqrt{t}}\,dt$$

$$\int_0^1 \frac{3x^2+1}{x^3+x^2+x+1}\,dx$$

$$\int \frac{\ln x}{x\sqrt{1+(\ln x)^2}}\,dx$$

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7.5 Strategy for Integration: Use as Review

What approach would you take to integrate?

$$\int_0^1 \frac{1+12x}{1+3x}\,dx$$

Long Division

$$\int_0^4 \frac{e^{\sqrt{t}}}{\sqrt{t}}\,dt$$

U-substitution

$$\int_0^1 \frac{3x^2+1}{x^3+x^2+x+1}\,dx$$

Partial Fractions

$$\int \frac{\ln x}{x\sqrt{1+(\ln x)^2}}\,dx$$

U-substitution

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7.5 Strategy for Integration: Use as Review

What approach would you take to integrate?

Trig Identity

$$\int \frac{\sin^3 x}{\cos x}\,dx$$

$$\int \frac{\sin^2 x \sin x}{\cos x}\,dx = \int \frac{(1-\cos^2 x)\sin x}{\cos x}\,dx$$

$$\int \frac{\sin x - \cos^2 x \sin x}{\cos x}\,dx = \int \left[\frac{\sin x}{\cos x}\,dx - \frac{\cos^2 x \sin x}{\cos x}\right] dx$$

$$-\ln|\cos x| - \int \cos x \sin x \,dx$$

$$-\ln|\cos x| + \tfrac{1}{2}\cos^2 x + c$$

or: $\ln|\sec x| - \tfrac{1}{2}\sin^2 x + c$

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7.5 Strategy for Integration: Use as Review

What approach would you take to integrate?

$$\int \ln(1+x^2)\,dx$$

$u = \ln(1+x^2)$ $dv = dx$ Parts

$du = \dfrac{2x}{(1+x^2)}\,dx$ $v = x$

$$= x\ln(1+x^2) - \int x \frac{2x}{(1+x^2)}\,dx = x\ln(1+x^2) - \int \frac{2x^2}{(1+x^2)}\,dx$$

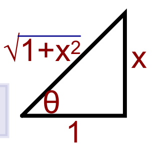
$$= x\ln(1+x^2) - \int \left[2 - \frac{2}{1+x^2}\right] dx$$

Long Division

$$\begin{array}{r} 2 \\ x^2+1 \overline{)\,2x^2} \\ \underline{2x^2+2} \\ -2 \end{array}$$

$$= x\ln(1+x^2) - 2x + 2\int \frac{dx}{1+x^2}$$

Trig Subst



$\tan\theta = x$

$\sec^2\theta\, d\theta = dx$

$\sec\theta = \sqrt{1+x^2}$

$$= x\ln(1+x^2) - 2x + 2\int \frac{\sec^2\theta\, d\theta}{\sec^2\theta}$$

$$= x\ln(1+x^2) - 2x + 2\int d\theta = x\ln(1+x^2) - 2x + 2\theta + c$$

$$= x\ln(1+x^2) - 2x + 2\arctan x + c$$

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7.5 Strategy for Integration: Use as Review

Partial Fractions

What approach would you take to integrate?

$$\int_0^1 \frac{3x^2+1}{x^3+x^2+x+1}\,dx$$

-1) 1 1 1 1
0 -1 0 -1
1 0 1 0

factors to $(x+1)(x^2+1)$

$$\frac{3x^2+1}{(x+1)(x^2+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+1}$$

$$\frac{3x^2+1}{(x+1)(x^2+1)} = \frac{A(x^2+1) + (Bx+C)(x+1)}{(x+1)(x^2+1)}$$

$$3x^2+1 = Ax^2+A+Bx^2+Bx+Cx+C = (A+B)x^2 +(B+C)x +(A+C)$$

$A + B = 3$
$B + C = 0$
$A + C = 1$
$A - B = 1$

A=2, B=1, C=-1

$$\int_0^1 \frac{3x^2+1}{(x+1)(x^2+1)}\,dx = 2\int_0^1 \frac{dx}{x+1} + \int_0^1 \frac{x-1}{x^2+1}\,dx$$

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7.5 Strategy for Integration: Use as Review

Partial Fractions

What approach would you take to integrate?

$$\int_0^1 \frac{3x^2+1}{x^3+x^2+x+1}\,dx$$

-1) 1 1 1 1
0 -1 0 -1
1 0 1 0

factors to $(x+1)(x^2+1)$

$$\int_0^1 \frac{3x^2+1}{(x+1)(x^2+1)}\,dx = 2\int_0^1 \frac{dx}{x+1} + \int_0^1 \frac{x-1}{x^2+1}\,dx$$

$$= 2\ln|x+1|\Big]_0^1 + \int_0^1 \frac{x}{x^2+1}\,dx - \int_0^1 \frac{dx}{x^2+1}$$

$$= 2\ln|x+1| + \tfrac{1}{2}\ln|x^2+1| - \arctan x\Big]_0^1$$

$$= 2\ln 2 + \tfrac{1}{2}\ln 2 - \arctan 1 - [\,2\ln 1 + \tfrac{1}{2}\ln 1 - \arctan 0\,]$$

$$= (5/2)\ln 2 - \pi/4$$

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