Goals:

- 1. Recognize an integrand that is the derivative of a composite function.
- 2. Generalize the Basic Integration Rules to include composite functions.
- 3. Use substitution to simplify the process of integration of composite functions.

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2.4 Chain Rule: Derivative of Composite Functions

Revisit, from 2.4:

Chain Rule

If:

1) y = f(u) is a differentiable function of u, and 2) u = g(x) is a differentiable function of x, then

y = f(g(x)) is a differentiable function of x, and

\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}

or

\frac{d}{dx}[f(g(x))] = f'(g(x))g'(x)

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Use the Chain Rule

G:
$$y = (x^2 + 2)^{10}$$

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5.5 Integration by Substitution

Use the Chain Rule

G:
$$y = (x^2 + 2)^{10}$$
 F: dy/dx

$$dy/dx = 10 (x^2 + 2)^9 (2x)$$

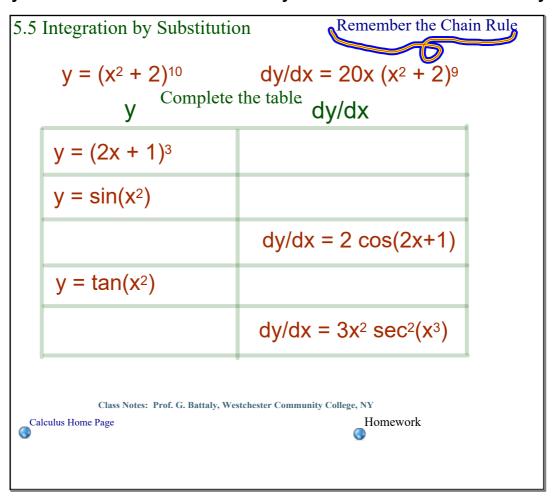
$$dy/dx = 20x (x^2 + 2)^9$$

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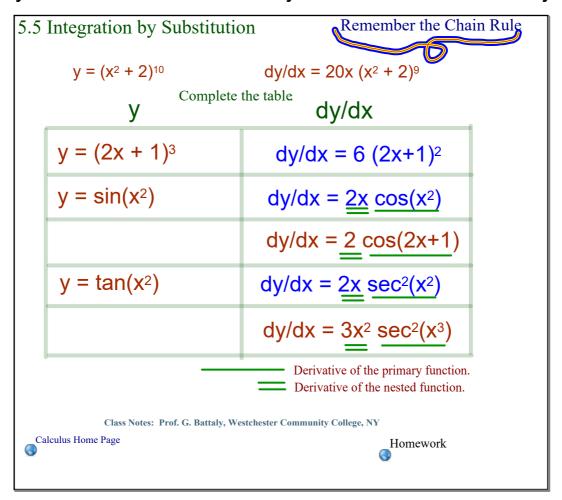
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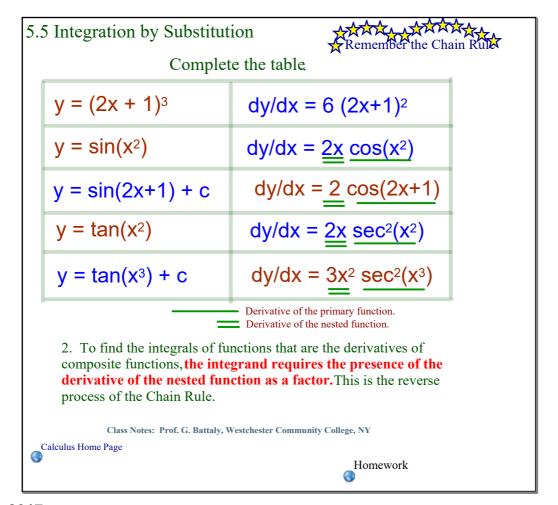
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5.5 Integration by Substitution Remember the Chain Rule	
$y = (x^2 + 2)^{10}$	$dy/dx = 20x (x^2 + 2)^9$
у	Complete the table dy/dx
$y = (2x + 1)^3$	$dy/dx = 6 (2x+1)^2$
$y = \sin(x^2)$	$dy/dx = 2x \cos(x^2)$
	$dy/dx = 2 \cos(2x+1)$
$y = tan(x^2)$	$dy/dx = 2x \sec^2(x^2)$
	$dy/dx = 3x^2 \sec^2(x^3)$
1. All derivatives here use the Chain Rule to find the derivative of composite functions.	
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$$\int (x^2 - 9)^3 (2x) dx$$

