6.3 Volume: Shell Method

Goals:
1. Understand volume as the sum of the areas of an infinite number of surfaces, called shells.
2. Be able to identify:
   - the bounded region
   - the reference rectangle
   - the surface that results from
     - revolution of the rectangle around an axis
     - when the rectangle is parallel to the axis
   - the area of that surface.
3. Set up the definite integral for finding the resulting volume, using area as the integrand.

Homework: Study 6.3 # 1, 3, 5, 7, 9, 13

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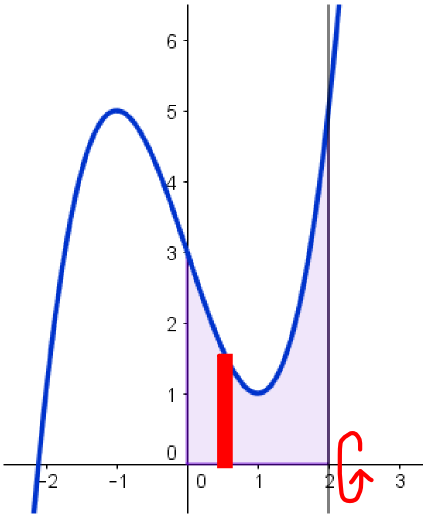
6.3 Volume: Shell Method

Consider: $y = x^3 - 3x + 3$, $x = 0$, $y = 0$, $x = 2$



Easy to revolve about x-axis. Use disk method.

What about revolving about y-axis? How to proceed?

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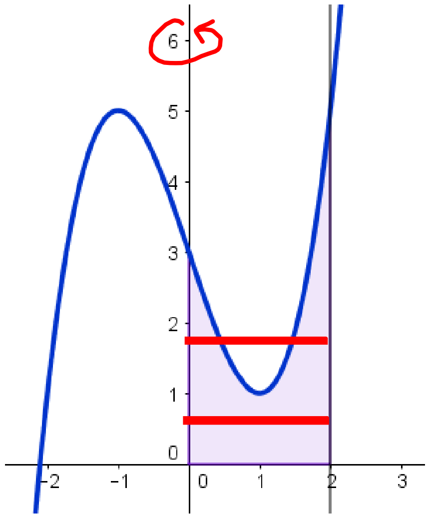
6.3 Volume: Shell Method

Consider: $y = x^3 - 3x + 3$, $x = 0$, $y = 0$, $x = 2$



What about revolving about y-axis? How to proceed?

Rectangle of disk method is not consistent.
Need to divide into 3 parts.

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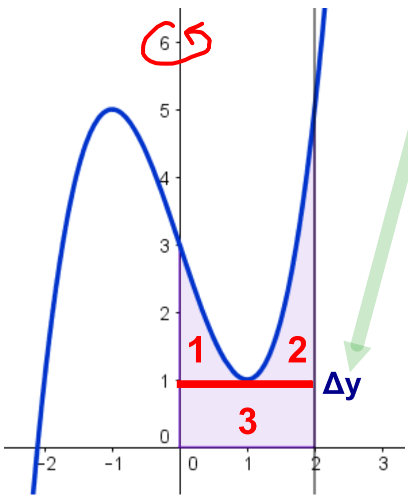
6.3 Volume: Shell Method

Consider: $y = x^3 - 3x + 3$, $x = 0$, $y = 0$, $x = 2$



Need to use $\pi \int_c^d R^2\, dy$

or $\pi \int_c^d (R^2 - r^2)\, dy$

This is integration with respect to y.
So, need to find x as a function of y ------ but, not so easy!

$y = x^3 - 3x + 3$
$x = ?$

Can divide into 3 parts?

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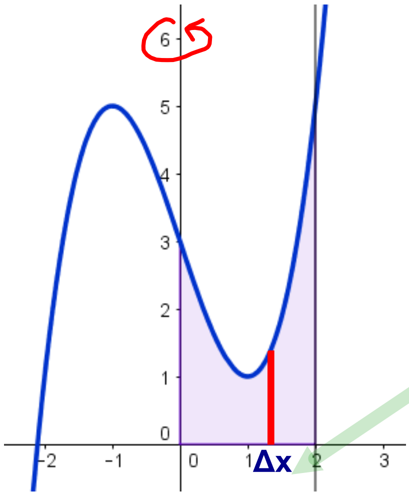
6.3 Volume: Shell Method

Consider: $y = x^3 - 3x + 3$, $x = 0$, $y = 0$, $x = 2$



Alternate Method:
Shell Method
Uses **Rectangle** that is **Parallel** to the **axis of revolution**

