6.2 Volume: Disk and Cross Section Methods

Goals:

1. Understand volume as the sum of the areas of an infinite number of surfaces.
2. Be able to identify:
   the bounded region
   the reference rectangle
   the surface that results from
   revolution of the rectangle around an axis
   or forms a cross section upon a base
   the area of that surface.
3. Set up the definite integral for finding the resulting volume, using area as the integrand.

Homework: Study 6.2 #1, 3, 5, 7, 11, 15, 55, 57

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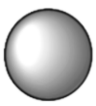
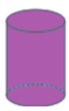
6.2 Volume: Disk and Cross Section Methods

From geometry, we find volumes of readily defined geometric figures. For example:

	Geometric Figure	Volume
	Sphere	$V = 4/3\ \pi r^3$
	Right Circular Cone	$V = 1/3\ \pi r^2 h$
	Right Circular Cylinder	$V = \pi r^2 h$

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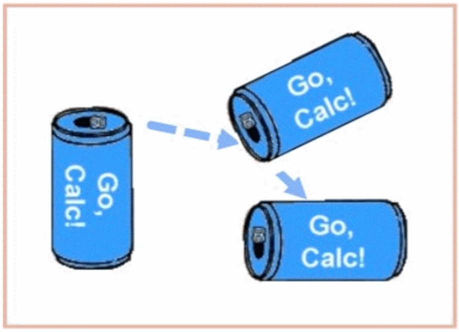
6.2 Volume: Disk and Cross Section Methods

Our focus: Volume of cylinder

Right Circular Cylinder $V = \pi r^2 h$



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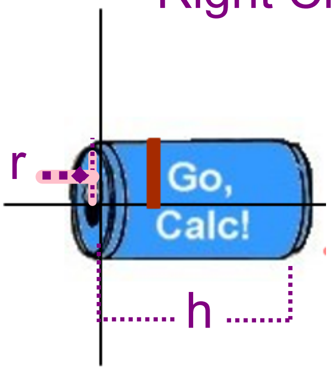
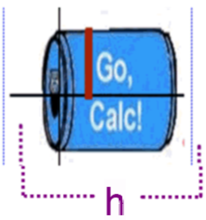
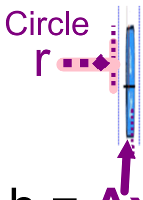
6.2 Volume: Disk and Cross Section Methods

Right Circular Cylinder $V = \pi r^2 h$



when h is small, have reference rectangle





let $h = \Delta x$

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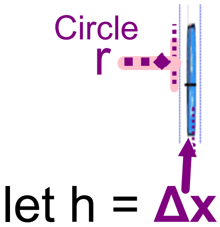
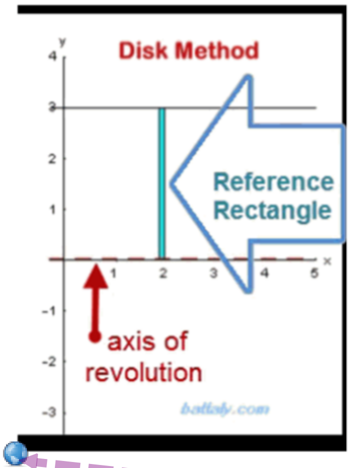
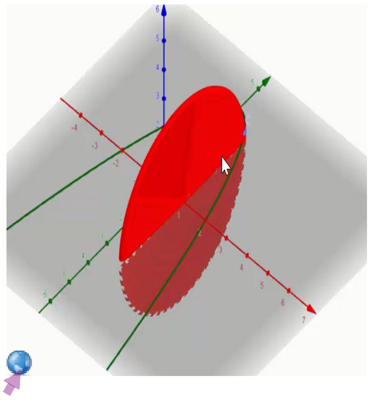
6.2 Volume: Disk and Cross Section Methods

Revolve reference rectangle around the axis of revolution, perpendicular to length of rectangle.







