## 7.1 Integration by Parts

Goals:

1. Understand that Integration by Parts (IP) is a method that reverses the product rule.
2. IP separates factors of the integrand into parts.
3. The IP Rule is: $\int u\,dv = u\,v - \int v\,du$
4. Select the u and dv that result in a solution (may require repeated IP).
   - i) If a part of the integrand is not readily integrable, let that part = u.
   - ii) Make selections that will reduce the complexity of the next integrand.
5. Find du and v, and complete the rule.

Study 7.1 8 to 9 problems of # 1-7, 11-23, 27-37



## 7.1 Integration by Parts

G: $y = e^x(x - 1)$ F: $dy/dx$

$dy/dx = e^x(1) + (x - 1)e^x$

$dy/dx = e^x + x\,e^x - e^x = x\,e^x$

## 7.1 Integration by Parts

What about.... $\int e^x x\,dx$ $\int e^u\,du$ ? No.

Used Product Rule to get derivative.

**Need a procedure of integration that reverses the product rule.**

**Product Rule:**

If u and v are both differentiable functions of x,
u = f(x) and v = g(x), then:

$$\frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

Therefore, integrating with respect to x:

$$\int \frac{d(uv)}{dx}dx = \int u\frac{dv}{dx}dx + \int v\frac{du}{dx}dx$$

$$uv = \int u\,dv + \int v\,du$$

definition of differential

$dy = \frac{dy}{dx}dx$

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## 7.1 Integration by Parts

$$uv = \int u\,dv + \int v\,du$$

Rewrite as an integral:

| | |
|---|---|
| $\int u\,dv = uv - \int v\,du$ | Integration by Parts |

So, we need to separate the integrand into parts: **u and dv**

Try again: $\int e^x x\,dx$

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## 7.1 Integration by Parts

$\int u\,dv = uv - \int v\,du$

$$\int e^x x\,dx$$

u = ______ dv = ______

du = ______ v = ______

Which is the best way to assign *u* and *dv*?

| | |
|---|---|
| u = ______ dv = ______<br>du = ______ v = ______ | u = ______ dv = ______<br>du = ______ v = ______ |

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## 7.1 Integration by Parts

$\int u\,dv = uv - \int v\,du$

$$\int e^x x\,dx$$

u = ______ dv = ______

du = ______ v = ______

Which is the best way to assign *u* and *dv*?

| | |
|---|---|
| u = $e^x$ dv = $x\,dx$<br>du = $e^x\,dx$ v = $\frac{x^2}{2}$<br>$\int u\,dv = uv - \int v\,du$<br>$\int e^x x\,dx = e^x \frac{x^2}{2} - \boxed{\int \frac{x^2}{2} e^x\,dx}$<br>*more complicated*<br>*--> **not** useful* | u = $x$ dv = $e^x\,dx$<br>du = $dx$ v = $e^x$<br>$\int u\,dv = uv - \int v\,du$<br>$\int e^x x\,dx = x\,e^x - \boxed{\int e^x\,dx}$<br>*less complicated*<br>*and **easy***<br>$\int e^x x\,dx = x\,e^x - e^x + c$<br>$= e^x(x - 1) + c$ |

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## 7.1 Integration by Parts

$\int u\,dv = uv - \int v\,du$

$$\int \frac{2x}{e^x}dx$$

u = ______ dv = ______

du = ______ v = ______

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## 7.1 Integration by Parts

$\int u\,dv = uv - \int v\,du$

$$\int \frac{2x}{e^x}dx = \int 2xe^{-x}dx$$

u = $x$ dv = $e^{-x}dx$

du = $dx$ v = $-e^{-x}$

$$= 2\int x\,e^{-x}dx$$

$$= 2\left[x\cdot -e^{-x} - \int -e^{-x}dx\right]$$

$$= 2\left[-xe^{-x} - e^{-x}\right] + c$$

$$= -2xe^{-x} - 2e^{-x} + c$$

$$-2e^{-x}(x+1) + c$$

*check by finding the derivative:*

$$-2e^{-x}(1) + (x+1)(+2e^{-x})$$

$$-2e^{-x} + 2e^{-x}x + 2e^{-x}$$

$$2e^{-x}x = \frac{2x}{e^x}$$

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## 7.1 Integration by Parts

$$\int u\,dv = uv - \int v\,du$$

$$\int x^4 \ln x\,dx$$

u = ________ dv = ________

du = ________ v = ________

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## 7.1 Integration by Parts

$$\int u\,dv = uv - \int v\,du$$

$$\int x^4 \ln x\,dx$$

u = $\ln x$ dv = $\int x^4\,dx$

du = $\frac{1}{x}\,dx$ v = $\frac{x^5}{5}$

$$= \frac{x^5}{5}\ln x - \int \frac{x^5}{5}\cdot\frac{1}{x}\,dx$$

$$= \frac{1}{5}x^5 \ln x - \frac{1}{5}\int x^4\,dx$$

$$= \frac{1}{5}x^5 \ln x - \frac{1}{5}\cdot\frac{x^5}{5} + C = \frac{1}{5}x^5 \ln x - \frac{1}{25}x^5 + C$$

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## 7.1 Integration by Parts

$$\int \frac{e^{1/t}}{t^2}\,dt$$

$e^u du$?