$$\int 2x (x^2 - 9)^3 dx$$

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5.5 Integration by Substitution

$$\int (x^2 - 9)^3 (2x) dx$$

$$u = x^2 - 9$$

$$du = 2x dx$$

$$\frac{(x^2 - 9)^4}{4}$$
 + c

$$\int 2x (x^2 - 9)^3 dx$$
 same as original

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$$\int x^2 \sqrt{x^3 + 1} \, dx$$

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5.5 Integration by Substitution

$$\int x^2 \sqrt{x^3 + 1} \, dx$$

$$u = x^3 + 1$$

$$\int x^{2}\sqrt{x^{3}+1} \, dx \qquad u = x^{3}+1$$

$$\frac{1}{3} \int (x^{3}+1)^{1/2} \, 3x^{2} dx$$

$$\frac{1}{3} \int U^{1/2} \, du$$

$$du = 3x^2 dx$$

$$\frac{1}{3}\int U^{1/2} du$$

$$\frac{1}{3} \frac{U^{3/2}}{\frac{3}{2}} + c = \frac{2}{9} (x^3 + 1)^{3/2} + c$$

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$$\int 2 \sec 2x \tan 2x dx$$

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5.5 Integration by Substitution

$$\int 2 \sec 2x \tan 2x \, dx$$

$$u = 2x$$

$$du = 2 dx$$

 \int sec u tan u du

$$sec u + c$$

$$sec 2x + c$$

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$$\int \sec 2x \tan 2x dx$$

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5.5 Integration by Substitution

$$\int \sec 2x \tan 2x \, dx \qquad \qquad u = 2x$$

$$u = 2x$$

$$du = 2 dx$$

$$\frac{1}{2} \int \sec 2x \tan 2x \quad 2dx$$
 $\frac{1}{2} \int \sec u \tan u \, du$

$$\frac{1}{2}$$
 sec $u + c$

$$\frac{1}{2} \sec 2x + c$$

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$$\int \sin x \cos x \, dx \qquad \qquad u = \underline{\qquad}$$

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5.5 Integration by Substitution

$$\int \sin x \cos x \, dx$$

$$u = \sin x$$

$$du = \cos x dx$$

$$\int u \, du$$

$$\frac{u^2}{2} + c$$

$$\frac{1}{2} (\sin x)^2 + c$$

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$$\int \sin x \cos x \, dx \qquad \qquad u = \cos x$$

$$u = \cos x$$

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5.5 Integration by Substitution

$$\int \sin x \cos x \, dx \qquad \qquad u = \cos x$$

$$u = \cos x$$

$$-\int \sin x \cos x \, dx$$

$$du = - \sin x dx$$

$$-\frac{u^2}{2} + c$$

$$-\frac{1}{2} (\cos x)^2 + c$$

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5.5 Integration by Substitution