Click globe links

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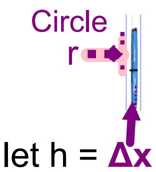
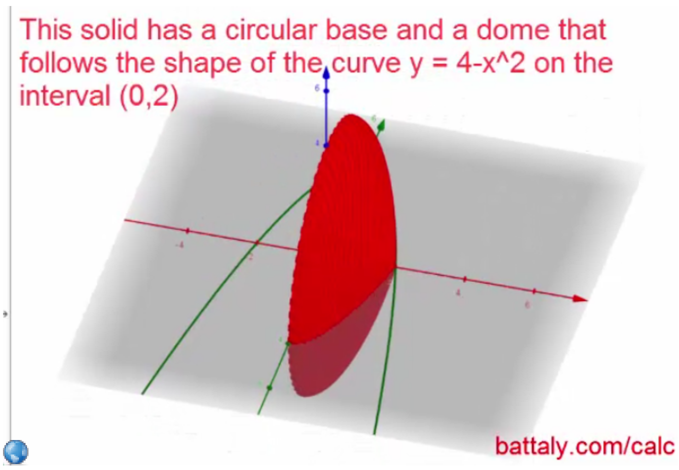
6.2 Volume: Disk and Cross Section Methods

Revolve reference rectangle around the axis of revolution, perpendicular to length of rectangle.





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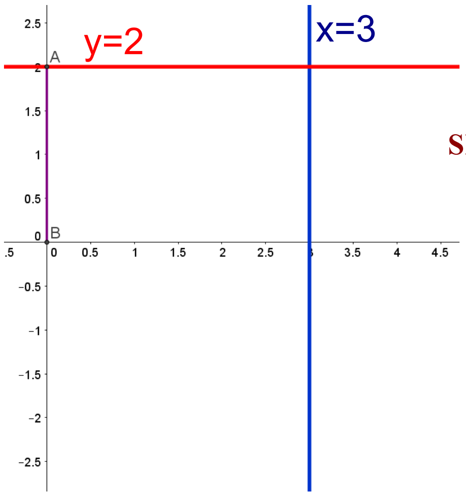
6.2 Volume: Disk and Cross Section Methods

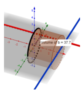
G: x=3, y=2, x-axis, y-axis F: V, ↻ x-axis



generates a cylinder, can use
$V = \pi r^2 h = A_{circle} h = \pi (2^2)(3) = 12\pi$

Now, use calculus:

Sketch the curves, id region, pts of intersection
Locate the axis of revolution (ARev)
Horizontal or vertical rectangle.? ⊥ ARev
Sketch rectangle; decide variable of integration
If vertical, use function of *x* and *dx*.
Determine the integrand: R^2



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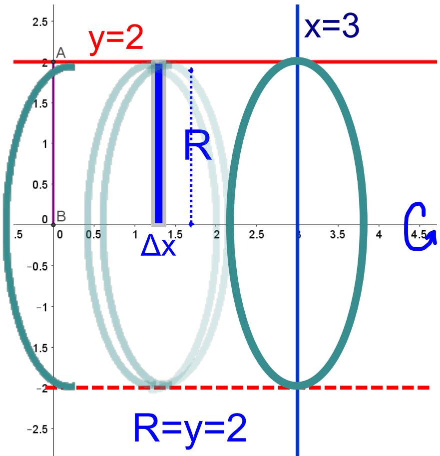
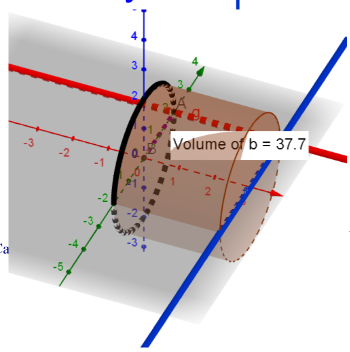
6.2 Volume: Disk and Cross Section Methods