## 7.1 Integration by Parts

$$-\int \frac{-e^{1/t}}{t^2}\,dt \qquad e^u du?$$

$$u = \frac{1}{t} = t^{-1}$$

$$du = -t^{-2}dt = -\frac{1}{t^2}\,dt$$

$$-\int e^u du = -e^u + C$$

$$= -e^{1/t} + C$$

*Integration by Parts is not needed.*



 

## 7.1 Integration by Parts

$\int u\,dv = uv - \int v\,du$

$$\int \frac{x^3 e^{x^2}}{(x^2+1)^2}\,dx$$

u = ________ dv = ________

du = ________ v = ________

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## 7.1 Integration by Parts

$\int u\,dv = uv - \int v\,du$

$$\int \frac{x^3 e^{x^2}}{(x^2+1)^2}\,dx$$

u = $x^2 e^{x^2}$ dv = $\frac{1}{2}\int \frac{2x}{(x^2+1)^2}dx$

du = $[x^2 e^{x^2}(2x) + e^{x^2}(2x)]dx$ v = $-\frac{1}{2}(x^2+1)^{-1}$

$$= -\frac{x^2 e^{x^2}}{2(x^2+1)} - \frac{1}{2}\int \frac{-2xe^{x^2}(x^2+1)}{(x^2+1)}dx$$

$$= -\frac{x^2 e^{x^2}}{2(x^2+1)} + \frac{1}{2}\int 2xe^{x^2}\,dx \qquad u = x^2 \quad du = 2x\,dx$$

$$= -\frac{x^2 e^{x^2}}{2(x^2+1)} + \frac{1}{2}\int e^u\,du = -\frac{x^2 e^{x^2}}{2(x^2+1)} - \frac{e^{x^2}}{2} + C$$

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## 7.1 Integration by Parts

$$\int u\,dv = uv - \int v\,du$$

$$\int \frac{xe^{2x}}{(2x+1)^2}\,dx$$

u = ________ dv = ________

du = ________ v = ________

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## 7.1 Integration by Parts

$$\int u\,dv = uv - \int v\,du$$

$$\int \frac{xe^{2x}}{(2x+1)^2}\,dx$$

u = $xe^{2x}$ $\int$dv = $\frac{1}{2}\int \frac{1 \cdot 2}{(2x+1)^2}\,dx$

du = $(2x\,e^{2x} + e^{2x})\,dx$ v = $\frac{-1}{2}(2x+1)^{-1}$

$= e^{2x}(2x+1)\,dx$

$$= \frac{-1\,xe^{2x}}{2(2x+1)} - \int -\frac{e^{2x}(2x+1)\,dx}{2(2x+1)}$$

$$= \frac{-1\,xe^{2x}}{2(2x+1)} + \frac{1}{2}\cdot\frac{1}{2}\int e^{2x}\,2\,dx$$

$$= \frac{-1\,xe^{2x}}{2(2x+1)} + \frac{1}{4}e^{2x} + C$$

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## 7.1 Integration by Parts

$$\int u\,dv = uv - \int v\,du$$

$$\int e^{2x}\sin x\,dx$$

u = ________ dv = ________

du = ________ v = ________

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## 7.1 Integration by Parts

$$\int u\,dv = uv - \int v\,du$$

$$\int e^{2x}\sin x\,dx$$

u = $\sin x$  $\int$dv = $\int e^{2x}dx$

du = $\cos x\,dx$  v = $\frac{1}{2}e^{2x}$

$$= \frac{1}{2}e^{2x}\sin x - \frac{1}{2}\int e^{2x}\cos x\,dx$$

u = $\cos x$  dv = $e^{2x}dx$

du = $-\sin x\,dx$  v = $\frac{1}{2}e^{2x}$

$$= \frac{1}{2}e^{2x}\sin x - \frac{1}{2}\left[\frac{1}{2}e^{2x}\cos x - \int -\frac{1}{2}e^{2x}\sin x\,dx\right]$$

$$= \frac{1}{2}e^{2x}\sin x - \frac{1}{4}e^{2x}\cos x - \frac{1}{4}\int e^{2x}\sin x\,dx$$

$$\int e^{2x}\sin x\,dx = \frac{1}{2}e^{2x}\sin x - \frac{1}{4}e^{2x}\cos x - \frac{1}{4}\int e^{2x}\sin x\,dx$$

$$\frac{5}{4}\int e^{2x}\sin x\,dx = \frac{1}{2}e^{2x}\sin x - \frac{1}{4}e^{2x}\cos x$$

$$\int e^{2x}\sin x\,dx = \frac{4}{5}\left[\frac{1}{2}e^{2x}\sin x - \frac{1}{4}e^{2x}\cos x\right] + C$$

$$\int e^{2x}\sin x\,dx = \frac{2}{5}e^{2x}\sin x - \frac{1}{5}e^{2x}\cos x + C$$

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## 7.1 Integration by Parts

Practice:

$$\int (x-1) \sin \pi x \, dx$$

$$\int_0^1 (x^2+1) e^{-x} \, dx$$

$$\int \ln \sqrt{x} \, dx$$

$$\int x \tan^2 x \, dx$$

$$\int e^{-\theta} \sin 2\theta \, dx$$

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