\int \sin x \cos x \, dx \qquad \text{Trig Identity: } \sin 2x = 2 \sin x \cos x

\frac{1}{2} \int 2\sin x \cos x \, dx \qquad \qquad u = 2x

\frac{1}{2} \int \sin 2x \, dx \qquad \qquad du = \underline{\qquad}

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5.5 Integration by Substitution
\int \sin x \cos x \, dx
\int \sin x \cos x \, dx
\int \sin 2x \cos x \, dx
\int \sin 2x \cos x \, dx
\int \sin 2x \, dx
\int \cos 2x \, dx
```

$$\int \sin x \cos x \, dx$$

Trig Identity: $\sin 2x = 2 \sin x \cos x$

3 solutions look different, but they are equivalent, different only by constants

$$\frac{1}{2} (\sin x)^2 + c_1$$

$$-\frac{1}{2}(\cos x)^2 + c_2$$

$$-\frac{1}{4}\cos 2x + c_3$$

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5.5 Integration by Substitution

Rules of Integration

Rules of Integration
$$\int k \, dx = kx + c$$

$$\int u^n \, du = u^{n+1} + c$$

$$\int e^u \, du = e^u + c$$

$$\int du = \ln |u| + c$$

$$\int \sin u \, du = -\cos u + c$$

$$\int \cos u \, du = \sin u + c$$

$$\int \sec u \, \tan u \, du = \sec u + c$$

$$\int \sec^2 u \, du = \tan u + c$$
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5.5 Integration by Substitution $\int (x+1)\sqrt{2-x} \, dx$ $u = \underline{\qquad}$ $du = \underline{\qquad}$

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S.5 Integration by Substitution
$$\int (x+1)\sqrt{2-x} \, dx$$

$$u = 2-x$$

$$du = -dx$$
Note: $x+1$ is neither part of u nor du , but we need to convert all x to u before continuing. The conversion of u to a function of u must be consistent with $u=2-x$:
$$u = 2-x; \text{ so } x = 2-u, \text{ and } x+1=2-u+1=3-u$$

$$-\int (3-u)(u)^{1/2} \, du = -\int (3u^{1/2}-u^{3/2}) \, du$$

$$= -\frac{3u^{3/2}}{3/2} + \frac{u^{5/2}}{5/2} + c$$

$$= -2(2-x)^{3/2} + \frac{2}{2}(2-x)^{5/2} + c$$
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Rules Integration

$$\int x^2 e^{x^3} dx$$

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5.5 Integration by Substitution

Rules Integration

$$\int x^2 e^{X^3} dx$$

$$u = x^3$$

$$du = 3x^2 dx$$

$$\frac{1/3\int e^{x^3}}{3x^2} \frac{3x^2}{4x}$$

$$\frac{1/3\int e^u}{4u} du$$

$$\frac{e^u}{3} + c = \frac{e^{X^3}}{3} + c$$

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$$\int \frac{x}{x^2 + 4} dx$$

Rules Integration

Rules Integration

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5.5 Integration by Substitution # 32

$$\int \frac{x}{x^2 + 4} dx$$

$$u = \chi^2 + 4$$

$$\int \frac{x}{x^2 + 4} dx$$

$$1/2 \int \frac{2x}{x^2 + 4} dx$$

$$du = 2x dx$$

$$\frac{1}{2}\int \frac{du}{u}$$

$$\frac{1}{2} \ln |x^2 + 4| + c = \frac{1}{2} \ln (x^2 + 4) + c$$

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$$\int_0^1 x e^{-x^2} dx$$

Rules Integration