G: x=3, y=2, x-axis, y-axis F: V, ↻ x-axis





using $V = \pi r^2 h = A_{circle} h$
$V = \pi (2^2)(3) = 12\pi$

using calculus:

$$V = \int_a^b A_{circle}\, dx$$
$$= \int_a^b \pi R^2\, dx$$
$$= \pi \int_0^3 2^2\, dx = 4\pi\, x \Big]_0^3$$
$$= 4\pi\, [3-0] = 12\pi$$

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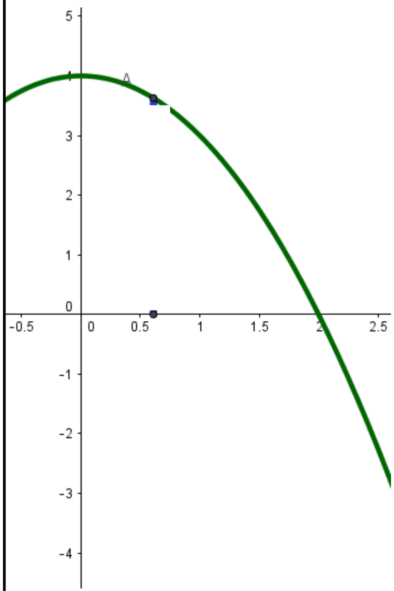
6.2 Volume: Disk and Cross Section Methods

For nonstandard geometrical figures, no formula, so need calculus.

G: $y= 4-x^2$, x-axis, y-axis, Q I F: V, ↻ x-axis



Sketch the curves, id region, pts of intersection
Locate the axis of revolution (ARev)
Horizontal or vertical rectangle.? ⊥ ARev
Sketch rectangle; decide variable of integration
If vertical, use function of *x* and *dx*.
Determine the integrand: R^2

geogebra

video

R

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6.2 Volume: Disk and Cross Section Methods

For nonstandard geometrical figures, no formula, so need calculus.

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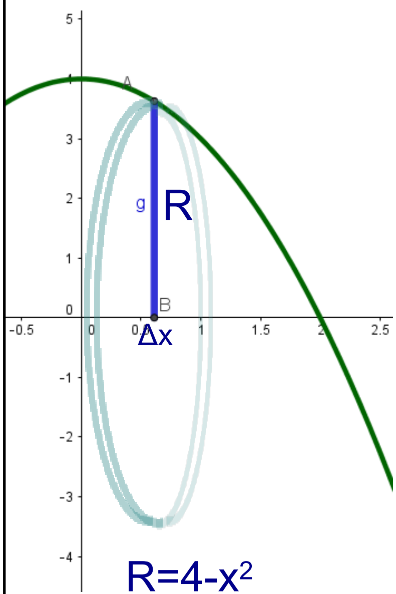

$$V = \int_a^b A_{circle}\, dx$$
$$= \int_a^b \pi R^2\, dx$$
$$= \pi\int_0^2 (4-x^2)^2 dx$$
$$= \pi\int_0^2 (16-8x^2+x^4) dx$$
$$= \pi[16x-(8/3)x^3+(1/5)x^5]\Big]_0^2$$
$$= \pi[32-64/3+32/5-(0)]$$
$$= \pi[32+32(-2/3+1/5)]$$
$$= 32\pi[1+(-2/3+1/5)]$$
$$= 256\pi/15$$

geogebra

video

R

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6.2 Volume: Disk and Cross Section Methods

Volumes of Revolution - Disk Method

1. **Sketch the curves and identify the region, using the points of intersection.**
2. **Locate the axis of revolution on the sketch.**
3. **Decide whether to use a horizontal or vertical rectangle.**
   The rectangle should be perpendicular to the axis of revolution.
4. **Sketch the rectangle and determine the variable of integration.**
   *If the rectangle is **horizontal**, then integrate with respect to y (use *dy*).
   The integrand must be in terms of y.
   *If the rectangle is **vertical**, then integrate with respect to x (use *dx*).
   The integrand must be in terms of x.
5. **Determine the integrand: R^2, or $R^2 - r^2$?**
   ***If the rectangle touches the axis of revolution**,
   identify R as the length of the rectangle. Find **R** in terms of the appropriate variable (see above), and **use R^2** in the integrand.