This uses Δx and results in integration with respect to x

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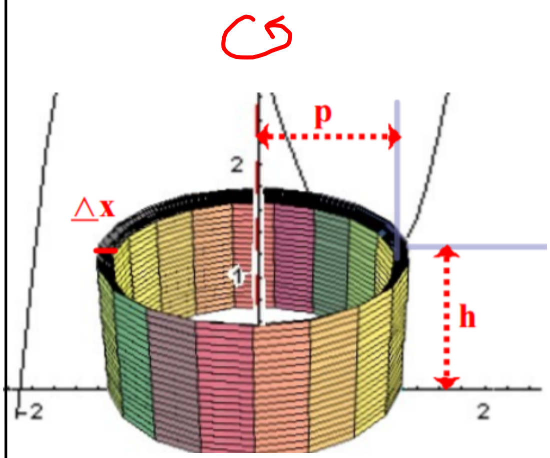
6.3 Volume: Shell Method

Consider: $y = x^3 - 3x + 3$, $x = 0$, $y = 0$, $x = 2$



video of shell

Alternate Method: **Shell Method**
Uses **Rectangle** that is **Parallel** to the **axis of revolution**

p = average radius of shell

h = height

dx or dy = thickness

$$V = 2\pi \int_a^b p(x)\, h(x)\, dx$$

or

$$V = 2\pi \int_c^d p(y)\, h(y)\, dy$$

Δx

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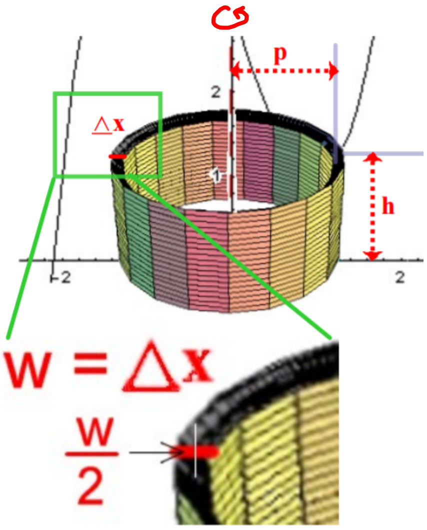
6.3 Volume: Shell Method

Consider: $y = x^3 - 3x + 3$, $x = 0$, $y = 0$, $x = 2$



p = average radius of shell
h = height
dx or dy = thickness

Volume of shell =
volume of outer cylinder -
Volume of inner cylinder

$$V = \pi\left(p+\frac{w}{2}\right)^2 h - \pi\left(p-\frac{w}{2}\right)^2 h$$

$$= \pi h\left[\left(p+\frac{w}{2}\right)^2 - \left(p-\frac{w}{2}\right)^2\right]$$

$$= \pi h\left[\left(p^2+wp+\frac{w^2}{4}\right)-\left(p^2-wp+\frac{w^2}{4}\right)\right]$$

$$= \pi h(2wp) = 2\pi p h w$$

$$V = 2\pi\int_a^b p(x)h(x)\,dx$$

w (Δx) is the width of the reference shell. Add the volumes of adjacent shells, and let Δx ->0. Results in representation of the thickness of the shell as dx or dy.

$V = 2\pi \int_c^d p(y)\, h(y)\, dy$

Δx $V = 2\pi \int_a^b p(x)\, h(x)\, dx$

6.3 Volume: Shell Method

Volumes of Revolution - Which Method?

1. Sketch the curves and identify the region, using the points of intersection.

2. Locate the axis of revolution on the sketch.

3. Decide whether to use a horizontal or vertical rectangle. Select the orientation that requires the least number of separate sections.

4. Decide whether to use the Disc Method or the Shell Method:

a) If the rectange is **perpendicular** to the axis of revolution, use the **Disc Method**.

b) If the rectangle is **parallel** to the axis of revolution, use the **Shell Method**.

Δx $V = 2\pi \int_a^b p(x)\, h(x)\, dx$ $V = 2\pi \int_c^d p(y)\, h(y)\, dy$

6.3 Volume: Shell Method

Volumes of Revolution - Shell Method

1. Complete Steps 1 to 4 in *Volumes of Revolution, which Method?* noted above.

2. Be sure that your rectangle is parallel to the axis of revolution.

3. Determine the variable of integration:

a) If the rectangle is horizontal, then integrate with respect to y (use *dy*). The integrand must be in terms of y. Δy

b) If the rectangle is vertical, then integrate with respect to x (use *dx*). The integrand must be in terms of x. Δx

4. Determine the integrand: p(x)h(x) or p(y)h(y) ?

a) **If the rectangle is horizontal**, identify **p(y)**, the distance of the rectangle from the axis of revolution, & **h(y)**, the length of the rectangle. Use: $V = 2\pi \int_c^d p(y)\, h(y)\, dy$

b) **If the rectangle is vertical**, identify **p(x)**, the distance of the rectangle from the axis of revolution, and **h(x)**, the length of the rectangle. Use: $V = 2\pi \int_a^b p(x)\, h(x)\, dx$

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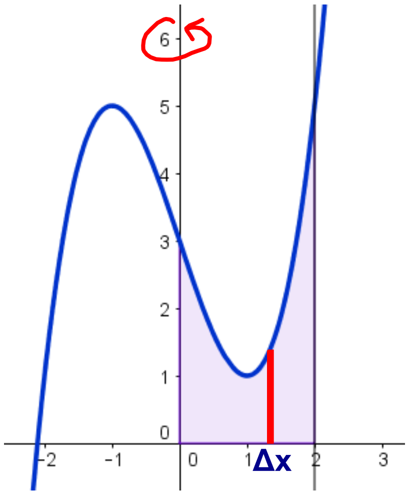
Δx

6.3 Volume: Shell Method

Consider: $y = x^3 - 3x + 3$, $x = 0$, $y = 0$, $x = 2$



Which Method?