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Rules Integration

#82 G: A bacteria population starts with 400 bacteria and grows at the rate in bacteria/hr as noted below.

$$r(t) = (450.268)e^{1.125673t}$$

F: How many bacteria will there be after 3 hours? Amt after 3 hours = original amt ± net change

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Homework

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5.5 Integration by Substitution
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#82 G: A bacteria population starts with 400 bacteria and grows at the rate in bacteria/hr as noted below.

$$r(t) = (450.268)e^{1.125673t}$$

F: How many bacteria will there be after 3 hours?

Amt after 3 hours = original amt ± net change

A in 3hr = 100 + net change
$$\int_0^3 (450.268)e^{1.125673t} dt$$
 u = 1.125673t
$$du = 1.125673 dt$$

$$\frac{450.268}{1.12567} \int_0^3 e^{1.125673t} (1.12567) dt$$

$$\frac{450.268}{1.12567} \int_{0}^{3.377019} e^{u} du = \frac{450.268}{1.12567} e^{u} \Big]_{0}^{3.377019} = \frac{3.377019}{11313.309}$$

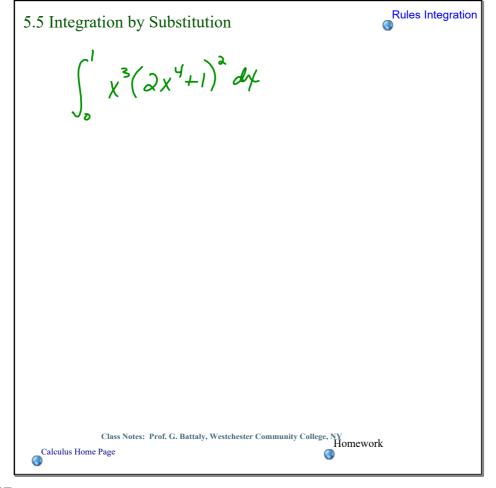
A in 3hr = 100 + 11313 = 11413 bacteria after 3 hours

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5.5 Integration by Substitution

$$\begin{cases}
\frac{1}{8} \chi^{3} (2 \chi^{4} + 1)^{2} d\gamma \\
\frac{1}{8} \chi^{3} (2 \chi^{4} + 1)^{2} d\gamma
\end{cases}$$

$$\begin{aligned}
u &= 2 \chi^{4} + 1 \\
u &= 8 \chi^{3} d\gamma
\end{aligned}$$

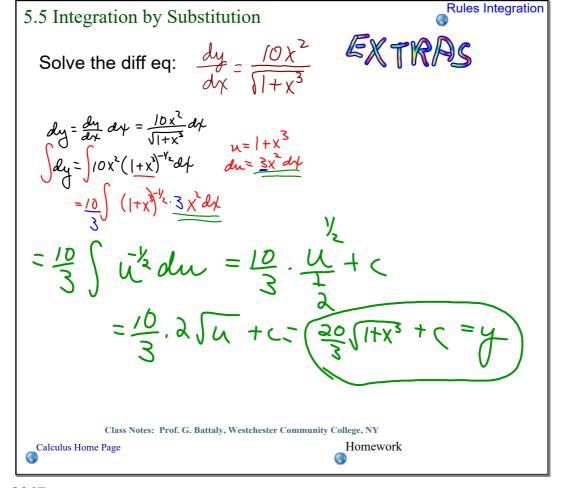
$$\begin{aligned}
u &= 2 \chi^{4} + 1 \\
\chi &= 0 \quad 0 + 1 = 1 \\
\chi &= 0 \quad 0 + 1 = 1
\end{aligned}$$

$$\begin{aligned}
\chi &= 0 \quad 0 + 1 = 1 \\
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\end{aligned}$$

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\end{aligned}$$

$$\begin{aligned}
\chi &= 0 \quad 0 + 1 = 1 \\
\chi &= 0 \quad 0 + 1 = 1
\end{aligned}$$
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Solve the diff eq:
$$\frac{dy}{dx} = \frac{x-4}{|x^2-8x+1|} = \frac{x^2-8x+1}{|x^2-8x+1|} = \frac{x-4}{|x^2-8x+1|} = \frac{x-4}{|x^2-8$$

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Rules Integration

Solve the diff eq, and find the equations thru (2,7):

$$\int_{1}^{1}(x)^{2} - 2x \sqrt{8 - x^{2}}$$

$$\int_{1}^{1}(x) dx = \int_{-2x}^{-2x} (8 - x)^{\frac{1}{2}} dx$$

$$= \int_{1}^{1} (x) dx = \int_{3}^{1} (8 - x)^{\frac{1}{2}} dx$$

$$\int_{1}^{1} (x) dx = \int_{3}^{1} (8 - x)^{\frac{1}{2}} dx$$

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$$\int_{1}^{1} (x) dx = \int_{3}^{1} (8 - x)^{\frac{1}{2}} dx$$

$$\frac{2}{3}(\sqrt{4})^{3}+c=7=\frac{16}{3}+c$$

$$C=7-\frac{16}{3}=\frac{21-16}{3}=\frac{5}{3}$$

$$\sqrt{(x)}=\frac{2}{3}(\xi-x^{2})^{2}+\frac{5}{3}$$

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