$$A = \pi \int_a^b R^2\, dx \qquad A = \pi \int_c^d R^2\, dy$$

   ***If the rectangle does not touch the axis of revolution**,
   identify **R** as the distance of the furthest end of the rectangle from the axis of revolution and **r** as the distance of the closest end of the rectangle from the axis of revolution. **Use $R^2 - r^2$** in the integrand.

$$A = \pi \int_a^b (R^2 - r^2)\, dx \qquad A = \pi \int_c^d (R^2 - r^2)\, dy \qquad R$$

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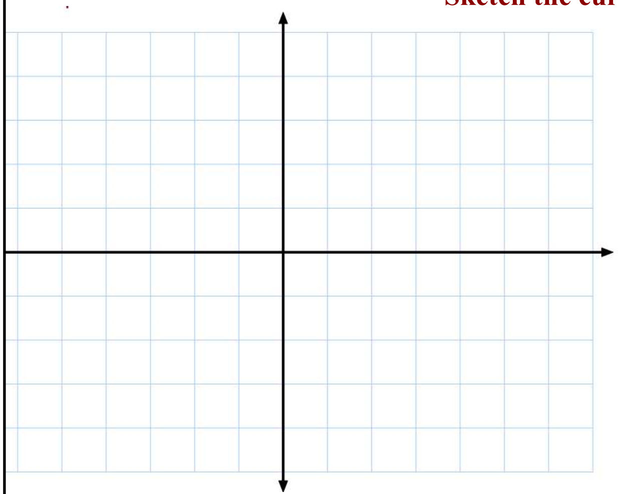
6.2 Volume: Disk and Cross Section Methods

G: $y = \sqrt{x}$, $y = 0$, $x = 4$ F: V, x-axis

Sketch the curves, id region, pts of intersection
te the axis of revolution (ARev)
or vertical rectangle.? ⊥ ARev
ıgle; decide variable of integration
vertical, use function of *x* and *dx*.
termine the integrand: R^2 or R^2-r^2



geogebra

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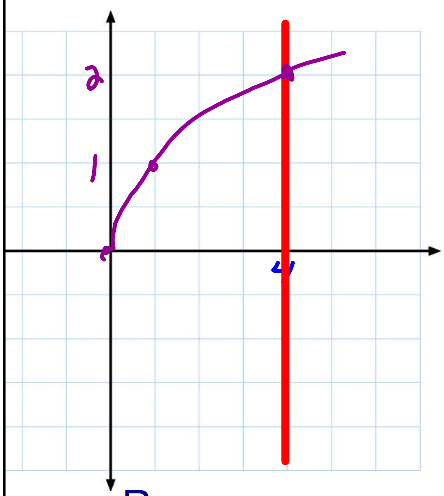
6.2 Volume: Disk and Cross Section Methods

G: $y=\sqrt{x}$, $y = 0$, $x = 4$ F: V, ↻ **y-axis**

Sketch the curves, id region, pts of intersection
Locate the axis of revolution (ARev)
Horizontal or vertical rectangle.? ⊥ ARev
Sketch rectangle; decide variable of integration
If horizontal, use function of y and *dy*.
Determine the integrand: R^2 or R^2-r^2



R = ______

r = ______

$A = \pi \int_c^d (R^2 - r^2)\, dy$

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6.2 Volume: Disk and Cross Section Methods