Shell Method

bec Disk Method requires:

1. 3 separate regions
2. x = f(y)

Which Integral?

$$V = 2\pi \int_a^b p(x)\, h(x)\, dx$$

$$V = 2\pi \int_c^d p(y)\, h(y)\, dy$$

* Use the formula with dx !

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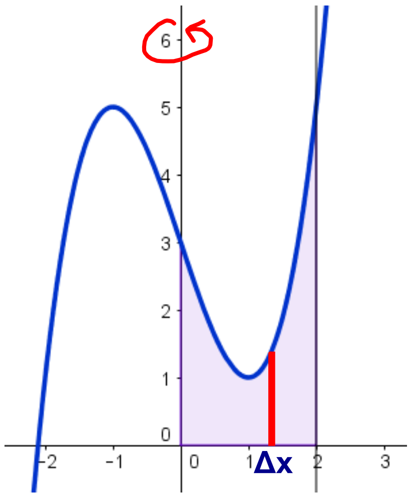
6.3 Volume: Shell Method

Consider: $y = x^3 - 3x + 3$, $x = 0$, $y = 0$, $x = 2$



F: Volume

$V = 2\pi \int_a^b p(x)\, h(x)\, dx$

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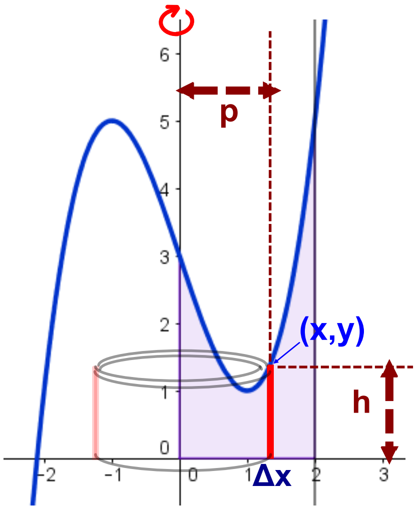
6.3 Volume: Shell Method

Consider: $y = x^3 - 3x + 3$, $x = 0$, $y = 0$, $x = 2$



$p = x$

$h = y = x^3 - 3x + 3$

* Use the formula with dx !

$V = 2\pi \int_a^b p(x)\, h(x)\, dx$

$V = 2\pi \int_0^2 [x(x^3 - 3x + 3)]\, dx$

$V = 2\pi \int_0^2 (x^4 - 3x^2 + 3x)\, dx$

$V = 2\pi \left[\frac{x^5}{5} - \frac{3x^3}{3} + \frac{3x^2}{2}\right]_0^2$

$V = 2\pi \left[\frac{32}{5} - 8 + 6 - 0\right]$

$V = 2\pi \left[\frac{22}{5}\right] = \frac{44\pi}{5}$

video, rotate ref. x=2

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6.3 Volume: Shell Method

G: $y_1 = x^2$, $y_2 = 4x - x^2$ F: V ↷ $x = 2$

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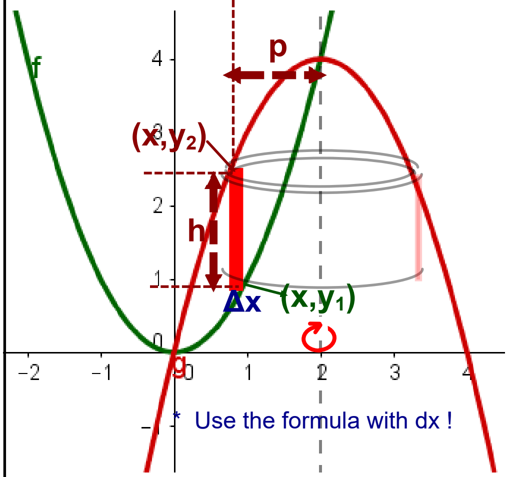
6.3 Volume: Shell Method

G: $y_1 = x^2$, $y_2 = 4x - x^2$ F: V ↷ $x = 2$



$$V = 2\pi \int_a^b p(x)\, h(x)\, dx$$

$$V = 2\pi \int_0^2 [x(4x-2x^2)]\, dx$$

$$V = 2\pi \int_0^2 (4x^2-2x^3)\, dx$$

$$V = 2\pi \left[\frac{4x^3}{3} - \frac{2x^4}{4} \right]_0^2$$

$$V = 2\pi \left[\frac{32}{3} - 8 - 0\right]$$

$$V = 2\pi \left[\frac{8}{3}\right] = \frac{16\pi}{3}$$

$p = x$

$h = y_2 - y_1 = (4x - x^2) - x^2$

$h = 4x - 2x^2$

video, rotate ref. x=2

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