G: $y=\sqrt{x}$, $y = 0$, $x = 4$ F: V, ↻ **y-axis**

Sketch the curves, id region, pts of intersection
Locate the axis of revolution (ARev)
Horizontal or vertical rectangle.? ⊥ ARev
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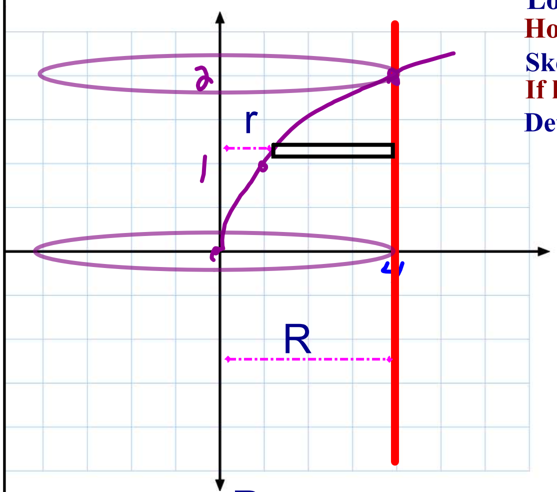

R = ______

r = ______

$A = \pi \int_c^d (R^2 - r^2)\, dy$

$A = \pi \int_c^d (R^2 - r^2)\, dy$

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6.2   Volume: Disk and Cross Section Methods

Practice Problem Setup

G: $y = 2x^2$, $y = 0$, $x = 2$

F: V,   Revolve about:

a) y-axis   b) x-axis   c) $y = 8$ d) $x = 2$

Click on globe above.
Then click on the
"Practice in Problem Setup" link.

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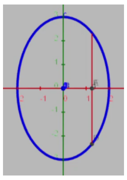
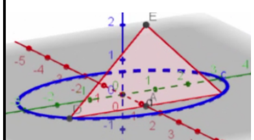
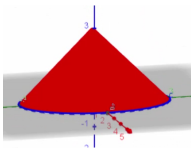
6.2   Volume: Disk and Cross Section Methods

Volume by Cross Section







Another approach to solids:

1. Start with a base: examples include circles, regular polygons, areas defined by a curve, etc.
2. Consider polygons perpendicular to the base
3. Describe the area of one of these polygons using data from the base
4. Integrate the area across the base to find its volume.

Volume by Cross Section

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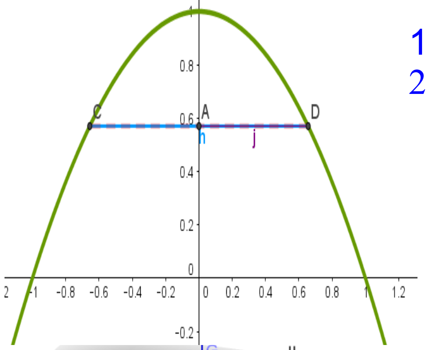
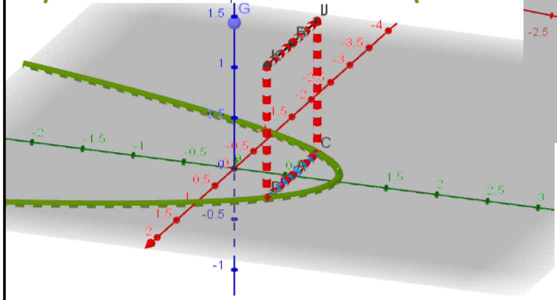
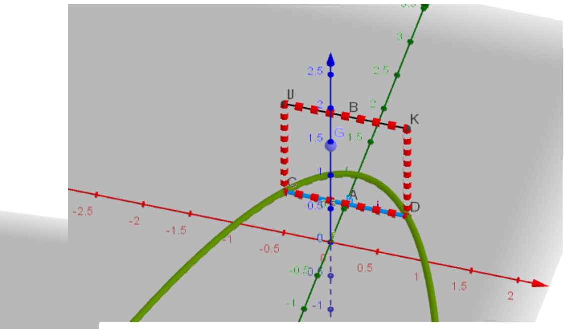
6.2 Volume: Disk and Cross Section Methods

G: base of solid S is bounded by $y = 1-x^2$ and the x axis. Cross sections ⊥ y axis are squares.
F: volume of S

1. Start with a base: $y = 1 - x^2$
2. Consider squares ⊥ to base







Cross Section: Squares on Parabola

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6.2 Volume: Disk and Cross Section Methods

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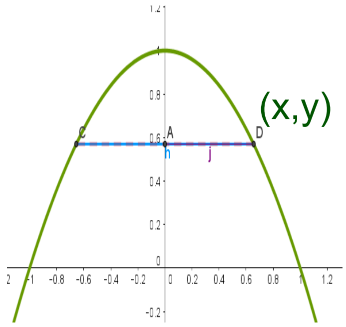
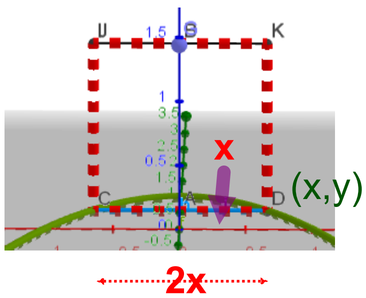
F: volume of S





1. Start with a base: $y = 1 - x^2$
2. Consider squares ⊥ to base
3. **Describe the area of the square**

$$A_{\blacksquare} = (2x)^2 = 4x^2$$

Cross Section: Squares on Parabola

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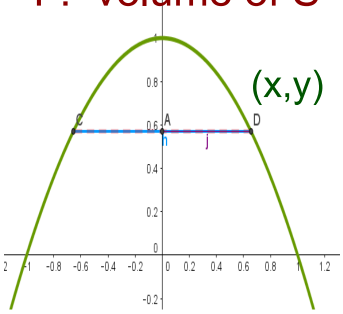
6.2 Volume: Disk and Cross Section Methods

G: base of solid S is bounded by $y = 1-x^2$ and the x axis. Cross sections ⊥ y axis are squares.

F: volume of S

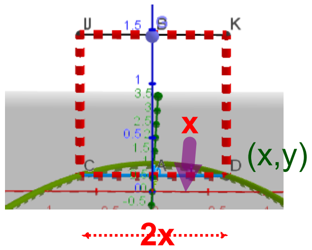


1. Start with a base: $y = 1 - x^2$
2. Consider squares ⊥ to base
3. Describe the area of the square
4. **Integrate the area across the base to find its volume.**

??? "across the base"

means letting y move from y=0 to y=1

BUT $A = f(x)$! and **need $A = f(y)$!**



$y = 1-x^2$

$x^2 = 1-y$

$A_{\blacksquare} = (2x)^2 = 4x^2$

$= 4(1-y)$

Cross Section: Squares on Parabola

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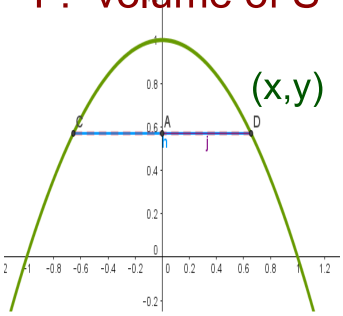
6.2 Volume: Disk and Cross Section Methods

G: base of solid S is bounded by $y = 1-x^2$ and the x axis. Cross sections ⊥ y axis are squares.

F: volume of S



1. Start with a base: $y = 1 - x^2$
2. Consider squares ⊥ to base
3. Describe the area of the square
4. **Integrate the area across the base to find its volume.**

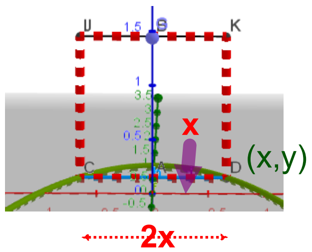
$A_{\blacksquare} = (2x)^2 = 4x^2 = 4(1-y)$

$$V = \int_0^1 4(1-y)\,dy$$

$$= 4\left(y - \frac{y^2}{2}\right)\Big]_0^1$$

$$= 4(1-1/2 - 0)$$

$$= 4/2 = 2 \text{ cubic units}$$



Cross Section: Squares on Parabola

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